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An Experimental Study of Polymer Drag Reduction and Boundary Layer Diffusion Characteristics for Incompressible Flow Over a Flat Plate

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PREFACE

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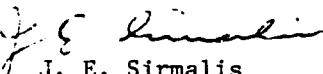
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Drag reduction by injection of high molecular weight polymers into boundary layers has been demonstrated repeatedly in the past. However, from a volume utilization tradeoff standpoint, the quantities of polymer required make the gains achieved by this process marginal. While indicating reduced polymer requirements for drag reduction, limited data obtained from pipe flow and external boundary layer flow experiments are conflicting and hard to interpret. Ambiguities in measurement techniques due to polymer effects on commonly used instrumentation and opposing features of varied flow facilities have also contributed to making

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20. ABSTRACT (Cont'd)

these earlier works contradictory and difficult to resolve.

Experiments performed in this research indicate that turbulence intensity distributions are altered by the addition of polymer in such a way that the peak of turbulence production is lowered and its location moved away from the wall. The transition region is delayed and extended by the addition of polymer to the boundary layer. The laminar sublayer of boundary layer profiles appears to have thickened due to the addition of polymer. When compared to the law of the wall corrected for developing flow, the velocity profiles also show evidence of a thickened sublayer.

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NOMENCLATURE

A	=	area	ft^2
A_s	=	ejection slot area	ft^2
a	=	acceleration	ft/sec^2
B	=	law of the wall constant	dimensionless
b	=	Batchelor's Constant	dimensionless
C	=	concentration	WPPM
C_i	=	injected concentration	WPPM
C_f	=	local skin friction coefficient	dimensionless
C_w	=	wall concentration	WPPM
C_w^*	=	critical wall concentration	WPPM
D_e-2	=	diameter of laser beam at $1/e^2$ intensity points	mm
d_ℓ	=	diameter of focused laser beam	mm
d_m	=	diameter of measuring volume	mm
d_f	=	fringe spacing	mm
F	=	force	lb_f
f_D	=	Doppler frequency	$1/sec$
g	=	gravitational acceleration	ft/sec^2
g_c	=	constant of proportionality	lb_m-ft/lb_f-sec^2
K	=	mixing length constant	dimensionless
L	=	plate length	ft
l_m	=	length of measuring volume	mm
m	=	mass	lb_m
p	=	pressure	lb_f/ft^2

Q	=	volume flow rate	ft^3/sec
Q_i	=	injection rate	ft^3/sec
Re	=	Reynolds number	dimensionless
Re_x	=	$\dfrac{U_o x}{\nu}$	dimensionless
t	=	time	second
T	=	temperature	^oF
T_i	=	temperature of injected fluid	^oF
T_w	=	temperature of water	^oF
U_e	=	velocity at edge of boundary layer	ft/sec
U_o	=	freestream velocity	ft/sec
u	=	local boundary layer velocity	ft/sec
u^+	=	u/V^* law of the wall variable	dimensionless
u'	=	turbulent velocity component	ft/sec
v	=	velocity	ft/sec
v_i	=	ejection velocity	ft/sec
v_{iy}	=	normal component of injection velocity	ft/sec
V^*	=	characteristic or friction velocity	ft/sec
V_m	=	volume of measuring volume	$(cm)^3$
V_o^*	=	critical shear velocity $=\sqrt{\tau_w/\rho}$	ft/sec
x	=	linear dimension	ft
X^*	=	x/L dimensionless distance	dimensionless
y	=	distance normal to plate	ft
y^+	=	yV^*/ν law of the wall variable	dimensionless
$\bar{y}$	=	mean vertical position of particle	ft

$\bar{Y}$	=	mean vertical position of single particle	ft
Z	=	defined by equation (58)	dimensionless
α	=	coefficient of polymer term in law of the wall	dimensionless
β	=	Clauser's equilibrium parameter defined by equation (11)	dimensionless
γ	=	exponent of polymer term in law of the wall	dimensionless
ϵ	=	eddy viscosity	$lb_f sec/ft^2$
ΔB	=	polymer constant for law of the wall	dimensionless
δ	=	boundary layer thickness	ft
δ_d	=	diffusion boundary layer thickness	ft
δ_1	=	boundary layer displacement thickness	ft
δ_2	=	boundary layer momentum thickness	ft
ϕ	=	laser beam intersection angle	degrees
λ_ℓ	=	wave length of laser beam	meters
λ	=	$(2/C_f)^{1/2}$	dimensionless
λ_c	=	characteristic height of diffusion boundary layer	dimensionless
ρ	=	density	lb_m/ft^3
μ	=	viscosity	$lb_f sec/ft^2$

τ_w	=	wall shear stress	lb_f/ft^2
ν	=	kinematic viscosity	ft^2/sec
η	=	$y\sqrt{U/\nu x}$ Blasius parameter	dimensionless

I. INTRODUCTION

General Considerations

During the thirty years since B. A. Toms first published his
work on the subject, it has been well established that consid-
erably reduced resistance to the turbulent shear flows of liquids
may be achieved through the addition of small quantities of high
molecular weight polymers into the fluid. This phenomenon of
drag reduction is of great importance and has far-reaching impli-
cations in the reduction of power required for the pumping of
fluids and the transport of bodies through liquids.

Early drag reduction efforts were primarily concerned with
well-developed pipe flows, the area in which the first observa-
tion was made. Until recently, researchers have directed their
efforts toward characterizing the manner in which the boundary
layer velocity profiles were affected in an attempt to quantify
the effect of polymer additives. Results similar to those
achieved in pipe flows have been obtained in investigations
dealing with external flows, such as on a flat plate. The
magnitude of the turbulent fluctuations in the boundary layer
are found by some researchers to decrease with a thickening of the
laminar sublayer and a lowering of the wall shear stress. Other

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researchers, however, have found no evidence of this thickened sublayer. A limit to the amount of drag reduction achievable with polymer use has been demonstrated and increases in the shear stress have been observed when excess polymer is used. Shear stress reduction for external flows over bodies has been achieved by the ejection of polymer solutions. The exact mechanism of shear stress reductions, however, still remains undefined. Ambiguities in measurement techniques due to polymer effects on commonly used instrumentation and confusing features of varied flow facilities have contributed to making past investigations contradictory and difficult to interpret.

It is evident that a better understanding of the mechanism of drag reduction in developing flows with polymer ejection is required if advantage is to be taken of the phenomenon for practical application. Most cases of interest in external flows would require that a carried polymer supply be used for ejection into the boundary layer. Presently, very little is known about the effect of ejection techniques on polymer distribution in developing boundary layers of external flows and no information is available on the effects of polymer on boundary layer transition phenomena.

Scope and Objectives of Present Study

The object of the research described herein was to: (1) develop a flow facility suitable for the study of polymer flows; (2) perform velocity measurements using non-flow-disturbing laser Doppler anemometry techniques with real-time data processing; (3) develop methods for the prediction of local skin friction values as a function of local polymer wall concentration; (4) determine the effects of ejection techniques on the distribution of polymer in the boundary layer; and (5) determine the diffusion characteristics of a polymer when ejecting it into a laminar boundary layer and also its effect on transition.

Approach

A flow facility was constructed under the constraints of limited funds and practicality of construction. The basic tunnel structure was designed with a width-to-height aspect ratio of 5:1 to minimize the effects of side wall boundary layer growth and corner flows in the rectangularly-shaped channel. Flow delivery was by constant head gravity feed to minimize vibration effects. The polymer delivery system provided minimum mechanical degradation of polymer solutions while allowing variations in polymer concentrations and ejection velocities. Transition from laminar to turbulent flow was achieved at midplate

position allowing ejection of polymer solutions into a developing laminar boundary at the plate leading edge and transitioning to turbulent flow along the plate. Boundary layer velocity profiles through transition to turbulent flow were taken with laser Doppler anemometry techniques, non-disruptive of the flow nor affected by polymer characteristics.

Using boundary layer sampling techniques, polymer diffusion characteristics were measured at five axial wall locations along the plate, as well as at eight positions normal to the plate at each wall station. The ejected polymer concentrations were contaminated with a fluorescent dye, which was later used to analyze the concentrations of the boundary layer samples. Thus it was possible to measure the change in wall concentration and concentration profile in the boundary layer during laminar, transitional, and turbulent flow.

Analytical methods were developed to predict skin friction coefficients as a function of local polymer wall concentrations for comparison with experimentally determined values.

II. LITERATURE REVIEW

Historical Background

B.A. Toms (1949) published the first data on the friction
reducing effects of polymers and showed that, for conditions of
turbulent flow with constant pressure gradient, the average
velocity of monochlorobenzene, flowing in a pipe, increased
substantially with the addition of dilute quantities of polymethyl
methacrylate. Friction reduction of up to 50% was achieved
(as compared with the pure solvent) by the addition of .25%
by weight of polymer to the solvent.

Oldroyd (1949) attempted to explain this phenomenon
(presently referred to by many as the "Toms effect") as a wall
effect due to exclusion of large polymer molecules from a region
near the wall due to their size. As a result of studies on the
flow of gasoline thickened with aluminum soaps at Edgewood Arsenal
during World War II (but not published at that time), Mysel (1949)
applied for and received a patent in 1949. He observed that in
tubulent flow the pressure loss per unit length of pipe was much
lower for thickened gasoline than that of the pure solvent.

The importance of drag reduction seemed to have been over-
looked until the late 1950s when Shaver and Merril (1959) and

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Dodge and Metzner (1959) published results indicating the low friction factors of non-Newtonian solutions, such as sodium carboxymethycellulose, in water. Similar observations of reduced friction were made in the oil well industry with the use of the plant derivative, Guar Gum. Solutions of the product were used to suspend sand in the high-pressure, sand-water mixtures utilized in oil well fracturing operations.

These experiences led to U.S. Navy studies in the early 1960's on possible military applications of friction-reducing effects. Among Navy reseachers, Hoyt (1963) made the first significant contribution followed by Fabula (1964) who discovered the spectacular friction reducing ability of polyethylene oxide, the most effective friction-reducing material known.

Crawford (1962) and Savins (1961) led the early efforts of industrial research centers. The years that have followed these early efforts have produced an ever-increasing number of international papers, reports, and conferences on the subject.

Excellent summaries of research efforts and results available in the field of drag reduction have been presented by Lumley (1969) and Hoyt (1972). Palyvos (1974), however, prepared the most extensive and detailed review of research efforts to date. This review reveals that after years of active research and the

combined efforts of hydrodynamicists, polymer chemists and
rheologists in many countries, there is yet no adequate
theoretical explanation for the mechanism of drag reduction, nor
is there a satisfactory correlation of the viscoelastic properties
of dilute polymer solutions with their friction reduction effect.

Literature pertaining to the current study will be reviewed
under four separate categories:

1. Turbulent boundary layer theory
2. Homogeneous polymer flow
3. Polymer ejection studies
4. Experimental facilities.

Turbulent Boundary Layer Theory

The concept of transition from laminar to turbulent flow was
first demonstrated in 1880 by Osborne Reynolds in his classic
pipe flow experiment. The next major step did not occur until
1904 when, by demonstrating the existence of a thin "boundary
layer" in fluid flows, L. Prandtl allowed the reconciliation of
viscous flow with classical frictionless hydrodynamic relations.
In 1914, by showing that boundary layers could be either laminar
or turbulent, Prandtl freed early investigations from the limi-
tation of considering only laminar boundary layers. The

introduction of the "Prandtl mixing length theory" in 1925 was a
major contribution toward the understanding of the development of
the boundary layer velocity profile. Since that time there have
been an ever-increasing number of contributions to the under-
standing of the flow of fluids.

Many attempts have been made to predict the conditions that
govern the transition of laminar to turbulent flow as well as to
predict velocity profiles and wall shear. H. Schlichting's classic
text, Boundary Layer Theory (1968), remains the most comprehensive
treatment of the subject to date. In this work, laminar and
turbulent boundary layers in both compressible and incompressible
flows over a wide range of application are addressed. However,
analyses of the turbulent boundary layer problem still rely heavily
on a combination of dimensional analysis, empirical data and flow
visualization techniques. Also, during 1968, an attempt was made
to introduce order into turbulent boundary layer research. A
conference of the world's leading boundary layer researchers was
called at Stanford University. Kline (1968) edited the proceed-
ings of that conference, in which some 29 methods and approaches
to the analysis of the turbulent boundary layer were presented
and discussed.

Viscous Fluid Flow, a text by F.M. White (1974), discusses
many of the more significant methods of analysis. Two major

approaches evolved: (1) integral methods averaged across the
boundary layer, and (2) finite difference techniques which attempt
to solve the full partial differential equations of the boundary
layer.

In the last analysis, it remains that the empirical dimen-
sional analysis approach is still the most useful for engineering
purposes. The scientific world is indebted to the physical insight
of Prandtl and Von Karman for the formulation of the turbulent
velocity profile concept. Consideration of regions of influence
where the relative importance of viscous shear or turbulent shear
dominate led to the formation of the three-layer concept for the
velocity distribution in a turbulent boundary layer. The layers
are:

Inner Layer: Viscous shear dominates

Outer Layer: Turbulent shear dominates

Overlap Layer: Viscous and turbulent shear equally
                important.

The mean velocity distribution in a two-dimensional turbulent
boundary layer, $u(y)$, depends upon four local parameters: the
local wall shear stress, τ_w; the fluid density, ρ; the fluid vis-
cosity, μ; and the boundary layer thickness, δ. For the inner
law, Prandtl (1933) deduced that the mean velocity depended on the

wall shear stress, the fluid properties, and the distance y from the wall which led to the following functional relation for the inner law:

$$u = f(\tau_w, \rho, \mu, y).$$ (1)

For the outer layer, von Karman deduced that the wall acts only as a source of retardation reducing the local velocity, u, below the freestream velocity, U_e, in a manner independent of the viscosity, μ. The outer law or velocity defect relation, as it is sometimes called, becomes

$$U_e - u = f(t_w, p, y, w).$$ (2)

Coles (1954) performed a dimensional analysis on these relationships, where V^* is a characteristic velocity called the wall shear velocity, and is defined as

$$V^* = \sqrt{\frac{\tau_w}{\rho}}.$$ (3)

Near the wall, Coles (1954) found that the velocity profile is unaffected by the boundary layer thickness, therefore

$$\text{Inner Law} \quad U^+ = \frac{u}{V^*} = f\left(\frac{yV^*}{\nu}\right)$$ (4)

Far from the wall, viscosity does not affect the velocity distribution. (The viscosity effects are concentrated near the wall.)

$$\text{Outer Law} \quad \frac{\left(U_e - u\right)}{V^*} = g(y/\delta) \quad (5)$$

In the overlap region between the inner and outer layers, the commonly known relation for the "law of the wall" (shown in figure 1) may be obtained by equating equations (4) and (5), such that

$$\frac{U}{V^*} = \frac{1}{K} \ln \frac{yV^*}{\nu} + B. \quad (6)$$

The constants K and B have been determined by the data of Nikuradse (1930) to be K = .40 and B = 5.5.

Coles (1954) presents a correlation of the dimensionless velocity profile, u/V^*, with dimensionless distance from the wall, yV^*/ν, up to a value of about 300. Neglecting separating flows, the data collapse into regions governed by the inner and logarithmic outer laws with the following limits:

Inner Law:

$$\frac{u}{V^*} = \frac{yV^*}{\nu}, \quad 0 < \frac{yV^*}{\nu} < 10 \quad (7)$$

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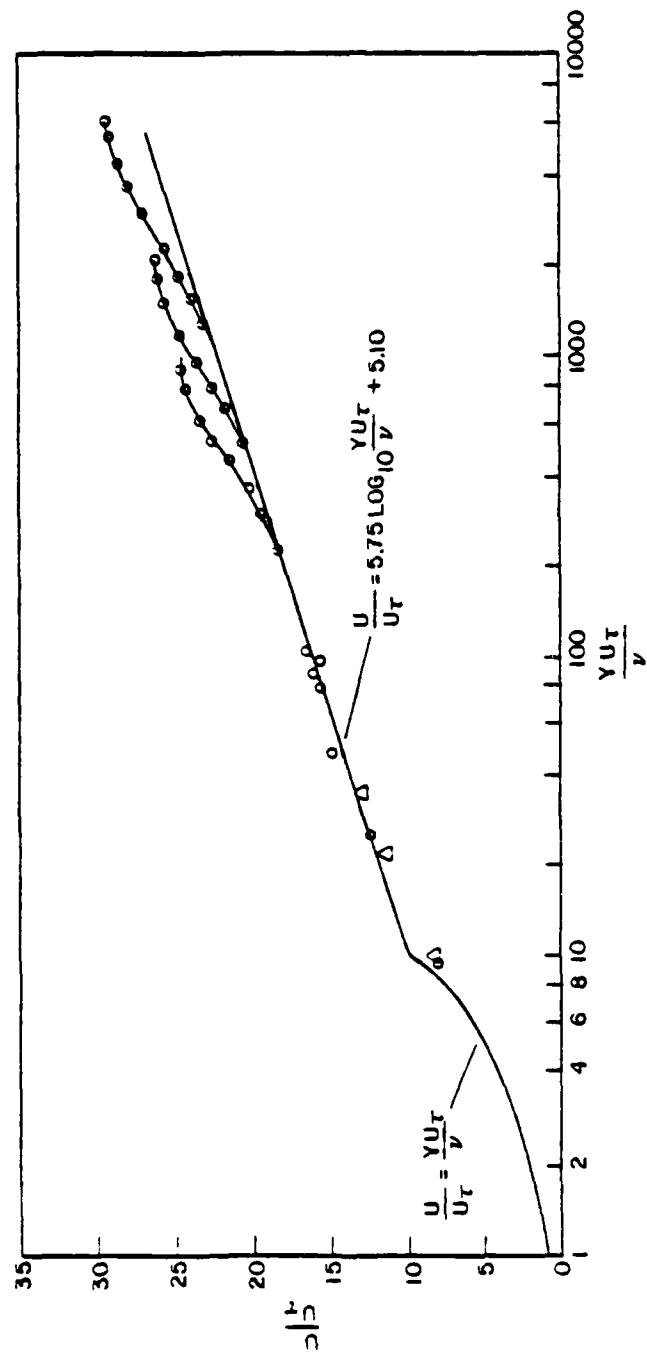
Figure 1. The law of the wall (from Coles (1954))

Logarithmic Law:

$$\frac{u}{V^*} = 2.5 \ln \frac{yV^*}{\nu} + 5.5, \quad 35 < \frac{yV^*}{\nu} < 300. \tag{8}$$

For values of $y^+ \equiv \frac{yV^*}{\nu} > 300$, the viscosity is of negligible importance in the determination of the shape of the boundary layer. The data will then correlate with the velocity defect law (shown in figure 2) in such a way that

$$\frac{U_e - u}{V^*} = f\left(\frac{y}{\delta}, \frac{u}{V^*}\right). \tag{9}$$

In a subsequent paper, Coles (1955) postulates that a two-dimensional boundary layer of an incompressible fluid may be represented by a linear combination of two functions:

$$\frac{u}{V^*} = f\left(\frac{yV^*}{\nu}\right) + \left(\frac{\pi}{k}\right) W\left(\frac{y}{\delta}\right). \tag{10}$$

The function $f(yV^*/\nu)$ is given by equations (7) and (8). Coles refers to the function $W(y/\delta)$ as the "law of the wake." The function, Π, is related to Clauser's (1954, 1956) equilibrium parameter, β, designed as follows:

$$\beta = \frac{\delta}{\tau_w} \cdot \frac{dP_e}{dx}. \tag{11}$$

After additional study, Clauser concluded that the typically fuzzy thickness, δ, should be replaced by the displacement
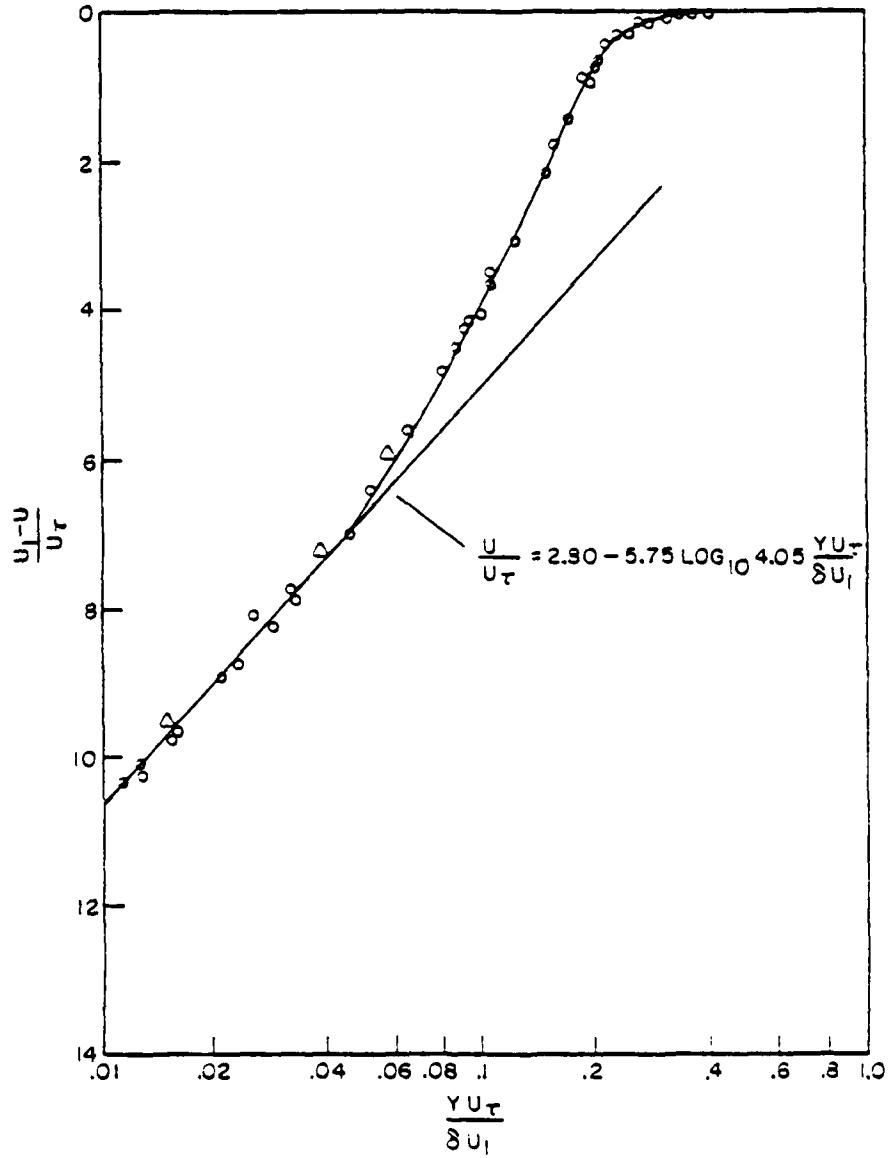
Figure 2. The velocity defect law (from Coles (1954))

thickness, δ_1, so that the accepted parameter is now

$$\beta = \frac{\delta_1}{\tau_w} \cdot \frac{dP_e}{dx} . \tag{12}$$

The "wake parameter," Π, has a reasonable fit to data when given by the relationship

$$\Pi \simeq 0.8(\beta + 0.5)^{0.75} . \tag{13}$$

Coles (1956) proposed the following curve fit for the wake function, W, as

$$W \frac{y}{\delta} \simeq 2\sin^2 \frac{\pi}{2} \cdot \frac{y}{\delta} . \tag{14}$$

Then in Cole's notation for the overlap and other layers, we have the following

$$u^+ = \frac{1}{k}\ln(y^+) + B + \frac{\Pi}{k} W (\frac{y}{\delta}) . \tag{15}$$

It may then be seen that, near the wall, the velocity profile is dominated by friction forces and the function $f(y \cdot u_\tau/\nu)$ controls. Away from the wall, the profile is dominated by inertia forces, with the function $w(y/\delta)$ controlling.

Kline, Runstadler, and Reynolds (1963) visually demonstrated the existence of three regions of turbulent boundary layers, using

a combination of pulsed hydrogen bubble techniques, hot film probes, and dye streak photographs. Shown in figure 3, the flow regions of the turbulent boundary layer model are:

1. A laminar flow region having a near regular, three-dimensional flow pattern for a range of y^+ from $0 < y(u_\tau/\nu) < 10$.

2. A fully turbulent region of randomly fluctuating eddies. This region, sometimes called the inner turbulent region, exists over a range of y^+ from $10 < y(u_\tau/\nu) < 370$.

3. An outer turbulent region, or wake region, extending from y^+ of approximately 370 to the outer edge of the boundary layer. In this region, large scale turbulent eddies are found.

Brady (1973) summarized the work of Kline et al. (1963) as follows:

> Kline et al. found the 'laminar sublayer' to be made up of a regular structure of low and high velocity longitudinal streaks which meander transverse and normal to the wall. A dimensionless streak spacing $\lambda^+ = \lambda v*/\nu = 76.5$ was found for zero pressure gradients. These streaks either break up or randomly burst from the sublayer into the fully turbulent region.

> The fully turbulent region is one of intense mixing and high dissipation of energy. Protruding from this region are intermittent large eddies -- visualized as peninsulas of turbulence.

REGIONS OF BOUNDARY LAYER FLOW | PHYSICAL PICTURE OF FLOW | FLOW STRUCTURE | VELOCITY PROFILE

U_∞

$Y/_\delta$ Y^+

FREE STREAM — EDGE OF BOUNDARY LAYER — UNIFORM FREE STREAM FLOW — 0.99 1000

INTERMITTENT WAKE REGION — PATH OF EJECTED EDDIES — PRIMARILY DECAY OF LARGE TURBULENT EDDIES — 0.4 370

FULLY TURBULENT REGION — IRREGULAR EDGE OF WAKE REGION — RANDOM INTER-ACTION OF EDDIES — 0.03 30

— BREAK-UPS — 0.01 10

WALL LAYER — WALL FLOW — WALL

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Figure 3. Pictorial sketch of boundary layer flow (from Kline (1963))

The peninsulas of turbulence which extend into the third boundary layer zone -- the outer turbulent region -- gives it a characteristic not unlike the wake behind a cylinder. It has rather large lumps of turbulence at intermittent spacings.

It is clear that the driving force for the entire turbulent boundary layer is the generation of velocity streaks in the sublayer and their subsequent bursting outwards. Kline presents the results of many other investigators, as well as his own, in support of this hypothesis.

Homogeneous Polymer Flow

Since the early work of Toms (1949), Oldroyd (1949), and Mysel (1949), many high molecular weight polymers have been shown to be effective drag reducers. Among the most prominent are: polysaccharides (Guar), polyethylene oxide, polyacrylamides, and sodium carboxymethyl cellulose. For the most part, investigators have considered mainly pipe flows, therefore, this effort will be considered first.

Pipe Flow

Table 1 from Hoyt (1972) shows the effectiveness of small concentrations of polymer on drag reduction. Listed are those concentrations required to achieve 67% drag reduction in pipe flow at Re $= 14 \times 10^3$.

TABLE 1

CONCENTRATIONS (WPPM) OF MATERIAL REQUIRED TO ACHIEVE 67%

DRAG REDUCTION IN PIPE FLOW OF Re = 14×10^3

(FROM HOYT (1972))

Polymer	Concentration (WPPM)
Gum Karaya	850
Guar	400
Polyacrylamide, Polyhall-250	20
Polyox WSR-301	10

Hoyt and Fabula (1964) and Virk (1971) present data that show a maximum drag reduction asymptote. For a smooth pipe, this asymptote corresponds to 80% of the friction reduction that would be attained if completely laminar flow were sustained at a given Reynolds number.

Meyer (1966) and Elata et al. (1966) have shown that drag reduction in pipes is due to a thickening of the laminar sublayer. It was shown that the constant B in the law of the wall equation (6) remained constant and equal to the Newtonian value until a critical threshold value of the shear velocity, V_o^*, was reached. Thereafter the value of B increased logarithmically with V^*:

$$B = 5.5 + Z \ln\left(\frac{V^*}{V_o^*}\right), \qquad (16)$$

$$B = 5.5 + \Delta B, \qquad (17)$$

where $\Delta B = Z \ln\left(\frac{V^*}{V_o^*}\right). \qquad (18)$

Using data from several investigators, F.M. White (1968) found that:

$$Z = \alpha C_w^\gamma, \qquad (19)$$

where $\alpha = 2.3$, and $\gamma = 0.5$, $\qquad (20)$

with a maximum value of Z of approximately 11. The data further indicated that the critical shear velocity, V_o^*, for the onset of drag reduction was .08 ft/sec. Virk (1966) deduced from experimental data that the onset shear stress is inversely proportional to the polymer molecular radius of gyration. The critical wall shear stress, τ_w^c, which must be exceeded for drag reduction to occur is given by

$$\tau_w^c = \rho(0.624 \times 10^6 \; \mu/R_G)^2 = \frac{\text{constant}}{R_G^2} \qquad (21)$$

where R_G is the rms radius of gyration of the molecule obtained from light scattering data.

Fabula et al. (1969) developed a criteria for the critical wall shear stress at the onset of drag reduction based on molecular deformation times and a molecular characteristic relaxation time. It is thought that individual molecules are too small by several orders of magnitude to interfere with the turbulence structure, indicated by the small value of the ratio of the polymer molecule scale to the scale of turbulent eddies at onset.

To circumvent this problem of length scales, Fabula postulated an interaction between the time scales of the periodic molecular deformation in the viscous sublayer, given by $\dot{\gamma}/2\pi$, where $\dot{\gamma}$ is the shear rate, and a molecular characteristic relaxation time, τ_2. The molecular charcteristic relaxation time, τ_2, may be determined by the Zimm or Rouse theories which relate a characteristic relaxation time of the solution, τ_2, to the solvent viscosity, μ_s; the solution viscosity, μ; the polymer molecular weight, M; temperature, T; and the concentration of polymer, C. It follows then that

$$\tau_2 = \frac{a(\mu-\mu_s)M}{CRT} \tag{22}$$

where a is a constant having a value between .4 and .6.

The resulting criterion for the critical wall sheer stress is
given by

$$2\Pi(\dot{\gamma}/2\Pi) \ \tau_\ell = 1$$

or

$$\tau_w^c = \frac{\mu}{\tau_\ell} = \frac{\mu CRT}{a(\mu-\mu_\sigma)M} \ . \tag{23}$$

Equation (23) may be written using a relation between intrinsic
viscosity, molecular weight, and molecular dimensions yielding:

$$\tau_w^c \simeq \frac{\text{constant}}{R_G^3} \simeq \frac{1.7 \ X \ 10^{10}}{R_G^3} \ . \tag{24}$$

Equation (24) of Fabula is similar to Virk's equation (21).
The former provides a better estimate of magnitude of onset shear
stress whereas the latter is considered more accurate when the
constant is determined from test data.

In their work, Kowalski and Brundrett (1974) support
Fabula's postulate that the effectiveness of very dilute solution
is due to entanglements or "blobs" of macromolecules rather than
individual molecules. A formula was developed relating the size
of the entangled molecules with the size of a dissipative eddy
and are tested to predict the onset of drag reduction in pipe
flows of homogeneous polymer solutions.

Darby (1972) reviews drag reduction theories comparing molecular hypotheses, continuum approaches, and conventional boundary layers length scales. Molecular time scale hypotheses are represented by the work of Virk, Fabula, and Kowalski. The continuum approach of Seyer and Metzner concludes that the presence of elastic properties in the dilute solution are sufficient criteria for drag reduction. Elastic properties are represented by a single parameter, the relaxation time, λ, incorporated into a dimensionless group, the Deborah number:

$$N_{De} = \lambda/t_1, \tag{25}$$

where t_1 is a characteristic time of the system. Seyer and Metzner (1969) chose this number to be the reciprocal of the dissipative turbulence frequency, ω_d,

$$\omega_d = \frac{\bar{V}}{D} N_{Re}^{3/4}. \tag{26}$$

Seyer and Metzner further offer the following phenomena as a possible explanation of drag reduction:

1. Particulate effects due to large molecules or agglomerates which may promote stability of laminar flow or dampen turbulence

2. The solid-like resistance of the fluid to sudden deformation or stretching may suppress the dissipative turbulent frequencies on the generation of turbulence

3. The laminar flow regime may be stabilized so that transition is delayed to higher Reynolds numbers.

Darby (1972) points out that, although Seyer and Metzner favor the second concept, no distinction is made between cause and effect. Dozens of different forms of the Deborah number are presented, illustrating the problems encountered with the present theoretical state of the art.

Boundary layer modification in terms of the Prandtl mixing length theory and law of wall relation indicates a concept of the flow field divided into three regions:

1. A laminar sublayer adjacent to the wall where turbulent fluctuations are absent and momentum transport is by viscous shear only

2. A transition or "buffer" zone adjacent to the laminar sublayer where both viscous shear and turbulent inertial fluctuations contribute to the momentum transport

3. A turbulent boundary layer or "core" outside the buffer zone where transport is dominated by turbulent inertial fluctuations.

Good results for friction factor correlations have been achieved by Elata, Lehner, and Kahanovity (1966) for Guar Gum solutions and Meyer (1966) and Wells (1965) for Polyox.

Many authors have described drag reduction as a "negative roughness effect" since polymers appear to thicken the sublayer while maintaining the same slope of the U^+ versus ln Y^+ curve in the overlap region. Nadolink (1968) demonstrated the existence of the thickened sublayer directly, using a high-speed motion picture camera and a microscope.

White and McEligot (1970), in their work on transition delay from laminar to turbulent flow, found a dependence on where the critical onset shear stress is reached. If the onset shear stress occurs in the laminar flow region, a delay in transition to turbulent flow can occur.

Polymer Injection Studies

Practical application of the phenomena of viscous drag reduction caused by dilute polymer solutions includes increasing

speed or power reduction requirements for naval vessels and increasing the flow through conduits. Realization of these advantages generally requires injection of polymer solutions into a developing boundary layer. Observation that drag reduction is related to changes in the sublayers implies that injecting polymer near the wall affects considerable drag reduction with relatively small amounts of polymer. Literature pertinent to this area of the field will now be reviewed.

Pipe Flows

Wells (1968) suggested uniform injection through a porous wall since it raises the additive concentration to the drag reducing level in the wall region only. Using a Reynolds-Prandtl analogy to analyze the diffusion process, he calculated that distributed injection would require 40 to 140 times less additive than slot injection to maintain equivalent drag reduction. The point was made that a continuously ablating additive coating would yield the maximum performance advantage if it could be made to ablate at the optimum rate.

Wells and Spangler (1967) performed a series of experiments in order to determine whether the presence of the additive only in the wall region could produce significant local shear stress reduction. The experiments utilized centerline injection and

circumferential slot injection of dilute polymer into fully-developed turbulent flow of a Newtonian fluid. It was found that the local pressure loss was reduced by an amount comparable to the flow of a uniform concentration, when the fluid was injected at the pipe centerline. However, no reduction in local pressure loss occurred until the fluid diffused into the wall region.

Maus and Wilhelm (1970) conducted polymer injection tests with a fully-developed flow in a 1.625-inch diameter smooth acrylic pipe. Five circumferential injection slots located 6 inches apart were used. Each slot was 0.050-inch wide and inclined 30° to the pipe centerline. The effects of Reynolds number, injection rate, number of injection points, and concentration of injected solution were studied. It was found that maximum drag reduction occurred when polymer was injected through the furthest upstream slot rather than being distributed over the test length.

Walters and Wells (1971) conducted tests using uniform ejection of polymer solution through a stainless steel porous pipe section into fully-developed turbulent pipe flow. Concentration profiles, velocity profiles, and wall shear stress data were obtained. Fluorometric techniques were used to obtain the concentration profiles. Uranine B was selected as the fluorescent dye over several others, as the laboratory apparatus could be easily decontaminated. The dye was mixed with the injection fluid and

then captured tracer sample concentrations were measured with a
fluorometer. Accuracies of the measurement instrumentation of
two parts per billion were reported. Uniform polymer injection at
the wall appeared to somewhat inhibit turbulent diffusion away
from the wall resulting in high wall concentrations. For certain
conditions of high polymer mass flux, a wall friction increase was
noted, possibly due to the higher viscosity in this region. As
compared to water injection, a one to two order of magnitude
reduction in total diffusivity in the ejection region was noticed.
Downstream of the ejection section, an order of magnitude reduction
of total diffusivity was noted along with a significant reduction
in wall friction.

Tullis and Ramu (1973) studied the characteristics of mean
turbulent flow in the entrance region of a rough pipe for water
and for polymer injection into a boundary layer. Polyox WSR-301
was injected through a perforated wall pipe section of a 12-inch
diameter, 200-foot-long steel pipe used for the study. Drag
reduction of up to 80% in the fully-developed region and 90% in
the inlet region were recorded. Comparison of water and dilute
polymer ejection showed the polymer concentration profiles
developed slower than that of the water alone, indicating lower
diffusion of the polymer solution. The inlet length needed for
flow to fully develop was found to be greater for polymer
injected flows that for the case of no injection. In the fully

developed region, the maximum drag reduction measured was 80% at $C_\infty = 25$ WPPM. Drag reduction in the inlet region appeared to be independent of injection concentration. Fluorescent techniques using rhodamine WT dye were used to measure the small concentration of polymer. Mean flow velocities between 5 and 50 ft/sec. ($R_e = 3 \times 10^5$ to 3×10^6) were used. Injection concentration varied from 100 to 2400 WPPM at rates of 30 to 220 gal/min at a location 3.5 pipe diameters from the pipe inlet.

Sellin (1974) reported drag reduction of 40-50% in large scale tests at R_e of 1.5×10^5. A Polyox WSR-301 solution was added to water flow in a 203-mm diameter pipeline, 4190 meters in length at the Bristol Sewage Treatment Work at Avonmount, England. Polymer powder, corresponding to a final concentration of 40-60 WPPM, was continuously mixed in a votex chamber and the resulting slurry pumped into the pipeline using a gear pump.

Open Channel Flows

Latto and Shen (1970) performed an experiment of slot injection over a flat plate suspended in a closed loop flume. Concentration of Polyhall-295 from 200 to 600 WPPM were ejected at a $20°$ angle. Using hot film anemometry it was found that momentum diffusion was less than that for pure water. The flow rate velocity, and angle of ejection of the polymer solution were

also found to be important. Large percentage drag reductions were reported by injecting high concentration aqueous polymer solutions into turbulent boundary layers at low flow rates. It was found that tangential injection at the lowest possible velocity was desirable.

Wetzel and Ripkin (1970) experientally studied the injection of Polyox WSR-301 into a developing boundary layer in a 9-foot, wide open channel. Pitot tubes were located at positions 16, 28, and 60 feet downstream of the injection slot. Polymer injection resulted in much fuller velocity profiles than did water injection. Several methods of polymer concentration determination were investigated including: polarography, the turbidimetric method, and the fluorimetric method. The fluorimetric method was found to give rapid, accurate results. Concentration profiles for water, 1000, 2000, and 3000 WPPM, were found to be in good agreement with the curves developed by Morkorin (1963).

A maximum drag reduction of 25% was experienced over the 40-foot boundary layer length, which resulted in a downstream wall concentration of 30 WPPM. At a distance of 16 feet from the slot, greater drag reduction was attained for the lower concentrations injected than for the larger quantities. At distances further downstream the reverse was true, with better drag reduction attained with the larger injection quantities.

The behavior was attributed to more complete mixing. Color-dyed polymer injection tests revealed that shortly after injection, the flow organized into a definite pattern of large wavering parallel streaks. This streaking was reported to be a secondary, three-dimensional vortex motion superimposed on the two-dimensional flow.

Fabula and Burns (1970) invoked the negative roughness analogy so that the outer layer mean velocity similarity is unaffected by friction reduction. The similarity law of mixing with polymeric friction is predicted to be the same as without polymeric friction reduction. The velocity defect law of Coles (1956) as well as his value for the law of the wake parameter Π of 0.55 were adopted. A relationship for calculating the local additive concentration at downstream stations along the wall was developed and compared to preliminary data on mixing in an open channel boundary layer injected with Polyox WSR-301.

Latto and Middleton (1970) reported on extensive velocity profile data taken with hot film probes for flow of homogeneous solution of Polyacrylamine MRL-402 over a flat plate suspended in a recirculating flume. Direct drag measurements were made over a period of time for concentrations of 0, 25, 50, and 75 WPPM which indicated no appreciable degradation of the polymer. The turbulent boundary layer profiles appeared "fuller" than corresponding

profiles for pure water. A friction velocity of $U_\tau^* \approx 0.074$ ft/sec
was found.

Fruman and Tulin (1974) performed a study of diffusion of a
thin tangential jet of Polyox WSR-301 solution injected into the
turbulent boundary layer of a flat plate vertically suspended in
a free-surface, high-speed water channel. Freestream Reynolds
numbers of the order of 3.6×10^7 were achieved. Drag measurement
by reluctance force gages were taken. Wall concentrations of
polymer were taken using a light-intensity, dyed-additive method.
The concentration distribution along the wall was found to be
represented by two regions.

Within the first region, the wall concentration was prac-
tically constant and equal to the injected polymer concentration.
In the second region, the concentration varied inversely with the
distance from the injection slit. The length of the first zone
was found to be 15 to 20 times that of water injection. This
extended initial zone appeared to be directly related to the
thickening of the viscous sublayer, the reduction of shear stress,
and the decrease of molecular diffusivity.

In the region where turbulent diffusivity was predominant,
both polymer solution and solvent flows displayed similar behaviors.
The distribution of the wall temperature over a flat plate given

by Seban (1960) provided a heat transfer analogy for correlation of data. According to Seban:

$$\frac{T}{T_w} = 25.0 \quad \frac{(\rho_i \nabla_i)^{1.2}}{\rho u} (X/S)^{-0.8}. \tag{27}$$

Whereas, Tulin indicated $\rho = \rho_i$, and therefore

$$\frac{C_w}{C_i} = 17.01 \left(\frac{\nabla_i}{U}\right)^{1.06} (X/S)^{-0.711}. \tag{28}$$

Closed Channel Flows

Wu (1969) presented experimental data obtained by injection of various polymer concentrations along a flat plate, and trends similar to Poreh et al. (1971) were noted (lower initial concentrations caused higher drag reductions). The results of Wu indicate poor mixing between the injected fluid and its surroundings. Photographs of submerged jets confirm that additives suppress turbulent diffusion. The flat plate results also suggest that, for efficient drag reduction, the solution injected should be dilute and comparable to the discharge within the inner boundary layer.

Wu and Tulin (1970) performed experiments in a circulating water channel with ejection of various polymer concentrations along smooth and rough plates forming the top of the closed test

section. It was recommended that, for the most effective drag reduction, the slot ejection angle should be small with respect to the flow direction and the slot opening should be comparable to the thickness of the viscous sublayer. Injection rates were also comparable to the normal viscous sublayer discharge. Optimum additional concentrations were found to be $10^2 - 10^3$ WPPM for the smooth plate and an order of magnitude larger for rough surfaces, where mixing due to roughness causes increased dilution of the injected solution.

Poreh and Cermak (1962) studied two-dimensional turbulent mixing of ammonia gas from a line source near a wall. They envisioned a four-zone diffusion process that is worthy of description since the concentration profiles are considered representative of polymer diffusion and for this reason have come under much study. The four zones as described by Poreh and Cermak are:

1. An initial zone - This zone is very close to the source in which a large portion of the diffusion boundary layer is submerged in the viscous sublayer. The length of the region is determined by: the initial condition near the source, relative to the thickness of the sublayer; the injection velocity; and the magnitude of the molecular diffusivity. The extent of

this zone was not determined. Very little reliable data were
obtained in this region due to very large velocity and concen-
tration gradients.

2. The intermediate zone - In this zone the diffusion plume
is submerged in the momentum boundary layer and its thickness
is large when compared to that of the sublayer. The diffusion
rate in this region is relatively large and the concentration
profiles are found to be approximately similar in the sense
that

$$C/C_{Max} = f(Z/\lambda), \qquad (29)$$

where λ is a characteristic height of the diffusion boundary
layer defined as the distance from the wall where $C/C_{Max} = 0.5$.
Measurements in air indicate that the length of the intermediate
zone is 20 to 40 boundary layer thicknesses downstream from the
source.

3. The transition zone - In this zone, the diffusion rate is
slower due to the lower level of turbulence in the outer portion
of the boundary layer.

4. The final zone - The growth of the diffusion boundary
layer coincides with that of the momentum boundary layer in

this zone. In this stage the maximum concentration near the wall is inversely proportional to the thickness of the boundary layer and the ambient velocity.

Morkovin (1963) considered the experimental evidence of Poreh and Cermak (1964) for the final two zones of quasi-similarity and concluded that they were consistent with the concepts of eddy diffusivity. He described their data as follows:

Intermediate zone: $\quad C/C_w = e^{-0.693 \ (y/\delta_d)^{1.5}}$ (30)

Final zone: $\quad C/C_w = e^{-0.693 \ (y/\delta_d)^{2.15}}$. (31)

Figure (4) displays a plot of the concentration profiles in the intermediate and final zones.

Poreh and Hsu (1971) indicate that the most widely used method to predict gross diffusion patterns in turbulent flow employs an eddy diffusivity model. This model assumes that the flux of the diffused matter by turbulent fluctuation is proportional to an eddy diffusivity term, D_e, times the local concentration gradient:

$$q_y = -D_e \ (\partial c/\partial y).$$ (32)
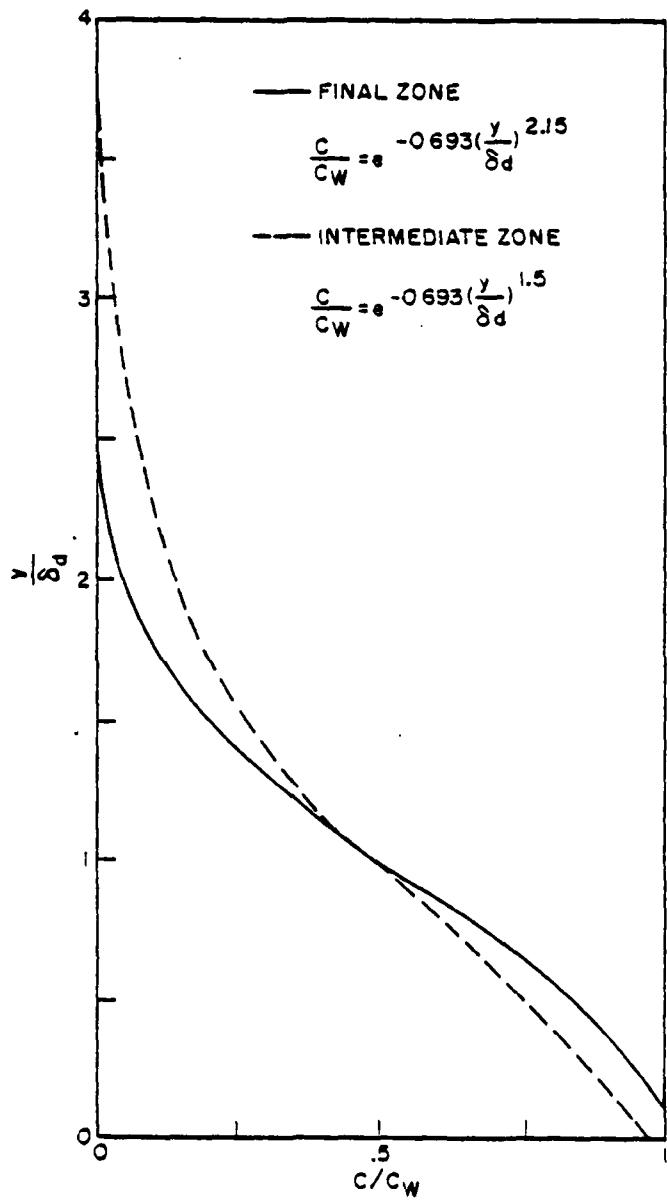
Figure 4. Concentration profiles in the intermediate and final zones

It is further assumed that D_e is a function of the flow field and its value can be specified at a point regardless of the position of the source. They point out that this assumption holds true only at distances from the source that are large compared to the Lagrangian integral scale of turbulence. Measurements made by Poreh and Cermak (1964) indicate that this limitation holds for diffusion in turbulent shear flows. Estimates of the Lagrangian integral scale suggest that it is of the order of boundary layer thicknesses.

Another method for treating diffusion patterns is based on Batchelor's (1957) Lagrangian similarity hypothesis used to predict the turbulent motion of particles in steady, self-preserving shear flows. Cermak (1962) applied the Lagrangian similarity hypothesis to predict diffusion from a continuous point and line source with the conclusion that results from the application of the Langrangian similarity hypothesis were significant for the modeling of diffusion.

Hsu (1971), and Poreh and Hsu (1971) applied these techniques to predict the diffusion boundary layer growth in the intermediate, transition, and final zones for polymer flow. The following equations that resulted describe the change of the mean vertical position, $\bar{y}$, and the mean longitudinal position, $\bar{x}$, for an ensemble

of single particle releases, in the logarithmic portion of the boundary layer:

$$\frac{d\bar{y}}{dt} = bV^*$$ (33)

where b is Batchelor's constant, and

$$\frac{d\bar{x}}{dt} = u(\bar{y}).$$ (34)

It follows, therefore, that

$$\frac{d\bar{y}}{dx} = \frac{bV^*}{u(\bar{y})}.$$ (35)

Poreh and Hsu (1971) further conclude that the boundary concentration downstream from a continuous line source may be given by

$$C_{Max} = \frac{Q}{\bar{y}\ u(\bar{y})}$$ (36)

where Q is the discharge of the source per unit width.

Ellison (1959) estimated that Batchelor's constant is given when b = K, where K is the Von Karman constant, $1/k = a_\ell$, in equation (39). His analysis further suggests that $\bar{y}$ (the mean position of particles at a given cross section, x) is equal to $\bar{Y}$, (the mean position of single particle releases when $\bar{x} = x$). The

mean position of a particle at any cross section, x, may be defined as:

$$\bar{y} = \frac{\int_0^\infty cy \, dy}{\int_0^\infty (c) \, dy} \ . \tag{37}$$

By replacing y with y/δ_d where δ_d is the value of y when C/C_{Max} is .5, and C by:

$$C/C_{Max} = f(y/\delta_d) \tag{38}$$

equation (37) may be integrated to obtain

$$\bar{y} = a_\lambda \, \delta_d. \tag{39}$$

Substituting equation (39) into (35) results in an expression for the diffusion boundary layer developing with distance x, such that

$$a_\lambda \frac{d\delta_d}{dX} = \frac{bV^*}{u_{(\bar{y})}} \ . \tag{40}$$

Hsu (1971) observed improved agreement with data when replacing the constant b with the expression

$$b = K(1 - \bar{y}/\delta). \tag{41}$$

Figure 5 presents a comparison of experimental data with results from equation (40) for b = K, b = .8K, and b = K(1-$\bar{y}$/δ). The growth of the diffusion boundary layer within the momentum boundary through several zones of diffusion is also shown in this figure.

Sampson (1969) constructed a recirculating water tunnel and an instrumentation system for investigation of turbulent, non-Newtonian, flat plate boundary layers. Non-Newtonian velocity profile data were obtained by laser velocimeter at Reynolds numbers in the region of 10^6; the 1/7th power law velocity profile was found to correlate well with the data.

In his studies with concentrations of 100 WPPM of the polymer Separan AP-30, Rudd (1972) found the drag reduction mechanism confined to the viscous sublayer close to the wall. The polymer did not appear to have any significant effect on the central region of flow. A test section of one-half inch square pipe, 7 feet 6 inches long, was used in a recirculating flow system driven by a peristaltic pump.

Velocity measurements were taken with a laser velocimeter. It was found that at concentrations of 100 WPPM, the polymer molecules produced quite a significant amount of light scattering, requiring no additional seeding.

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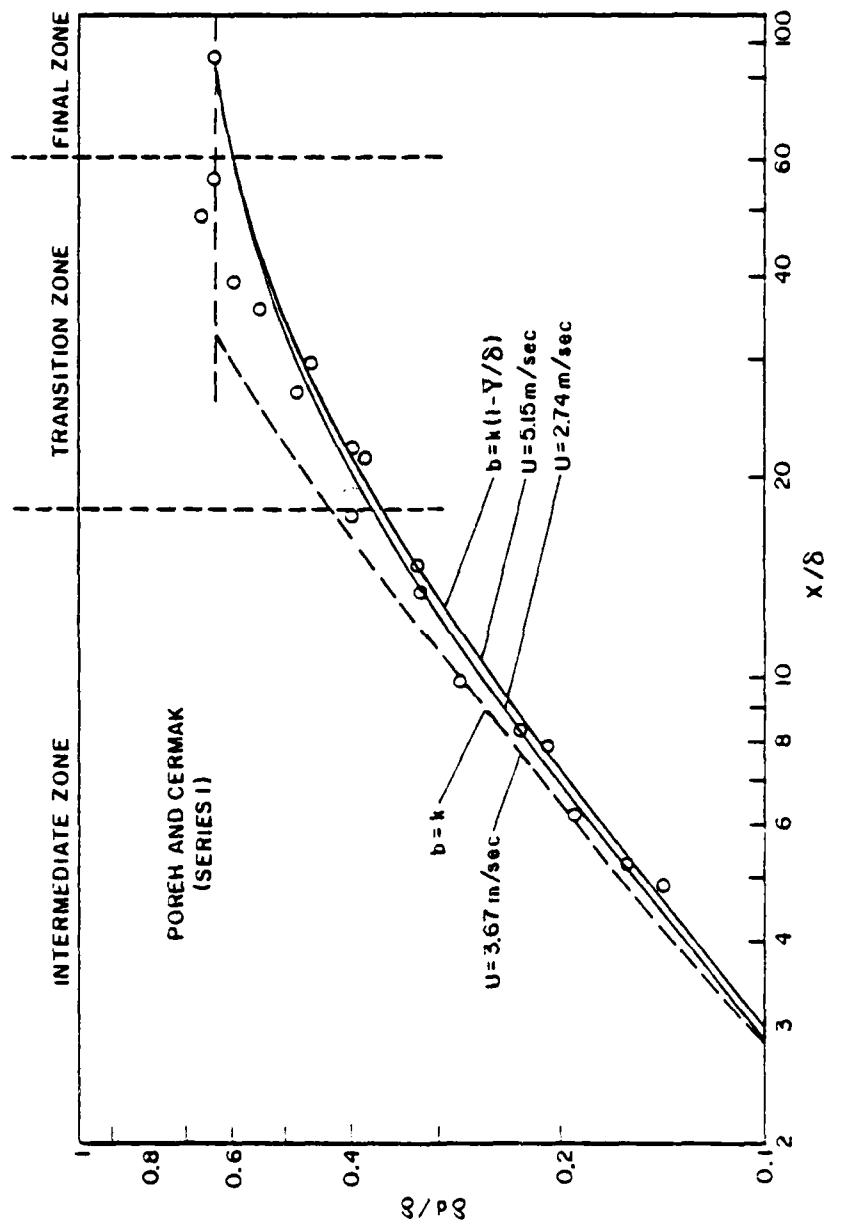
Figure 5. Growth of the diffusion boundary layer within the momentum boundary layer (from Hsu (1971))

Logan (1972), using a one-half inch square duct under gravity feed, confirmed Rudd's findings of increased streamwise turbulence intensities and decreased spanwise intensities.

Using the recirculating tunnel built by Sampson (1969), Kumor and Sylvester (1973) conducted a series of experiments on the time effects of polymer drag reduction degradation on mean velocity profiles. Sparan AP-30 was the polymer used. The experiments show that the velocity gradient at the wall increases with time, while the sublayer thickness decreases.

Reischman (1973) attempted to correct the flow facility deficiencies of previous investigations. He constructed a unique dimensional turbulent flow channel, in which the side walls were slightly bowed, to allow good spatial resolution at the wall in the center of the channel. The flow channel was constructed of 1/4-inch Plexiglas with a cross section of 1 inch by 12 inches (an aspect ratio of 12:1) and a length of 70 inches. Velocity measurements in homogeneous 100 WPPM solution of Magnifloc 837-A, Sepman AP 273, and Polyox WSR-301 were made using a laser Doppler anemometer measuring individual realizations. For drag reductions of 40% or less for the near wall velocity measurements, the viscous sublayer (indicated by a linear velocity profile) was found to be of the same extent as for a comparable solvent flow. Reischman found no evidence of a thickened sublayer.

Drop Tests, Rotating Disks and Cylinders

Crawford and Pruitt (1963) were among the first to observe the increased velocity of steel spheres falling in a solution of Guar Gum as compared to their velocity in plain water. Ruszczycky (1965), using steel spheres of a diameter up to 1 inch, performed tests in concentrated solutions of Guar Gum and Polyox WSR-301 up to 15,000 WPPM. For the 1-inch spheres, the maximum drag reduction observed was approximately 28% at Guar Gum concentration of 5000 WPPM and a Polyox concentration of 7500 WPPM.

Lang and Patrick (1966) found that the drag of spheres falling in Polyox WSR-301 was reduced by about 70% while the drag of other shapes, such as cones and cylinders, was only slightly reduced. Dyed-wake photographs of falling spheres indicated a significant change in the shape of the wake, with the laminar separation point being shifted rearward (Re range 6×10^3 to 2.5×10^5).

Hoyt et al. (1965, 1964) showed that addition of Guar Gum lowers the torque by as much as 70% for concentrations of 300 to 400 WPPM for turbulent rotating disk flow.

Using a multiple disk apparatus, Whitsitt et al. (1969) showed that rough disks gave higher torque reduction of about 60%

in Guar Gum solution at 250 WPPM than did smooth disks. A poly-
acrylamide solution lost its torque-reducing ability very quickly.
However, the work of Huang and Santelli (1972) showed that poly-
acrylamide solutions were more resistant to shear degradation
when compared to polyethylene oxide.

Sirmalis (1974) conducted an extensive study of drag reduc-
tion on axisymmetric torpedo-shaped bodies having 6° and 12°
spherical tail cones. During the study, experiments covering a
range of Reynolds numbers from one to five million in plain water
and polymer oceans of 1 to 60 WPPM of Polyox WSR-301 were conducted.
Photographic studies revealed that, at very low concentrations,
(2.5 WPPM), the fine structured turbulence was absent and only
large scale turbulence remained. Computer routines tended to
overpredict boundary layer thickness while indicating proper growth
shapes. No shift in the boundary layer separation region was noted.

Testing in fresh water with polymer ejection from the body
showed similar results but with better visual representation at
the higher polymer concentration. Dyed water ejection tests
displayed the characteristic course and fine turbulence structure.
Ejection of 50 WPPM dyed polymer solution eliminated most of the
fine structured turbulence leaving only the course structure.
Dye streaking was apparent at the higher concentration levels.
The streak spacing, consistent with laminar sublayer streaks,

was also in agreement with the number of ejection holes in the screen ejector in the nose of the body. A maximum of 70% reduction in skin friction was achieved with the 6° tail body in a polymer ocean of 20 WPPM.

Polymer diffusion tests were conducted at a fixed rate of 20.6 in/sec and a nominal velocity of 27 ft/sec. Wall samples and boundary layer samples at .025 inch and .055 inch were taken and analyzed by fluorometric techniques. Tests were conducted with water and concentrations of 5, 10, 20, 50, 500, and 1000 WPPM of polymer. All polymer tests displayed a drastically reduced diffusion resulting in an extended initial mixing zone. Similarity concentration profiles were found to have like forms, and therefore

$$C/C_w = e^{-0.693(y/\delta_d)^{K_3}}.$$ (41)

Profile coefficient, K_3, has an experimentally determined value of 0.75. Sirmalis indicates that the true value is believed to be below this value, since data were not obtained in the critical region of within approximately 0.010 inch of the wall. These results were considered significant in that they imply substantially reduced polymer quantities may be required with proper ejection techniques.

Review of Experimental Apparatus

The vast number of investigations concerning the phenomena of drag reduction, and the continuing attempts to conduct a definitive experiment, indicate the scientific community's interest in the problem and the difficulties involved. From these studies, it is known that drag reduction additives include soap solutions, algae, plant derivatives, and high molecular weight polymers. Additionally, a maximum drag reduction asymptote of approximately 80% has been shown. In spite of these efforts, the exact nature of the phenomena of drag reduction remains elusive to the various techniques applied to its study.

Flow visualization experiments have provided some insight from a qualitative view point, while pressure drop-flow rate experiments have contributed an indication of the magnitude and scope of the phenomena. The bulk of these efforts are, however, still contradictory and difficult to interpret due to the confusing features of the flow facilities and ambiguities in the measurement instrumentation employed.

Reischman (1975) points out that, in general, experimental techniques for velocity measurement may be separated into four groups: pitot tubes, bubble tracing, hot-film probes, and laser Doppler anemometers.

Pitot probe investigations comprise the largest group and are subject to errors requiring complex corrections. Smith et al. (1967) have demonstrated that measurements in identical flow situations made by various size pitot probes yield different results. Metzner and Astaria (1967) attribute these errors to the influence of additional viscoelastic normal stress terms. Corrections for pitot probe techniques are generally omitted, however, as being too complicated. In spite of this, Tomita (1970) applied viscoelastic corrections to his pitot probe measurements.

Hot film probes, the next largest group, depend upon the heat transfer characteristics of the medium. Frieche and Schmartz (1969) have shown that hot-element sensors are difficult to calibrate because polymer additives alter the heat transfer characteristics of the medium and collect on the sensors. It is pointed out that the calibration drift problem is particularly severe in dilute polymer solutions where the sensitivity of the probe to velocity changes is lower than it is in the water alone.

Donohue et al. (1972) point out that bubble tracing techniques are hampered by large uncertainties as well as being an extremely tedious process.

The fourth group comprise laser Doppler anemometer efforts. A relatively new discovery by Yeh and Cummings (1964), the laser Doppler anemometer is a linear, non-interfering instrument which does not depend on the rheological or intensive properties of the working fluid. The laser anemometer was first used in drag reducing flows during four investigations almost simultaneously: Chung and Graebel (1969), Goldstein et al. (1967), Rudd (1969), and Shankan (1969). The measurements of Shankan (1969) have insufficient sample size, however, for accurate results, while Goldstein et al. (1967) only made measurements on the tube center-line.

Chung and Graebel (1969) experimented in a small, .47-inch diameter pipe where the spatial resolution of their laser anemometer was very poor. Subsequently, five researches have made acceptable laser velocimetry measurements in drag-reducing flows. This group consists of Rudd (1969), Logan (1972), Sampson (1969), Kumor and Sylvester (1973), and Reischman (1975). Of these experiments, all were conducted in a square cross-section apparatus, except the work of Reischman who used a 12:1 aspect ratio tunnel. Rudd and Logan utilized a one-half inch square duct and Sampson, and Kumor and Sylvester, used an 8-inch square tunnel with a submerged off-center flat plate. White law (1973) has shown that non-symmetric secondary flows are significant in square duct flows.

Sampson (1967) reported that pump vibrations were significant, prohibiting meaningful tests at higher flow rates and that the effect of these vibrations on the laser system were unknown. Additionally, accommodations in both equipment and system design were necessary to minimize costs of the construction of the laser system. Although the electronic output was noisy with a relatively poor signal-to-noise ratio, usable data were obtained by use of a storage oscilloscope.

Kumor and Sylvester utilize the same facility as Sampson but offer no water data to allow verification of the suitability of this facility.

Reischman avoided the detrimental recirculation aspects of the previous researchers by using his flow facility in a blow down mode. In contrast to the previous researchers, Reischman used an individual realization-type laser Doppler anemometer, which measures the period for 10 cycles of a Doppler burst scattered from a single particle. However, the orientation of the laser light beam optical system was in an undesirable side scatter mode where scattered light intensity is predictably weak. The singular results of Reischman, indicating that the viscous sub-layer does not thicken, were obtained in an improved flow facility under weak signal conditions for homogeneous polymer flow.

This is in direct conflict with the conclusion of all previous
investigators.

Rudd, Logan, and Sampson used continuous wave anemometers
employing a frequency tracker which converts Doppler frequency to
an analog signal proportional to the instantaneous velocity.

To date, all of the work using laser Doppler anemometers
has been performed with homogeneous polymer flows, not with
ejection of polymer solutions. A goal of the present work was
to improve the deficiencies of previous researchers and obtain
data on velocity profiles, with and without injection of polymer
solutions into the boundary layer on a submerged flat plate in
a high aspect ratio, rectangular water tunnel. Laser Doppler
anemometry velocity measurements in this experiment were made
in the forward scatter mode where scattered light intensity
distributions are strongest.

III. EXPERIMENTAL APPARATUS

The experimental apparatus in this research consists of a
flow facility and water tunnel, a polymer ejection system, a
laser Doppler anemometer system for velocity measurement, a
fluorometer to measure boundary layer concentration profiles,
and a minicomputer for real-time data processing. The character-
istics of each element are discussed in detail in the sections
that follow.

Flow Facility

The overall flow facility designed for this experimental
effort is shown schematically in figure 6. Filtered water is fed
to a 15,000-gallon capacity water tank having a constant head
overflow, providing 5 psi flow head for the tunnel. Various
filter cartridges allow filtration of particles ranging in size
from 5 microns to .5 micron. A high pressure steam heat exchanger
allows deaeration of the water by heating. The flow rate of
water from the storage tank to the tunnel is measured by a Cox
Instruments Co. turbine flowmeter and controlled by a manual ball
valve in the exit line. A round to rectangular transition section
provides entrance to a rectangular diffuser. The flow is then
straightened in a settling chamber designed to form small scale
turbulence of uniform distribution. The flow enters the water

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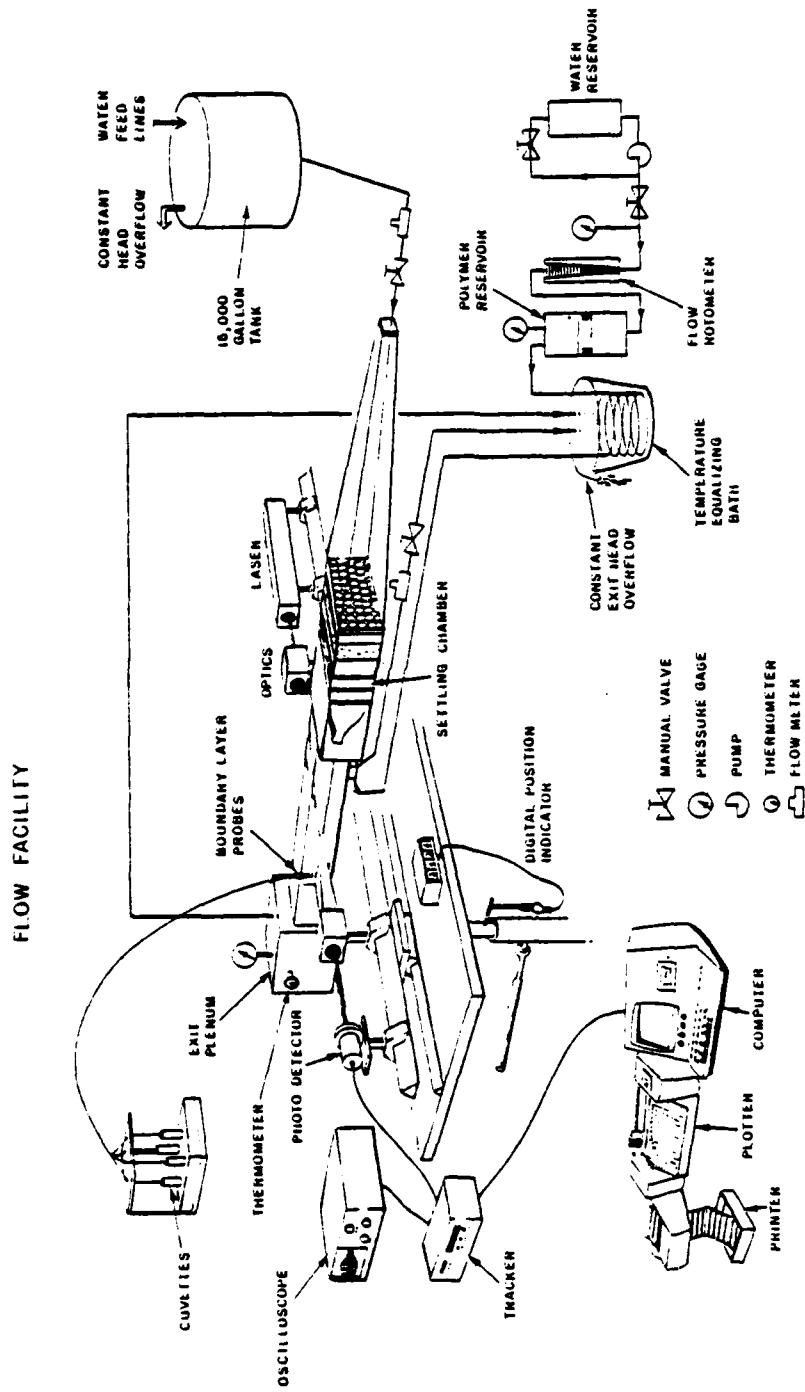
Figure 6. Experimental flow facility

tunnel through a specially designed transition nozzle. The
flat plate is attached to the floor of the tunnel allowing wall
samples to be taken through the bottom wall of the tunnel. The
boundary layer approaching the flat plate is bled off through a
suction port ahead of the leading edge of the flat plate.
Boundary layer sampling probes are lowered through ports in the
tunnel ceiling. Velocity measurements are taken throughout the
entire internal volume without disturbing the flow using the
laser Doppler anemometer techniques and a three-dimensional
traversing system. The flow then exits the tunnel through an
exit plenum. Constant tunnel exit pressure is provided by exiting
the flow into a container with constant exit head overflow. The
boundary layer bleed flow is also exited into this container.
Polymer solutions, for ejection over the flat plate, pass through
a delivery line coiled in the exit flow container. The temper-
ature of the polymer solution then approaches the temperature of
the water flowing through the tunnel. Figure 7 is a photograph
of the actual test setup.

Initial Attempts

In order to avoid the design of a round to rectangular
transition diffuser and because axial length was at a premium
in the test setup, a relatively large cross-section settling

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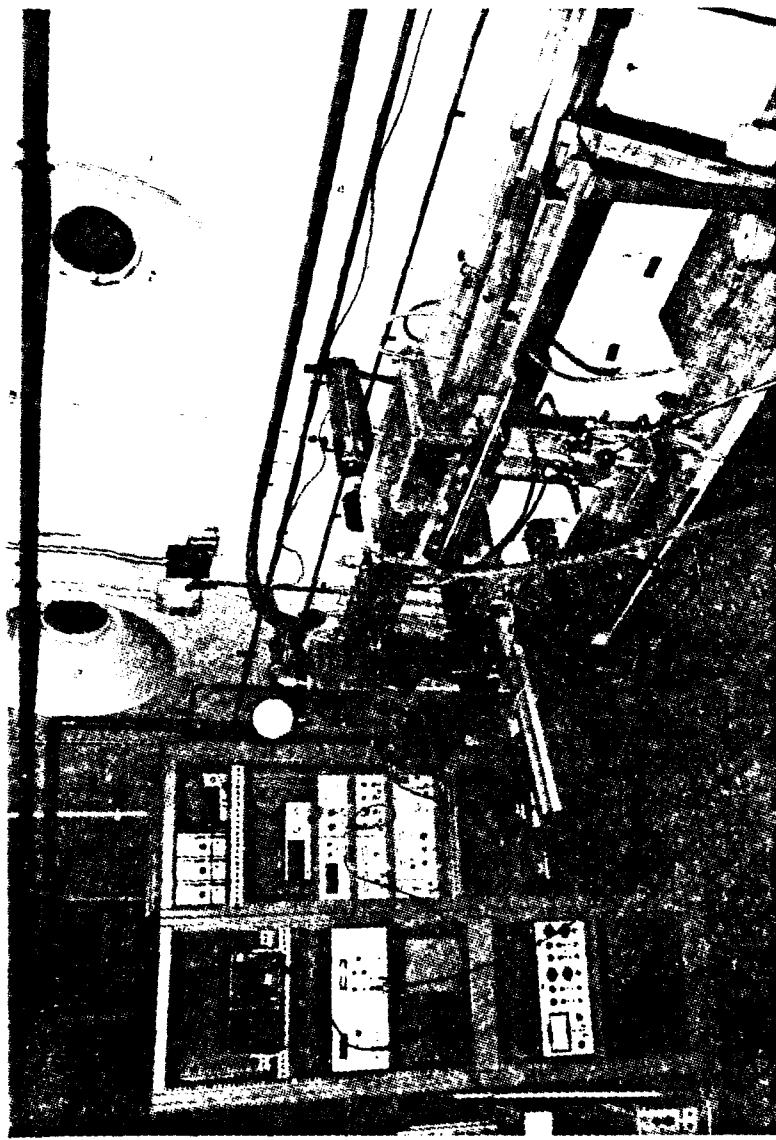
Figure 7. Experimental set-up photograph

chamber was designed and built of anodized aluminum as shown
in figures 8 and 9. The dimensions were: width, 8.25 inches;
height, 6.5 inches; and length, 11 inches. The top and rear side
were made of Plexiglas for measuring and photographing the flow.
Dye probe access ports were located in the top and the flow
entered through a 3/4-inch fitting. Two overlapping perforated
plates (0.25-inch diameter holes and 48% open area) were thought
to distribute the flow, followed by 1/8-inch HEXCELL honeycomb
(1 inch long) for flow straightening and four screens (24-mesh,
.007-inch diameter wire of 67.4% open area) for the even dis-
tribution of fine, small-scale turbulence. A rounded edge slot
formed the tunnel entrance.

The perforated plates were totally ineffectual as a high-
speed central core water jet maintained itself well into the
settling chamber. An impact distribution plate was attached to
the inlet fitting. This fix transformed the high-speed core flow
into violent turbulent recirculating flows. In addition, corner
vortices formed in the exit flow, as shown in figure 10. At this
juncture it became evident that a diffuser would be necessary to
decelerate the flow and an entrance nozzle to the tunnel would
also be required to eliminate the formation of corner vortices.
A redesign of the settling chamber to provide equal contraction
ratio in the horizontal and vertical planes was also performed in
an attempt to further minimize the formation of corner vortices.
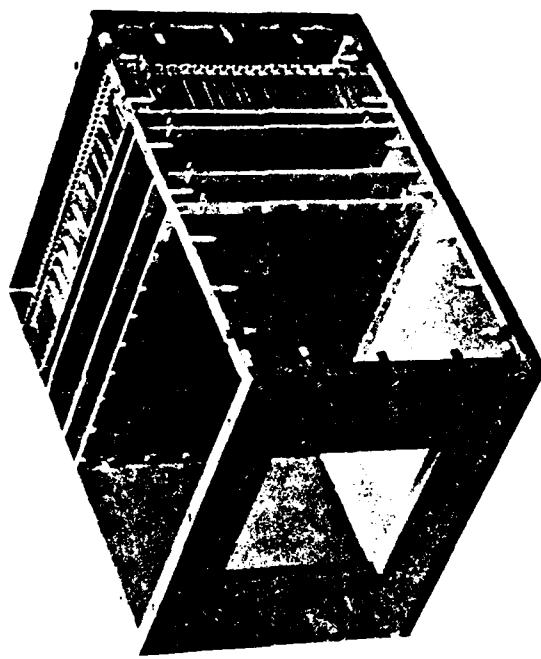
Figure 8. Initial settling chamber photograph

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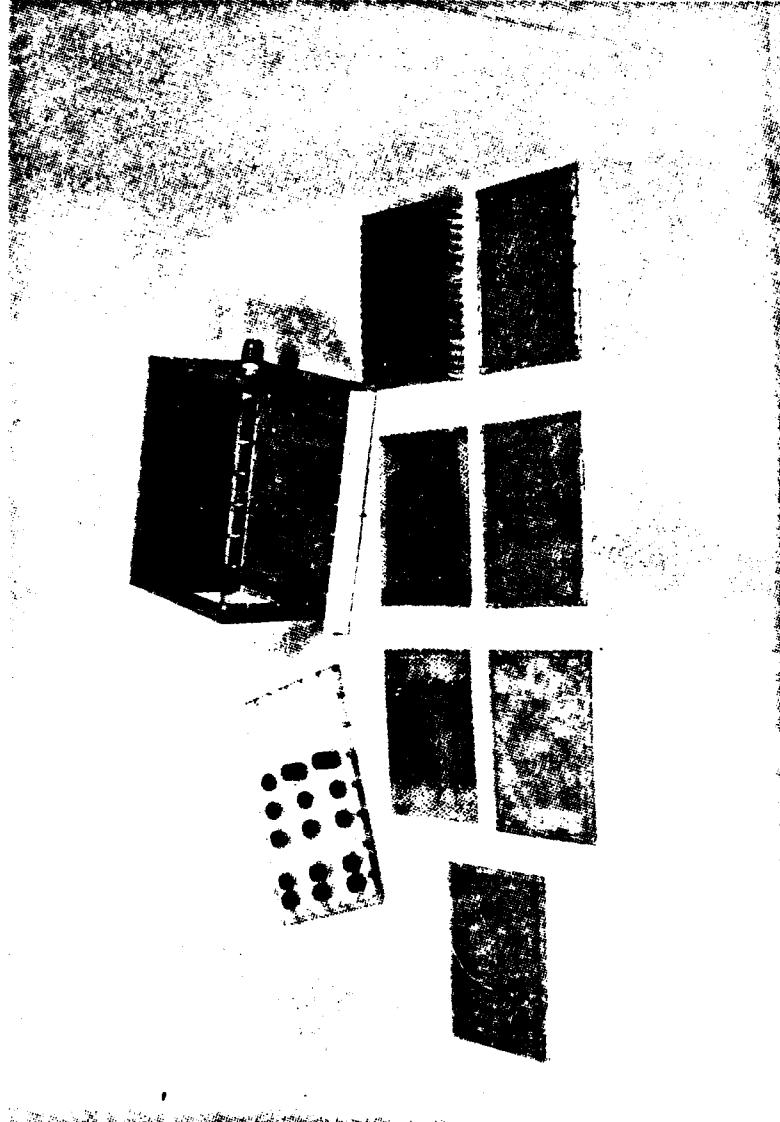
Figure 9. Initial settling chamber disassembled photograph
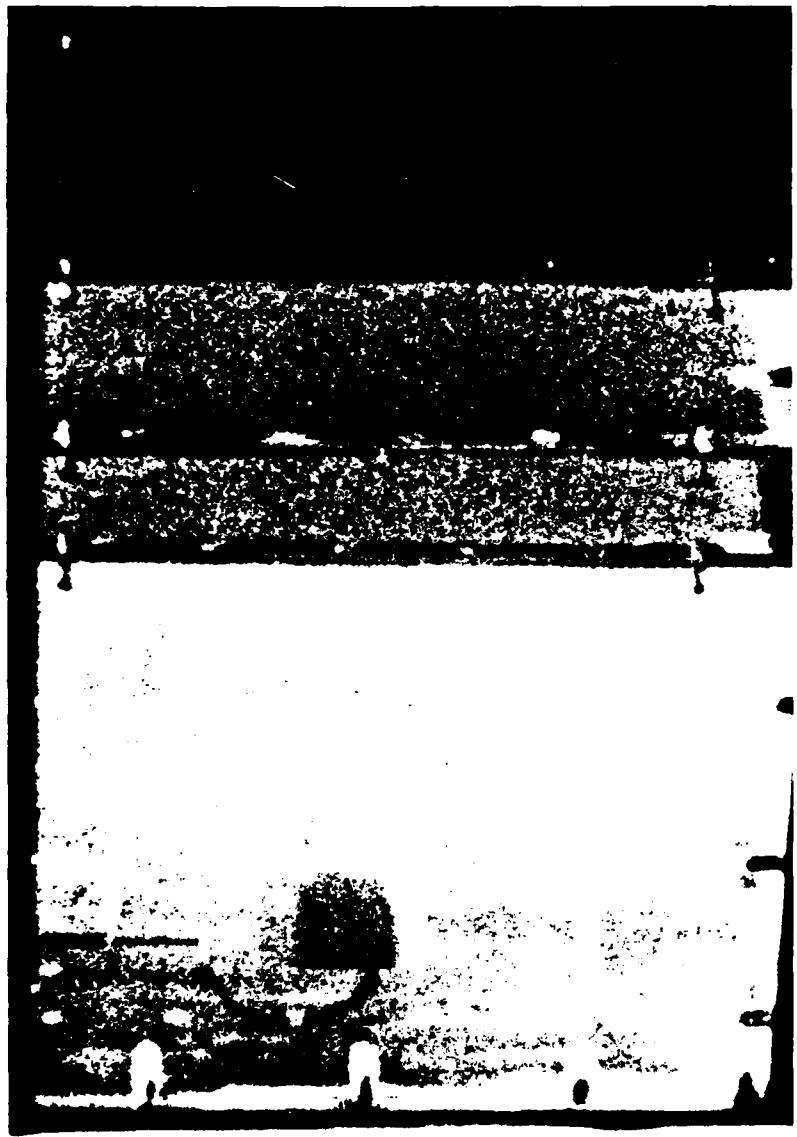
Figure 10. Initial settling chamber corner flow photograph

Settling Chamber

The second settling chamber design dimensions now remained firm at a width of 8.25 inches, a height of 2.75 inches, and a length of 18.5 inches. The contraction ratios from settling chamber to tunnel were fixed at 3.67 in both the horizontal and vertical planes, providing an overall contraction of 13.5:1. The settling chamber was constructed entirely of Plexiglas and had an internal configuration, shown (in order) in figure 11, consisting of a perforated plate, two screens, a honeycomb, and four additional screens, spaced at 1-inch intervals with 2 inches upstream of the honeycomb. The entrance to the tunnel was a well-rounded rectangular hole, shown in figure 12.

The configuration straightened the flow; however, corner vortices at the tunnel entrance remained (figure 13). A standoff rectangular nozzle was designed and tested with no success, as shown in figures 14 and 15. In addition, small flow oscillations appeared due to intermittent stall in the diffuser.

The corner vortices were finally eliminated by the design and fabrication of a transition nozzle in the form of a ninth order polynomial whose first four derivatives are zero at each end. The polynomial coordinates were placed on paper tape and used to cut templates for the vertical and horizontal contours in
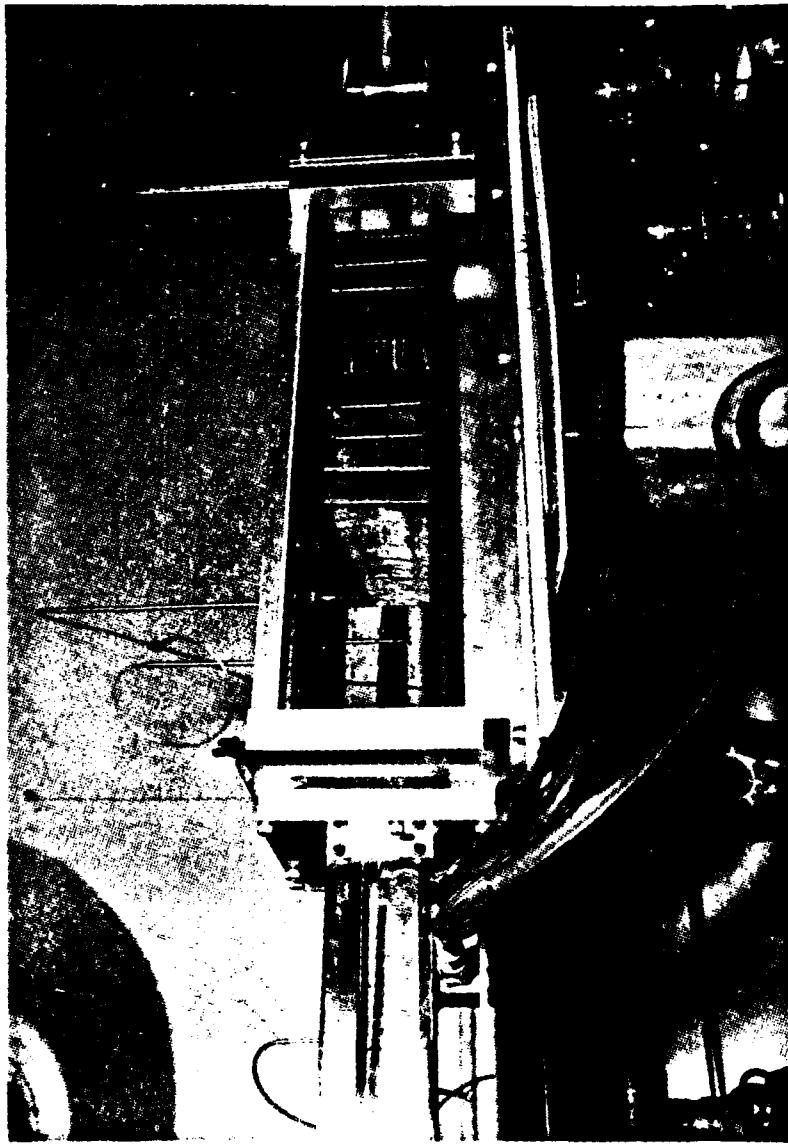
Figure 11. Settling chamber configuration photograph
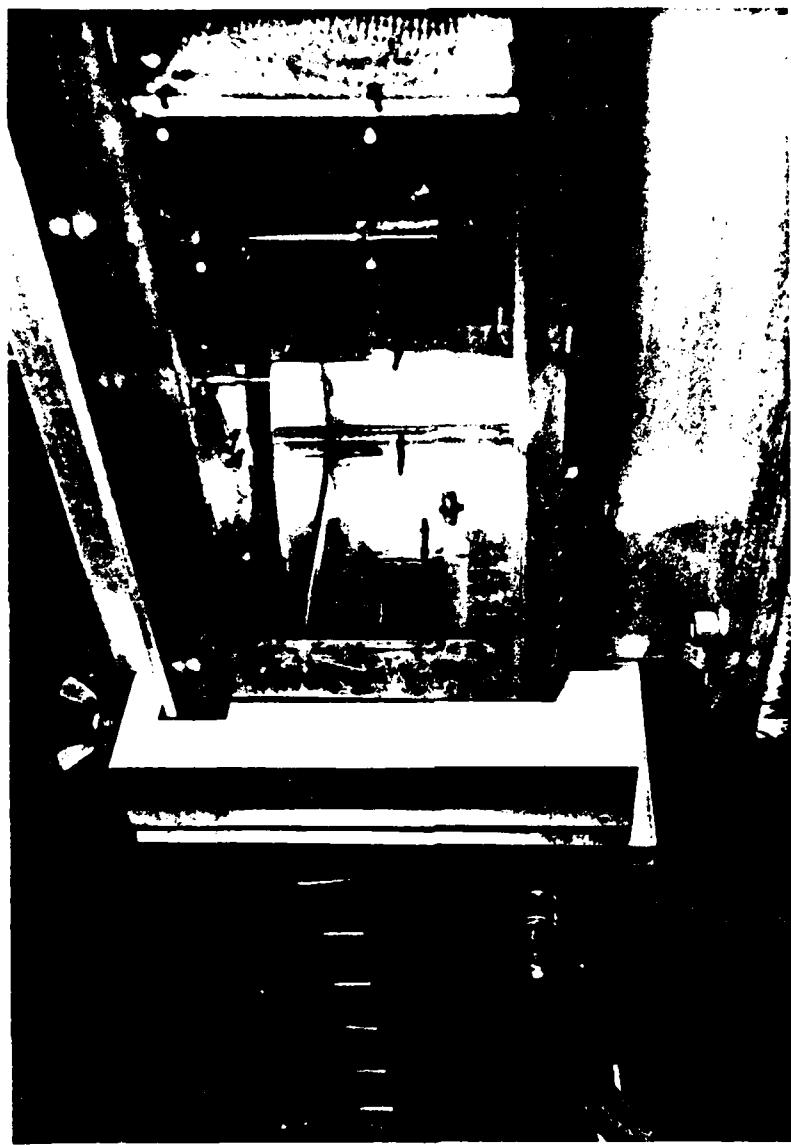
Figure 12. Rounded rectangular tunnel entrance photograph
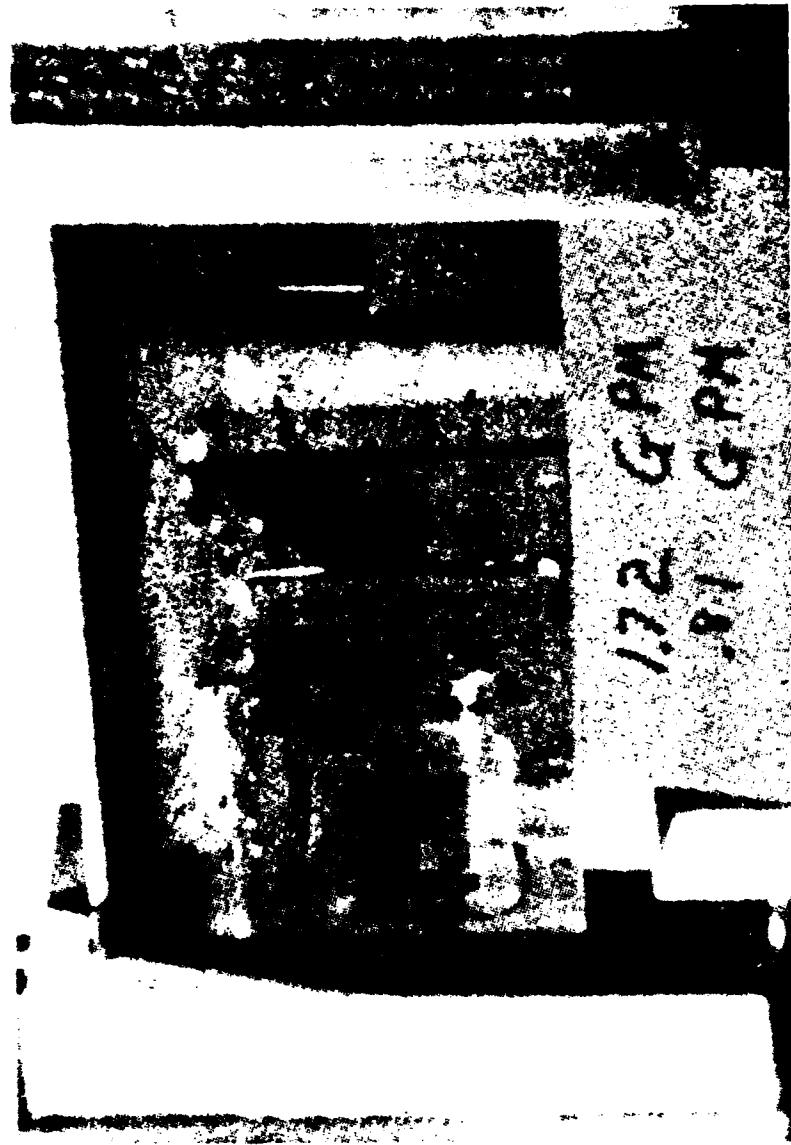
Figure 13. Settling chamber flow pattern photograph
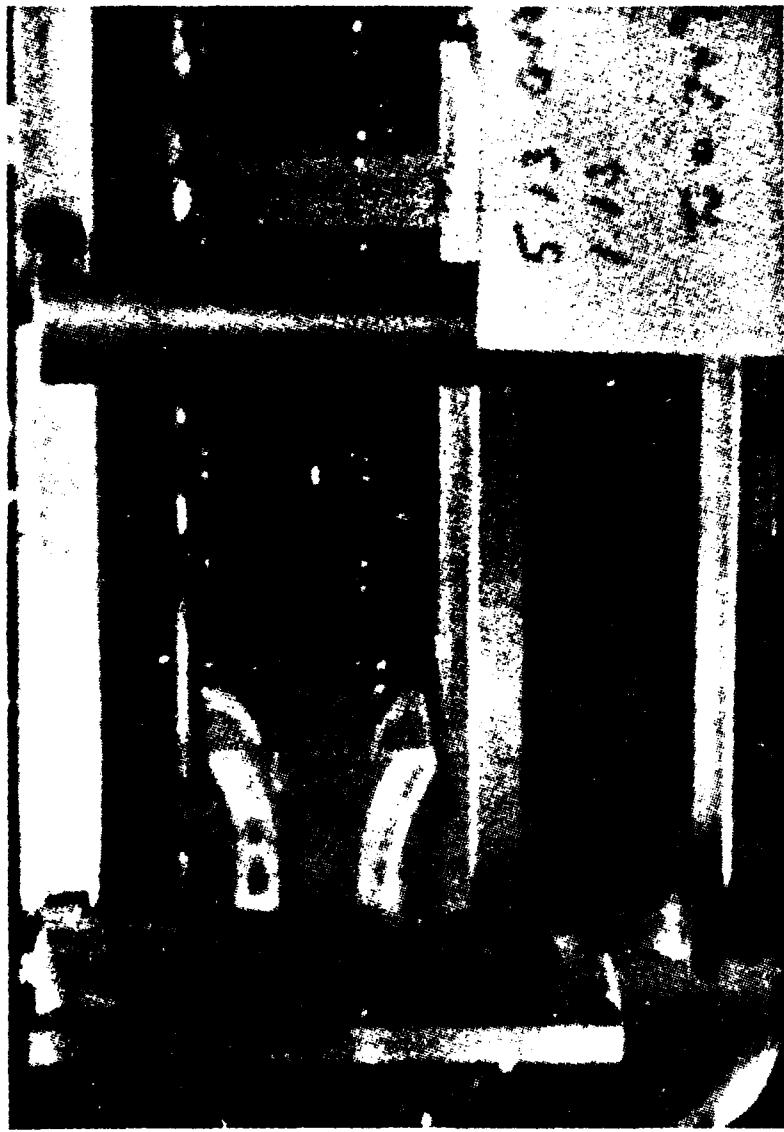
Figure 14. Rectangular stand-off tunnel entrance nozzle photograph

Figure 15. Rectangular stand-off tunnel entrance nozzle flow pattern photograph

a tape-driven automatic milling machine. These templates were used
to cut a mandril from a cast block of epoxy resin. This mandril
was placed in a casting box of the same internal dimensions as the
settling chamber. The transition nozzle block was then cast over
the mandril. Removal of the mandril after an appropriate curing
time yielded a block with proper internal flow passage configura-
tion that fit into the settling chamber. The transition nozzle
used is shown in figures 16 and 17.

Discussions on the internal configurations of the settling
chamber, with Drs. Andrew Chawat and Steven Barker at the
University of California at Los Angeles, provided the key element
for the removal of the final obstacle for flow conditioning. A
3-inch length of reticulated Scott foam quieted and steadied the
flow, removing any remaining flow disturbances.

The final settling chamber design consisting of a perforated
plate, Scott foam, honeycomb, two screens and a transition nozzle
is shown in figure 18 and schematically in figure 19.

Diffuser

A 36-inch long rectangular Plexiglas diffuser (figure 20) was
designed having no appreciable stall, according to Kline (1959).
The entrance dimensions were 0.75 inches by 0.75 inches with rectan-
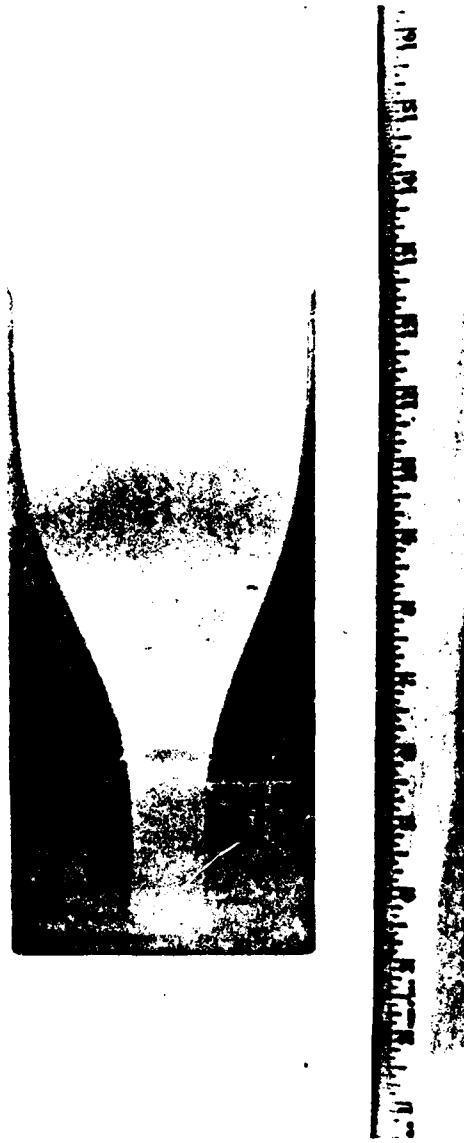
Figure 16. Transition nozzle side view photograph
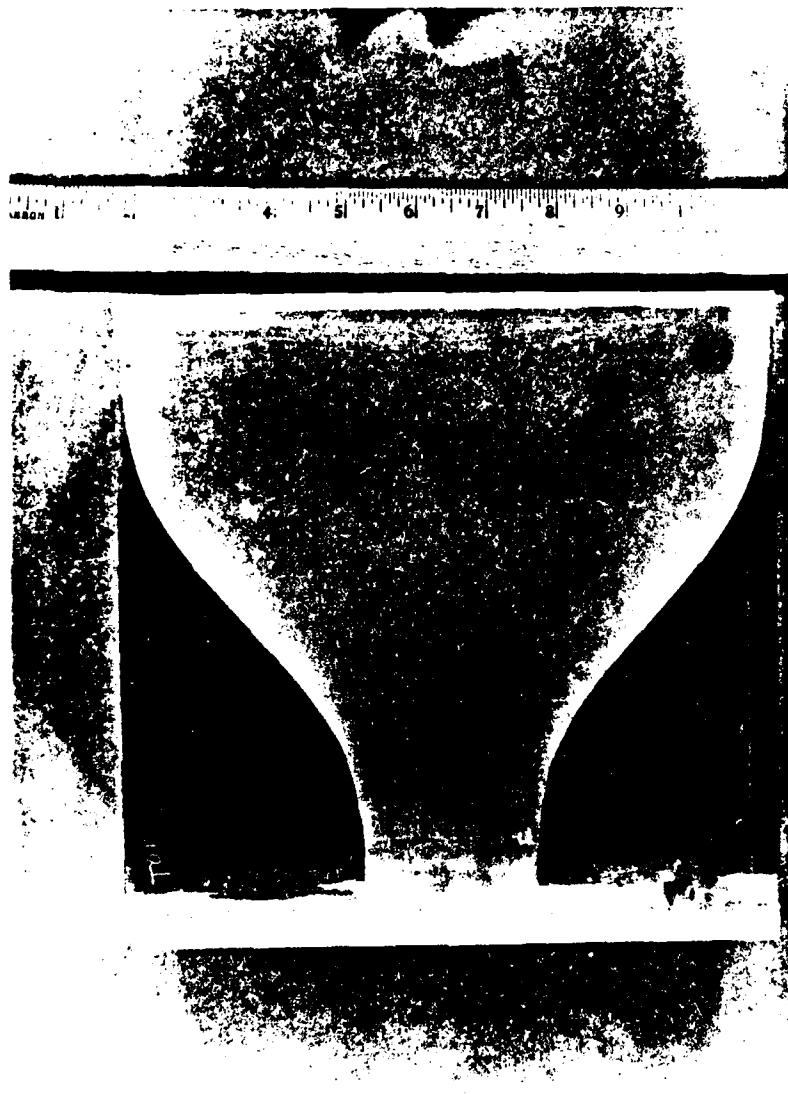
Figure 17. Transition nozzle top view photograph
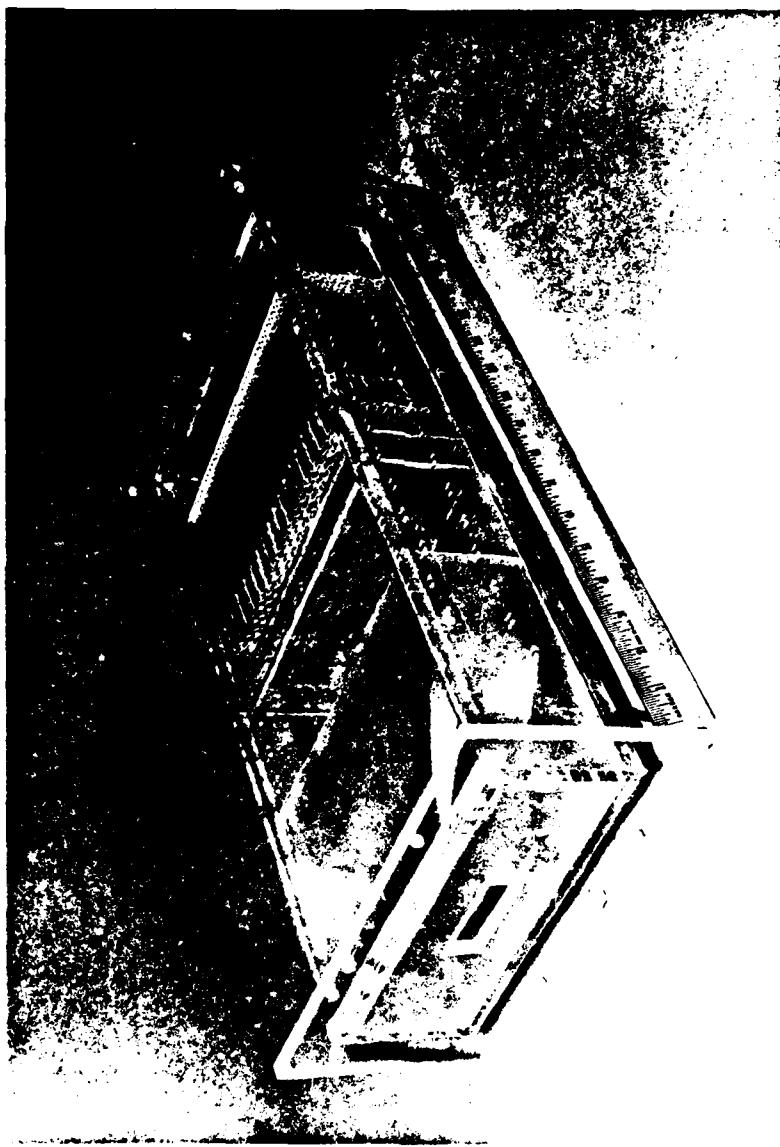
Figure 18. Final settling chamber configuration photograph
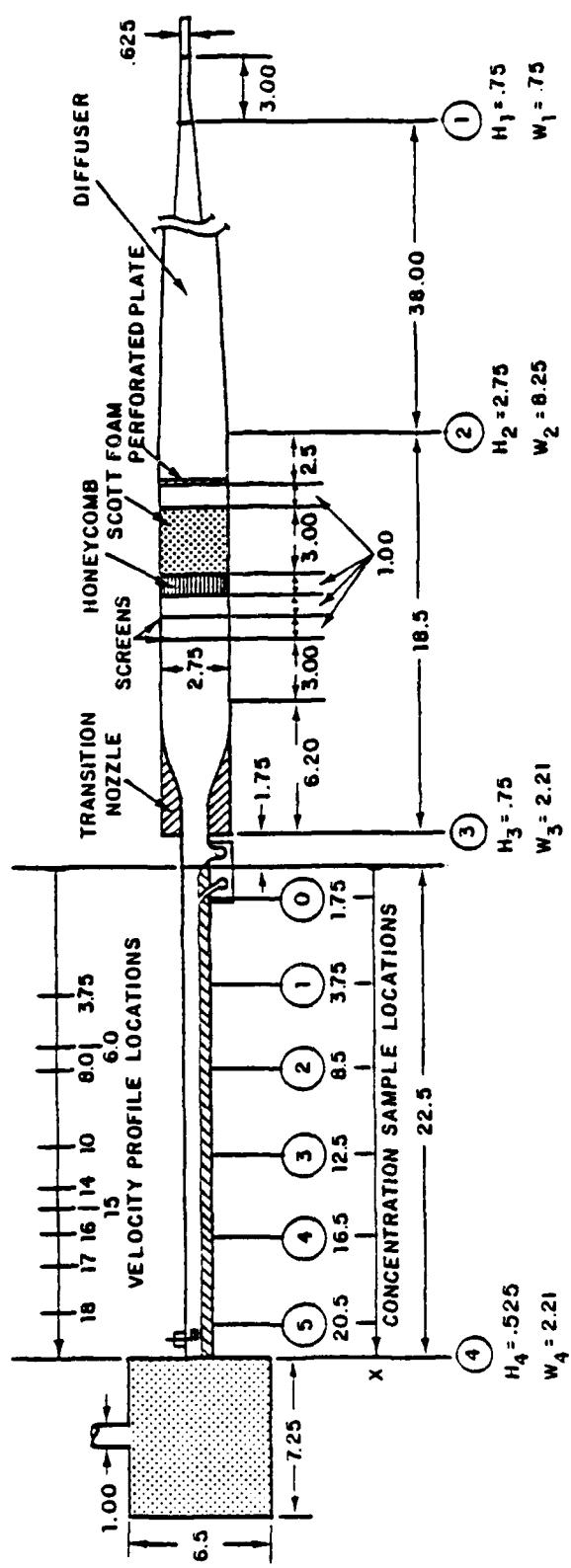
Figure 19. Test set-up schematic
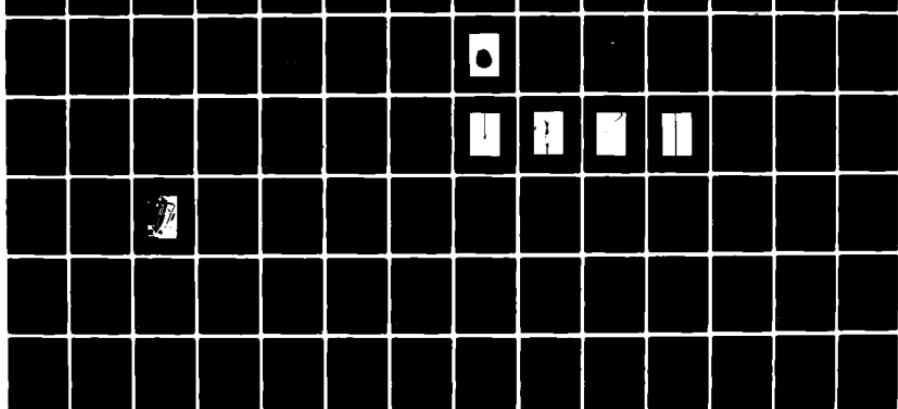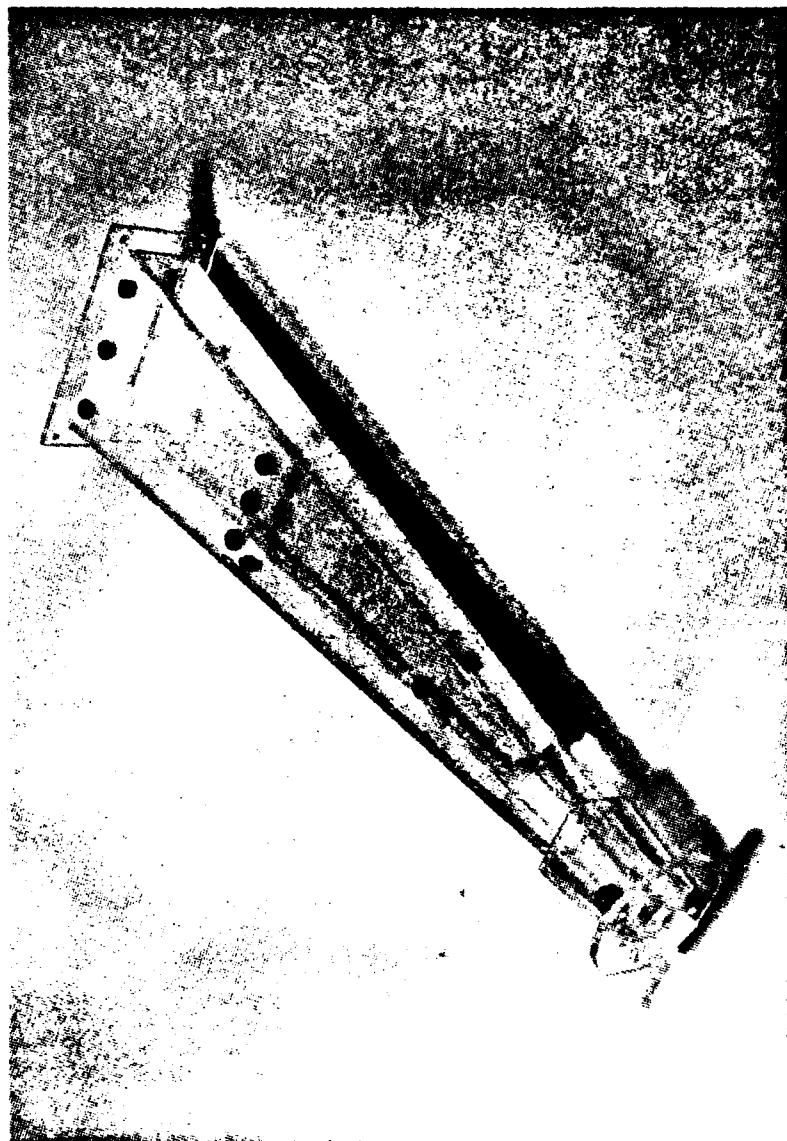
Figure 20. Rectangular diffuser photograph

gular exit dimensions of 8.25 inches in width and 2.75 inches in height to match the new settling chamber dimensions. In order to minimize separation effects in going from .625-inch (inside diameter) round pipe to a .75-inch square cross section, a special transition section was fabricated. The walls at the entrance of the diffuser were thickened with extra layers of Plexiglas and a round entrance hole of a diameter equal to the diagonal of the entrance was drilled. The interfaces where the round hole broke through the interior walls were hand-blended smooth. A conical transistion section from the .625-inch (inside diameter) inlet pipe to the rounded diffuser inlet was made and is shown in figure 21. A screen was incorporated to aid in maintaining flow distribution.

Tunnel Design

The tunnel is constructed of .375-inch thick Plexiglas walls and inverted aluminum channel sections forming the top and bottom of the tunnel. The interior dimensions are: height, .75 inches; width, 2.21 inches; and length, 24.25 inches. Ten access ports along the top of the tunnel allow insertion of boundary layer sampling probes into the tunnel flow. The five stations used in this experiment are shown in figure 19.

Figure 21. Conical diffuser inlet transition section photograph

Starting 1.75 inches from the entrance to the tunnel, an
anodized aluminum flat plate is fixed to the floor of the tunnel.
A one-half inch suction slot at the upstream edge of the flat plate
allows the incoming boundary layer to be bled off and a new bound-
ary to be started in the flat plate. An axially adjustable lead-
ing edge allowed the ejection slot height to be varied up to 0.050
inches in the horizontal, x, direction. Boundary layer wall
concentration sampling holes (.030 inch in diameter) were drilled
at locations corresponding to the probe sampling planes. The
polymer ejection slit was located 1.75 inches from the leading
edge and slanted at an angle of 15° to the flow.

Eight different leading edges, shown in figure 22, were tried
in an attempt to obtain transition in the center portion of the
plate. Of all the leading edges tried, the most effective one
(figure 23) consisted of an epoxied sand surface (average grain
size 0.020 inch) sloped at 7.5°.

Boundary Layer Probes

Boundary layer probe rakes were fabricated and are shown
assembled in figure 24 and as an exploded view in figure 25. The
mounting fixture has a slot machined in its side to allow the rake
to be retracted up into the roof of the tunnel out of the flow
passage. Figure 26 is a close-up of the probe tips showing their

Figure 22. Flat plate leading edge photograph

Figure 23. Sand roughened leading edge photograph
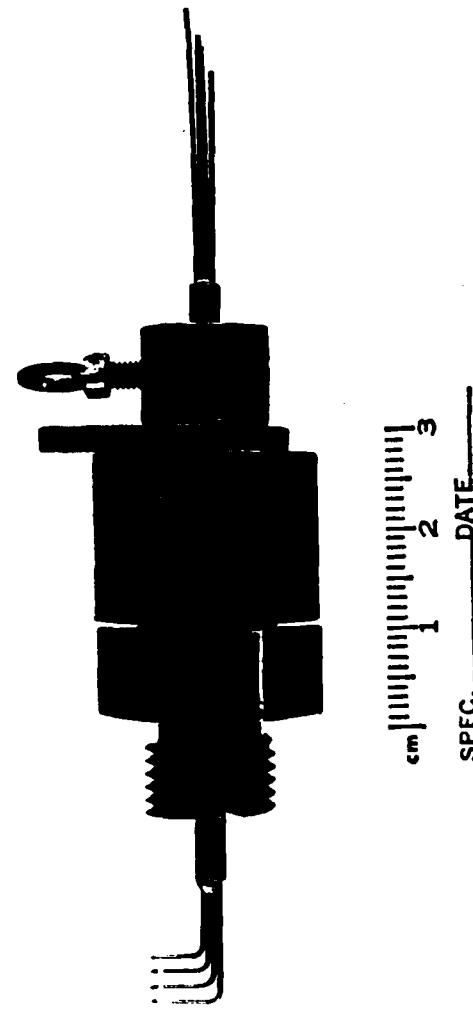
Figure 24. Boundary layer probe rake photograph
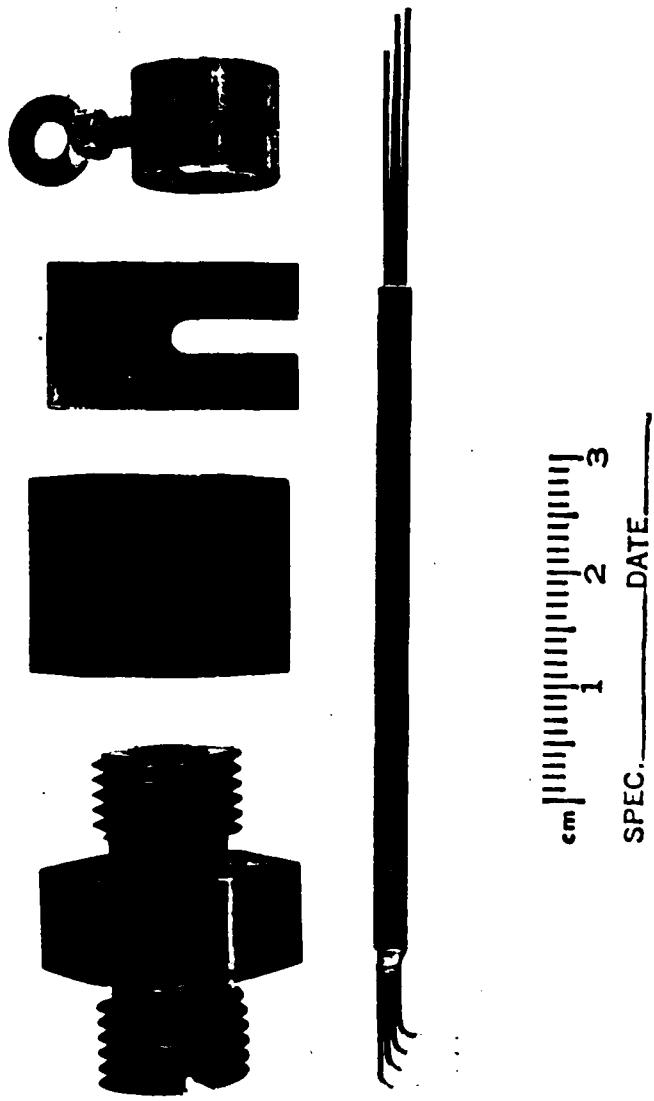
Figure 25. Disassembled boundary layer probe rake photograph
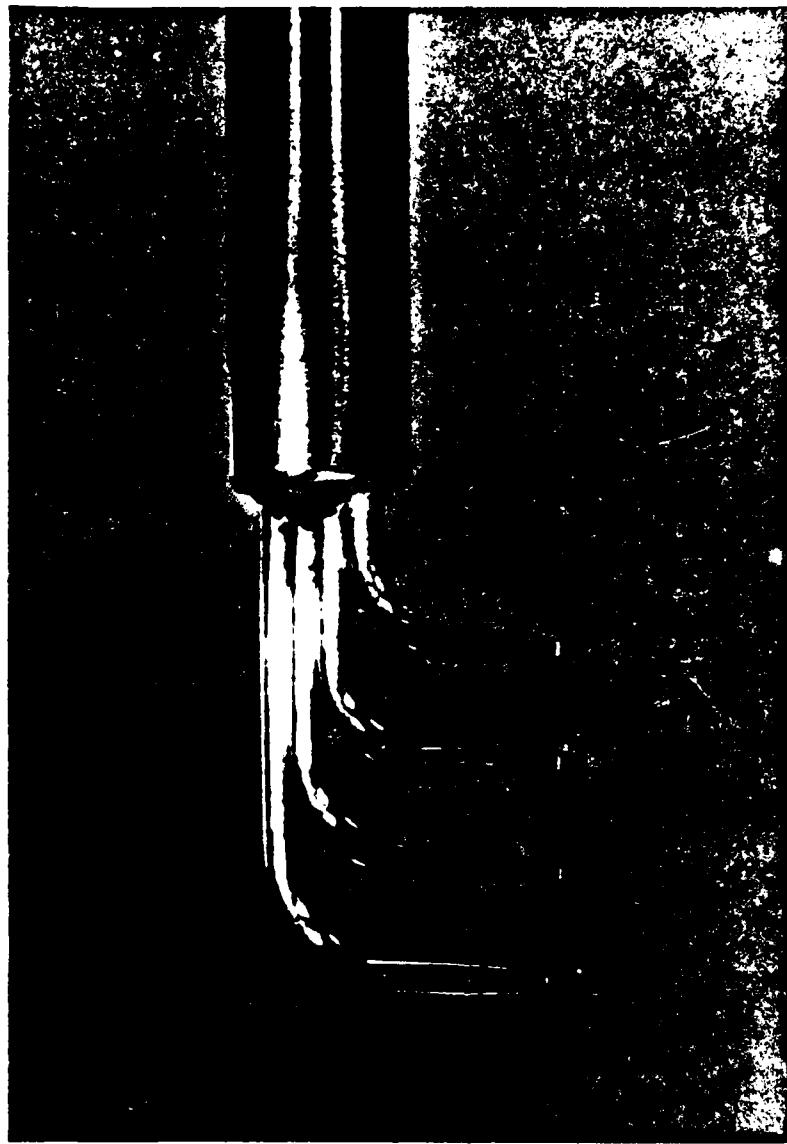
Figure 26. Boundary layer probe rake tip close-up photograph

tapered rectangular shape of .005 inch high by .015 inch wide.
Figure 27 presents probe dimensions and table 2 presents the
location of the sampling points with and without the spacers used
to reposition probes in the vertical, y, direction. The bound-
ary layer samples would pass through the probe and along a 36-inch
by .020-inch diameter flexible tube to a sampling stand containing
six cuvettes. Figures 28 and 29 show this arrangement with reser-
voirs in place of the cuvettes in the sampling stands.

Polymer Injection System

The polymer injection system was designed to minimize polymer
degradation during the delivery process. As shown in figure 6,
water is recirculated to a water reservoir. Part of the water
flow is bypassed through a water-calibrated rotometer to the poly-
mer reservoir. The polymer reservoir contains 13 liters of polymer
solution above a separation piston. The water flow pushes the
piston upwards. This action in turn pushes the polymer mixture
out of the reservoir into the delivery line to the tunnel. The
delivery line is coiled in the constant exit head container and
submerged in a bath of tunnel exit flow water to equalize the
polymer flow temperature with that of the main tunnel flow.
Figure 30 is a photograph of the polymer delivery system.

RECTANGULAR TIP

DIMENSION
IN INCHES

A = .020

B = .015

C = .010

D = .060

E = .500

F = .375

G = 2.375

H = .180

J = .036

K = .125

I.D OF
TUBE = .010
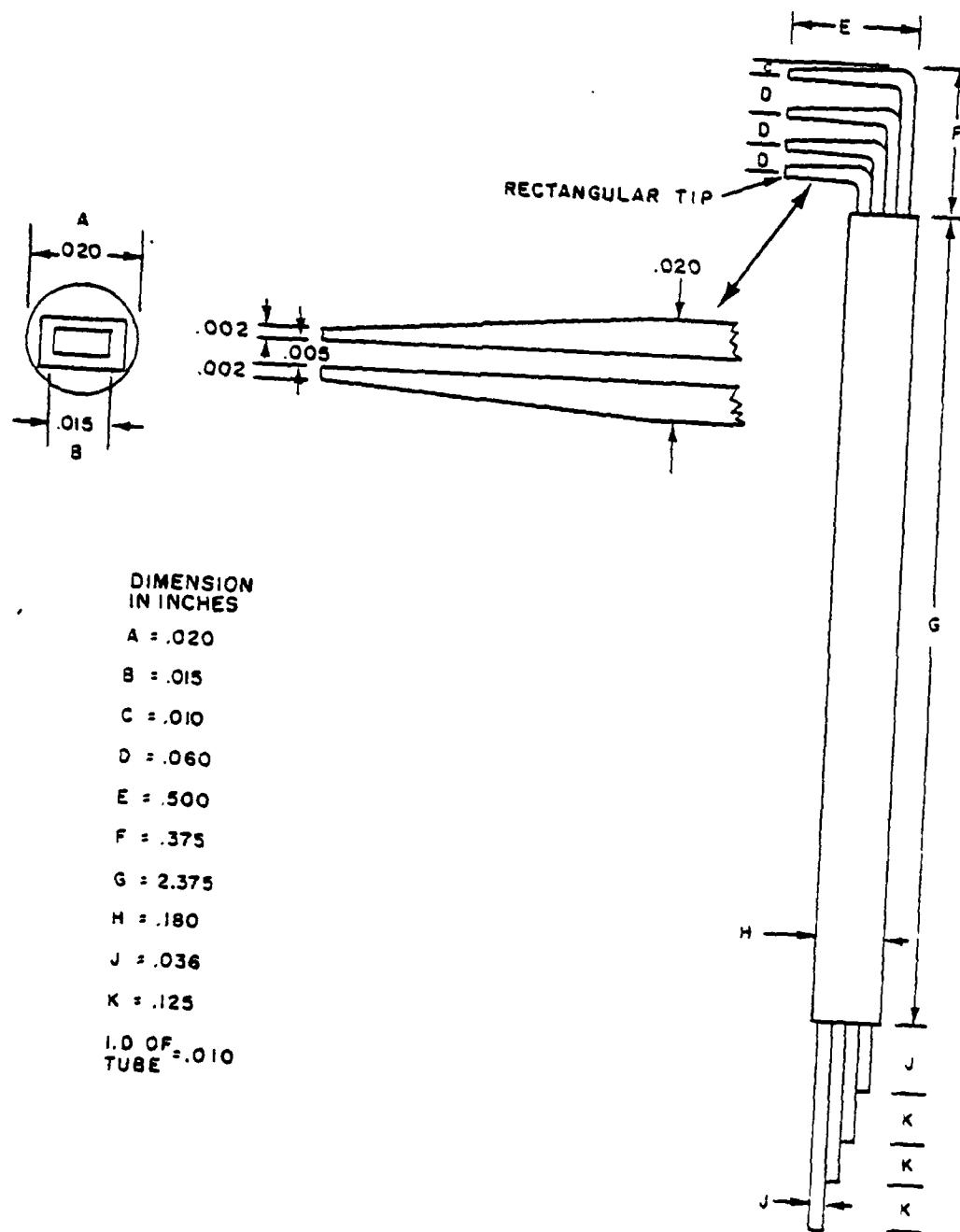
Figure 27.   Boundary layer probe rake dimensions

TABLE 2

CONCENTRATION SAMPLING LOCATIONS IN THE BOUNDARY LAYER

Probe Sample	Yc (Spacer .020-inch)	Yc (Spacer .103-inch)
0	0	0
1	.010	.010
2	.030*	.072
3	.072	.113*
4	.092*	.134
5	.134	.175
6	.154*	.196
7	.196	.237*
8	.216*	.299*

* Spacer in place

Figure 28. Boundary layer probe rake placement photograph
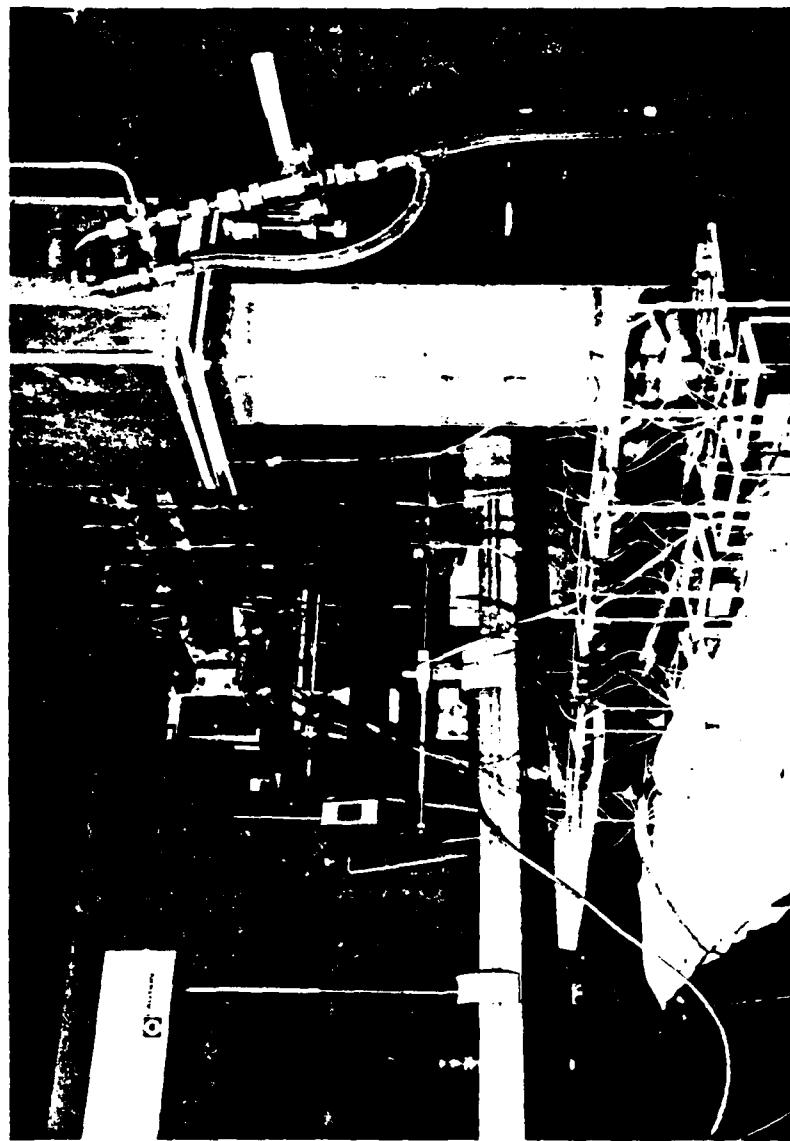
Figure 29. Boundary layer probe sampling system photograph
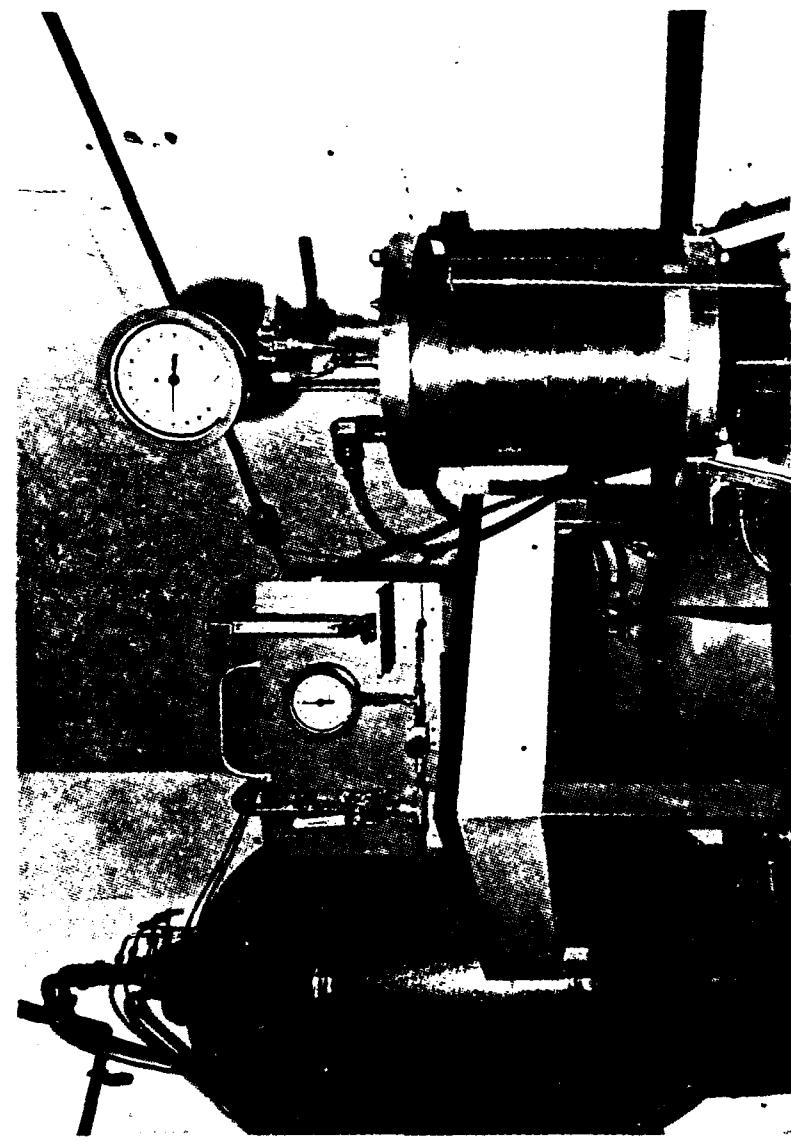
Figure 30. Polymer delivery system photograph

Boundary Layer Concentration Measurements

The methods used for the measurement of concentration distri-
butions of polymers in solution include: polarographic (investi-
gated by Goren (1965)); turbidity (investigated by Ruch et al.
(1965)); and opaqueness and fluorometric techniques. Wetzel and
Ripkin (1970) present a complete discussion of all but the
opaqueness method, which was used by Fruman and Tulin (1974).

Fruman and Tulin (1974) constructed a specially-built light
absorption system utilizing photocells and a recording oscillograph.
The intensity of the light source was chosen in order to achieve
the maximum possible output readings between zero dye and a dye
concentration corresponding to practically zero light transmission.
This dye concentration was chosen to be 1250 WPPM of India ink in
tap water. The voltage output of the photocell was 40.0 mV and
the equivalent spot displacement on the recorder was 12.0 cm.
When a continuously-running sample provided a steady state output,
the reading was compared to previous readings obtained during the
calibration of the system. The scatter in the data was attributed
to stability problems with the light intensity and galvanometer
of the recording oscillograph.

The method selected for use in this study is the fluorometric
method. This method has been used successfully by Wetzel and

Ripkin (1970), Walters and Wells (1971), Tullis and Ramu (1973),
and Sirmalis (1974). The method consists of injecting a tracer
dye into the fluid being analyzed and capturing a sample of the
fluid for analysis in a fluorometer.

Fluorescence, the basic phenomenon being measured, is the
instantaneous emission of light from a molecule or atom that has.
absorbed light. Consideration of the equation of fluorescence in
dilute solution shows that

$$F_m = KI_o \epsilon Cd\phi \qquad (42)$$

where

  F_m is the meter reading,

  K is the amplification factor of the photodetector,

  I_o is the intensity of the exciting light,

  ϵ is the extinction coefficient of the compound

   (a constant for any given wavelength of I_o),

  C is the concentration to be measured,

  d is the path length (a constant),

  ϕ is the quantum yield (a constant),

indicates that decreasing concentration, C, may be compensated by
increasing the amplification factor, K, or the light intensity, I_o.
This is done in fluorometry, yielding larger scale readings for
lower concentration. The fundamental principle of fluorescence

measurement is shown in figure 31, a simplified schematic

of a fluorometer taken from Turner (1972). The intensity of the

exciting light is controlled by a range selector having four

apertures or sensitivity ranges. The desired wavelength of

exciting light is selected by a primary monochromator or filter

placed between the light source and the sample. Emitted by the

sample, the wavelength of light to be measured is selected by

a second filter placed between the sample and a photodetector.

The output of the photodetector, a current proportional to the

intensity of the fluorescent light, is amplified to give a reading

on a meter or recorder.

In operation, a standard sample is placed in the fluorometer

and the sensitivity or intensity of exciting light adjusted to a

desired reading. Unknown samples and a blank are then read. The

net readings of the standard and unknown samples, with the blank

subtracted, are in the same proportion as their concentration.

Standard linear curves may be prepared and the concentrations of

samples read directly from the curve.

The basic assumption made in all polymer solution studies is

that the diffusion of the tracer dye is identical to that of the

boundary layer mixture. This is a reasonable assumption based

on data comparing various measurement methods by Wetzel and Ripkin
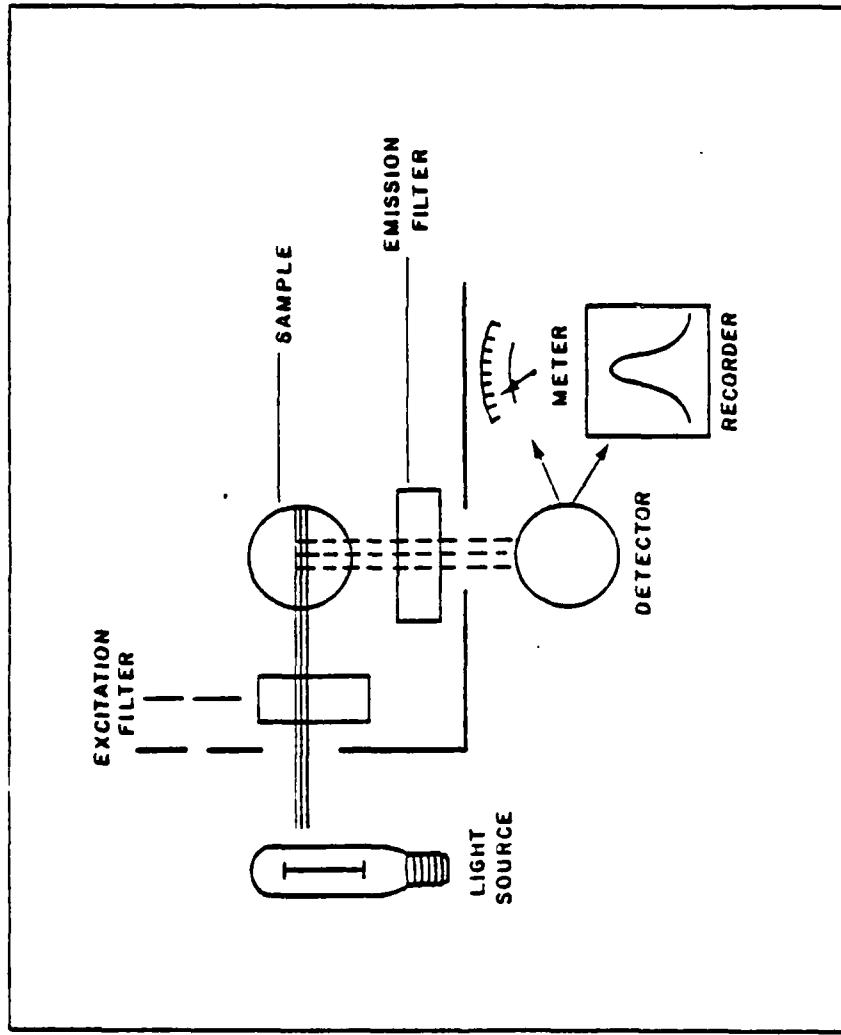
Figure 31. Fluorometer schematic

(1970). Tests of two widely used fluoroscent trace dyes, Rhodimine
and Uranine-B, found both to be compatible with Polyox. The
solutions were stable and their drag-reducing properties were
unaffected when tested in a capillary tube blowdown rheometer
similar to Hoyt (1966).

Selection of the fluorometric method for the present experi-
ment was based on the accuracy, ease, and rapidity in determining
dye concentrations reported by previous authors and the availabil-
ity of a G.K. Turner Model 111 Fluorometer, figure 32. The use of a
fluorometer of the type utilized by previous investigators should
minimize data correlation problems due to apparatus and technique
differences.

Although it is easy to use, the Rhodamine dye was rejected
because it is very difficult to clean off the apparatus. The
Uranine-B dye was selected, however, since it is readily adaptable
to laboratory use and gave a reproducible reading to 1 part in 10^9
on a weight basis. This range was adequate as the dye concentration
of the injected solution could be adjusted to produce samples
along the plate sufficiently above the background blank levels to
give reproducible results. Calibration curves were experimentally
developed for the fluorometer using known concentrations of
Uranine-B dye in the type water to be used in the experiment.
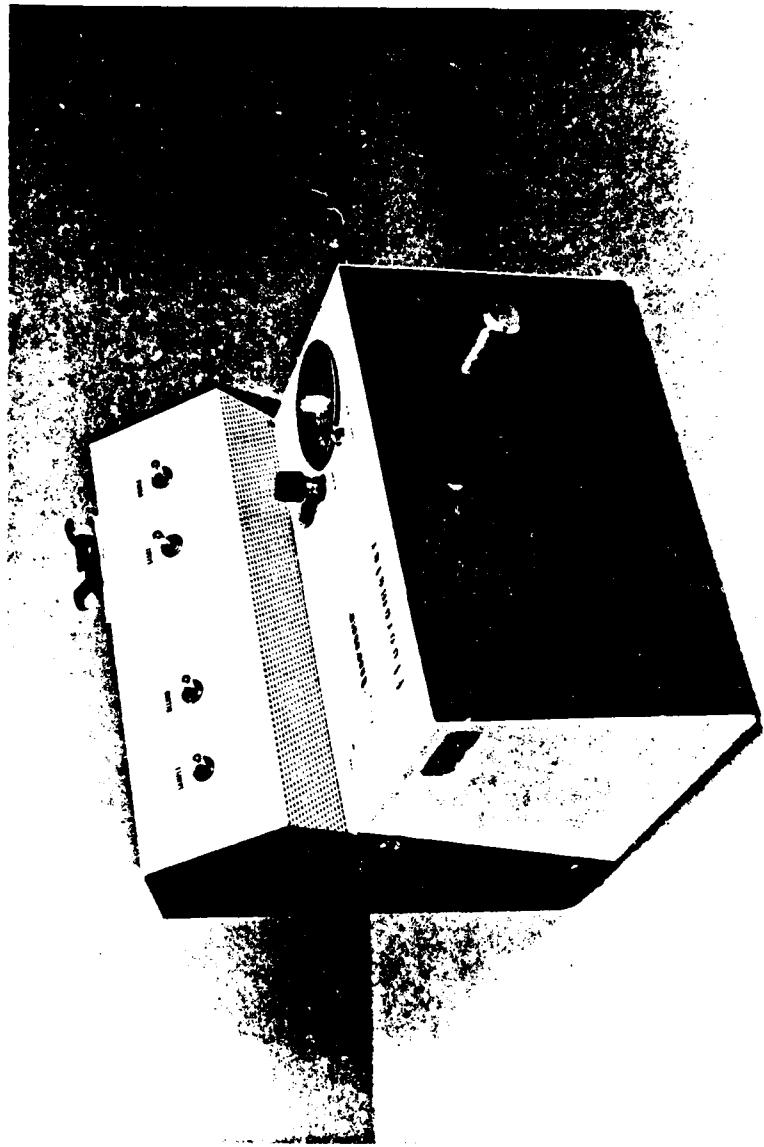
Figure 32. G.K. Turner model 111 fluorometer photograph

The G.K. Turner Model 111 Fluorometer has four sensitivity
aperture settings allowing calibration curves to be drawn over a
range of about 1 part per 10^6 to 1 part per 10^9 on a weight basis.
For concentrations greater than the lower value, controlled
dilution prior to measurement was performed. The calibration
curves used in this experiment are shown in figures 33 and 34.
Sample sizes of approximately 3.5 to 4.5 ml are required for
measurement.

The sample is placed in a small test tube called a cuvette
and inserted in the instrument. The measurement is automatically
taken when the door is closed. Cleanliness must be observed and
care taken to wipe the outside of the cuvette prior to measurement
as fingerprints affect the reproducibility of the sample readings.
Wiping the exterior of the cuvette with a clean paper towel was
found to be sufficient. All samples were allowed to achieve room
temperature prior to reading.

Table 3 provides a list of apparatus used in the concentra-
tion measurements.
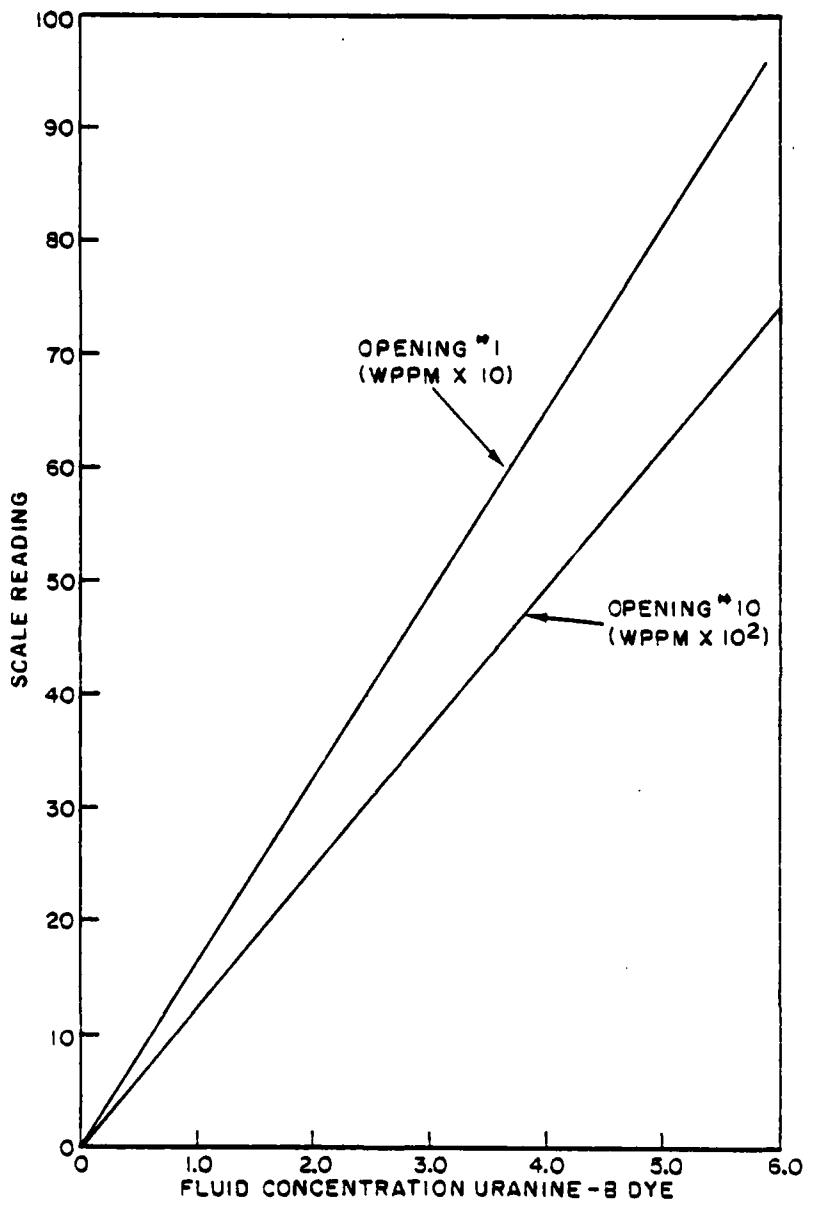
Figure 33. Calibration curves for model 111 fluorometer opening 1 and 10
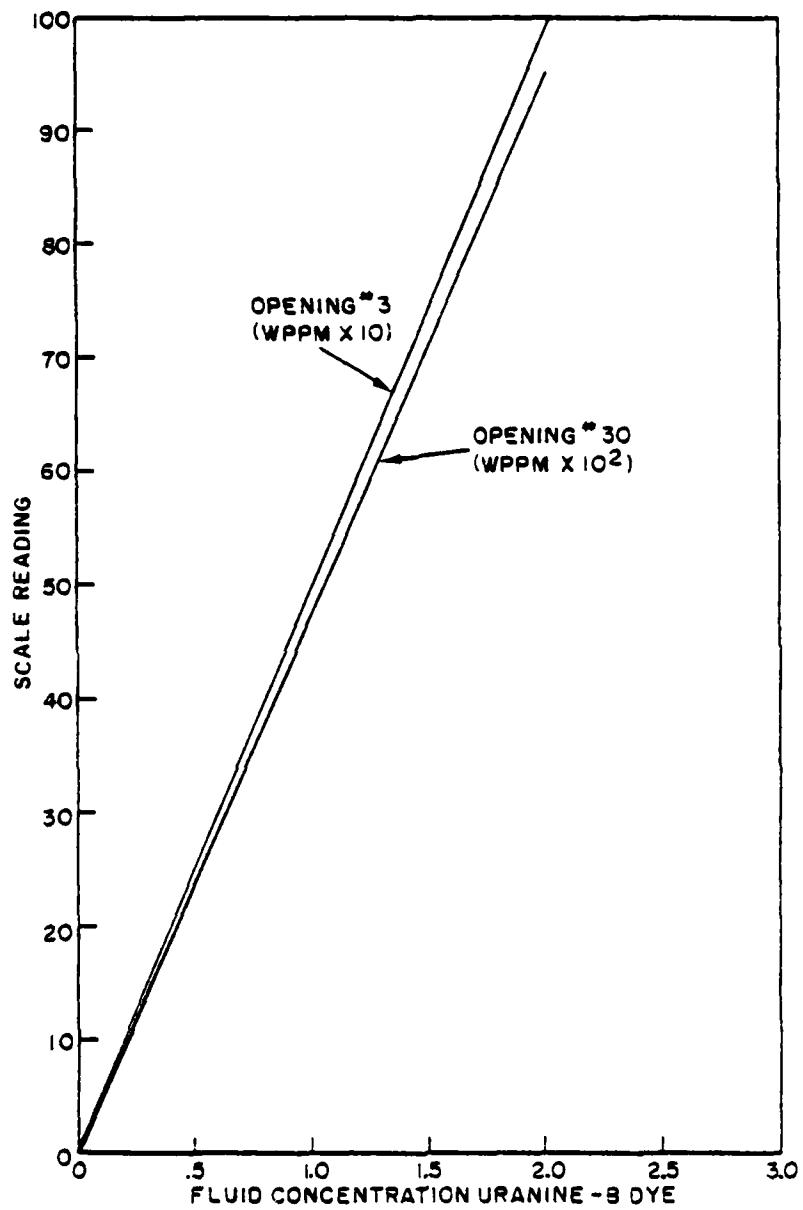
Figure 34.   Calibration curves for model 111 fluorometer
openings 3 and 30

TABLE 3

CONCENTRATION MEASUREMENT APPARATUS

Component	Specification
Fluorometer	G. K. Turner Model 111
Excitation lamp	G.E. # G4T4/1
Primary filter	2A (closet to lamp)
	47B
	Together these filters transmit only light at 435 nm wave length.
Secondary filter	2A-12
	This filter only passes light at wavelengths greater than 500 nm.
Cuvettes	Matched cuvettes
	Fisher Scientific No. 14-385-900A
Dye	Uranine-B
	Fluoresces at 470 nm and 515 nm

Laser Doppler Anemometer System

The velocity measurements in this experiment were taken with
a laser Doppler anemometer. (LDA will be used as short form
terminology. Operation of an LDA system is based on the prin-
ciple that coherent laser light scattered from a particle in a
moving fluid will be Doppler shifted in frequency. The basic
components of an LDA system, shown in figure 35, include a laser,
transmitting optics, receiving optics, photodetector, and signal
processing equipment. Table 4 presents a list of the laser
Doppler system components used in this experiment.

Among the advantages of LDA systems are the following:

1. The flow is not disturbed

2. No flow calibration is required

3. There is excellent spacial resolution due to small
   measuring volume

4. High frequency response occurs.

Constraints on the use of LDA systems require (1) an optically
clear medium, (2) a window to the flow (two windows are preferred),
and (3) particles in the flow. Because the LDA measures particle
velocity and not fluid velocity, there must be sufficient concen-
tration of particles of the proper size and density to provide the
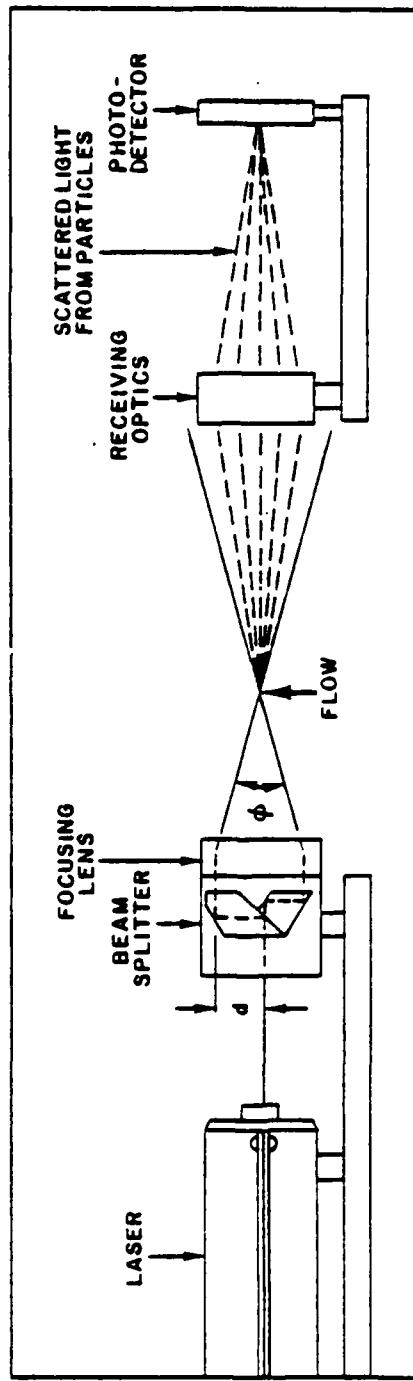
Figure 35. Basic components of a laser doppler anemometer system

TABLE 4

LASER DOPPLER ANEMOMETER SYSTEM COMPONENTS

Component	Specification
• Laser:	Spectra Physics model 120 Helium-Neon laser power - 5 milliwatts beam diameter - .8 millimeters (at $1/e^2$ point) wavelength - 632.8×10^{-9} meters
• Transmitting Optics:	TSI Model 910
Lens I:	Model 917 Focal length 103. mm Beam intersection angle 25.0°
Lens II:	Model 918 Focal length 241. mm Beam intersection angle 11.58°
Lens III:	Model 919 Focal length 598. mm Beam intersection angle 4.71°
• Beam separation:	50 mm
• Receiving Optics:	TSI Model 930
Lens I:	Model 937 Focal length 102. mm
Lens II:	Model 938 Focal length 242.2 mm
Lens III:	Model 939 Focal length 577.2 mm
• Photodetector:	TSI Photodiode model 960 Bandwidth 100 Mhz
• Aperture diameter:	0.25 dia mm
• Signal Processor:	TSI tracker model 1090
• Frequency response:	2kHz to 50 MHz

required signal while closely following turbulent fluctuations in the flow.

In general, LDA systems are characterized by the arrangement of their optical components as shown schematically in figure 36.

In the "forward scatter" arrangement, the receiving optics and photodetector are located opposite the laser to receive light scattered in the forward direction. This is the most favorable mode as the majority of incident light is scattered by the particles in the forward direction. A typical polar scattered light intensity distribution predicted by the Mie scattering theory for micron size particles is shown in figure 37. The scattered light intensity in the forward direction is typically of order 10^3 times greater than the backscattered light intensity. Requiring two windows on the flow, the forward scatter mode is ideal for water tunnel measurements. Lower laser power and particle concentrations may be used due to increased light intensity.

The "backscatter" arrangement has the receiving optics and photodetector located on the same side of the flow as the laser light source. This arrangement requires only one window on the flow. However, higher laser power, shorter focal lengths, increased particle concentration and/or size are required due to decreased light intensity scattered in the backwards direction.
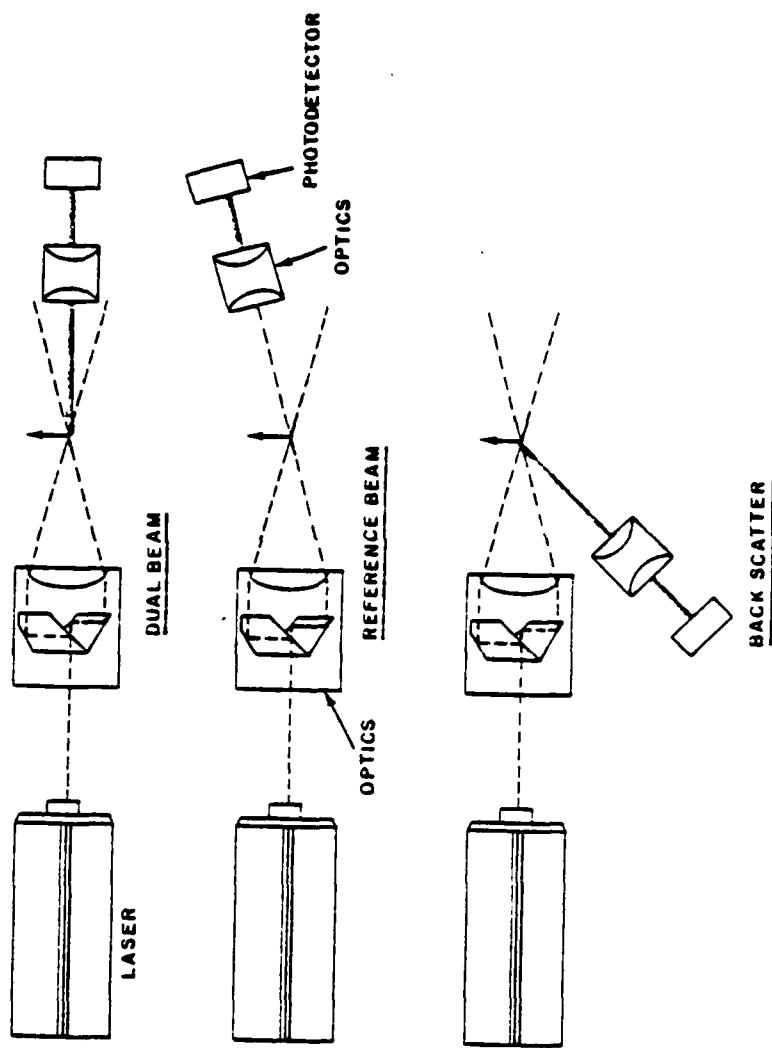
Figure 36. Optical configurations for laser doppler anemometer systems

DIRECTION
OF INCIDENT
LIGHT

10^3

10^2

10^1

10^0
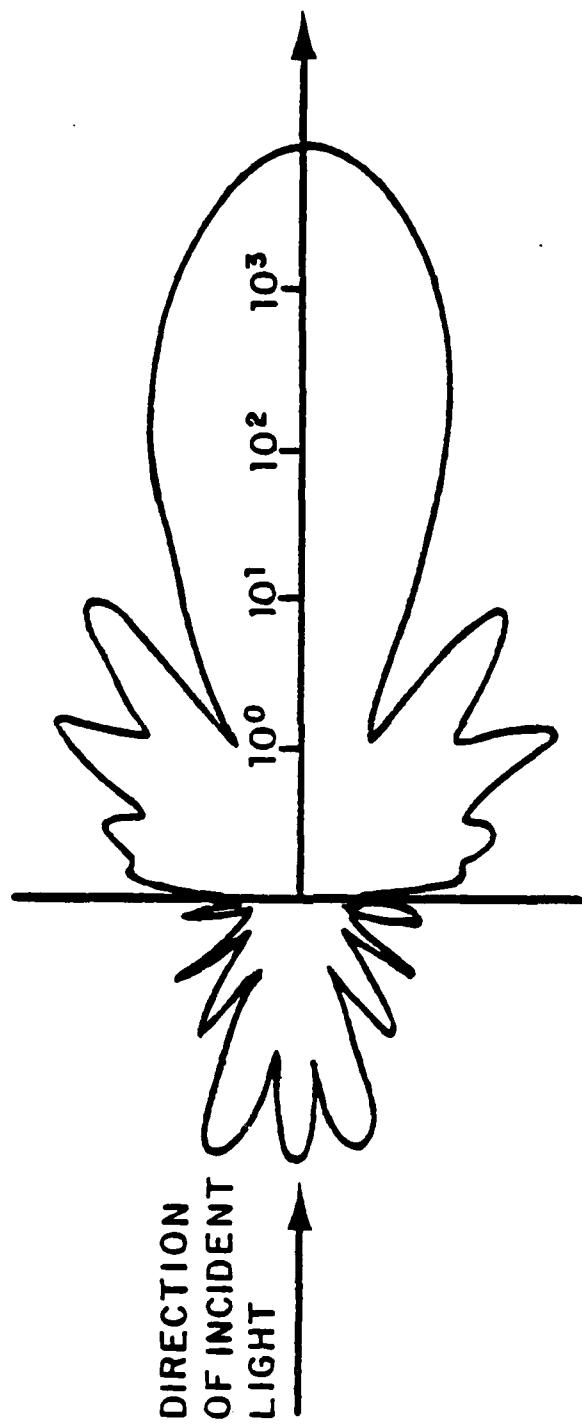
Figure 37. Polar scattered light intensity distribution

"Reference beam" mode refers to a single observed light beam, whereas the "dual beam" mode refers to the system where the light from two crossed beams is observed. Although the reference beam mode in forward scatter was developed first, the dual beam mode will be discussed in detail here as it is the actual mode used and the easiest to conceptualize.

In the dual beam mode, the laser beam is split into two beams of equal intensity, equidistant from the original beam. The beams are then focused at a crossover point where they interfere with each other to form a fringe pattern, shown schematically in figure 38. These fringes are caused by the interaction of the light in the two beams cancelling and reinforcing each other to form a standing electromagnetic interference pattern. The crossover region where the two beams are focused is called the measuring volume. Figure 39 is an actual photograph of a plane of this measuring volume magnified 600 times, containing over 100 fringes. A particle moving through the measuring volume in the plane of the two beams then passes through alternating regions of low light intensity (light cancelling) and regions of high light intensity (light reinforcing).

Light scattered from particles passing through the measuring volume is focused by the receiving optics onto a photodetector
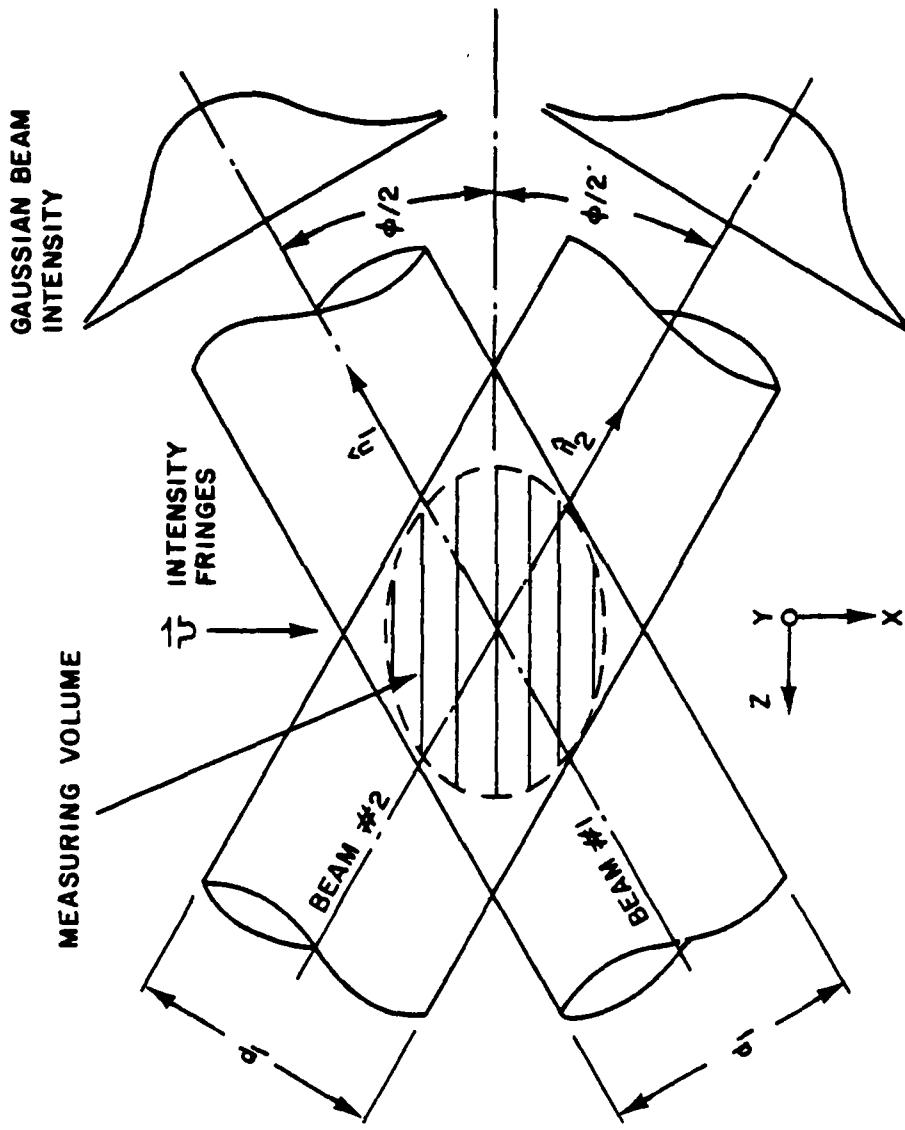
Figure 38. Fringe pattern schematic
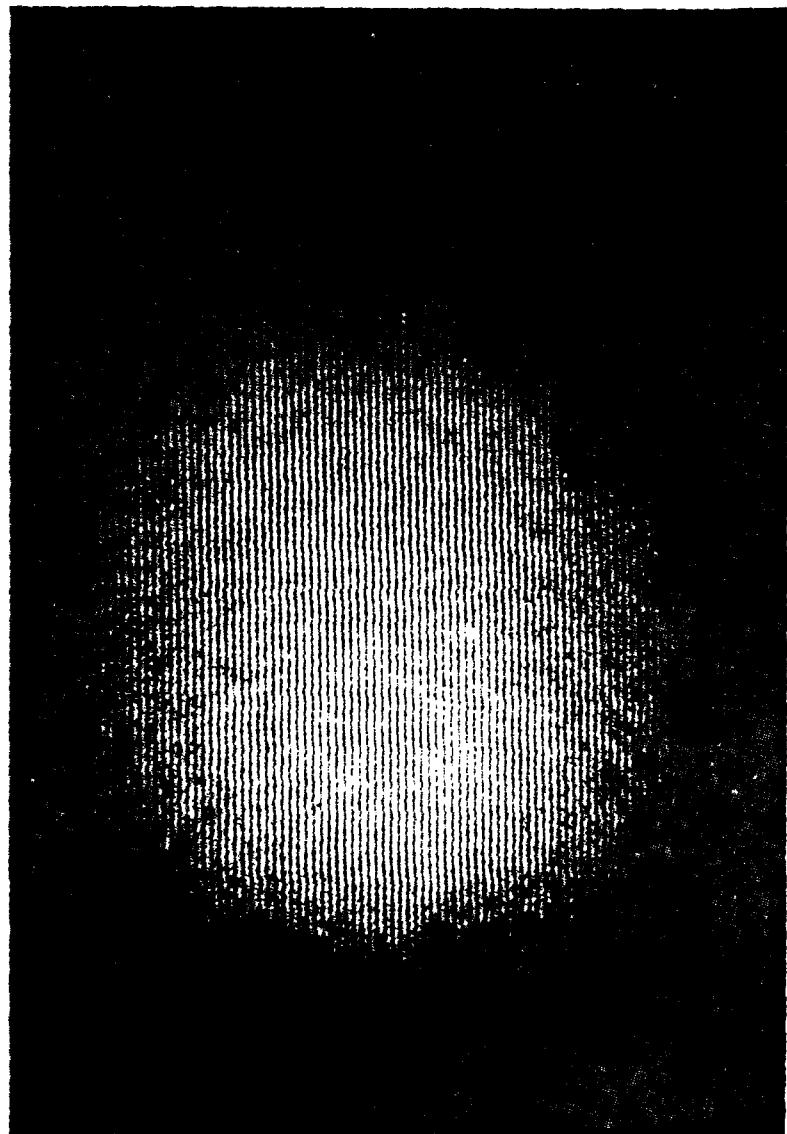
Figure 39. Fringe pattern photograph

whose output signal is shown schematically in figure 40. An
aperture is placed in front of the photodetector, blocking light
from locations other than those from the beam crossover region.
Due to the Gaussian intensity distribution, the amplitude will
vary with a maximum occurring in the center of the measuring volume.
The frequency, f_D, of this signal, produced by the scattered light
pulses from a particle travelling through the measuring volume,
is linearly proportional to the velocity with which the particle
is moving across the fringes (figure 41). This frequency is de-
termined by the frequency tracker of the LDA system. The distance
between fringes, d_f, is a function of the laser wavelength and the
angle of beam intersection only, such that

$$d_f = \frac{\lambda}{2 \sin(\emptyset/2)}.$$ (43)

The velocity of the particle may then be given by

$$U = f_D \cdot d_f.$$ (44)

Conversion of frequency to velocity requires only that the
laser wavelength and angle of beam intersection be known. A flow
direction ambiguity of 180° does exist, however, in that the fringe
system cannot distinguish reverse flow from forward flow. The
problem may be resolved for reversing flow by frequency shifting.
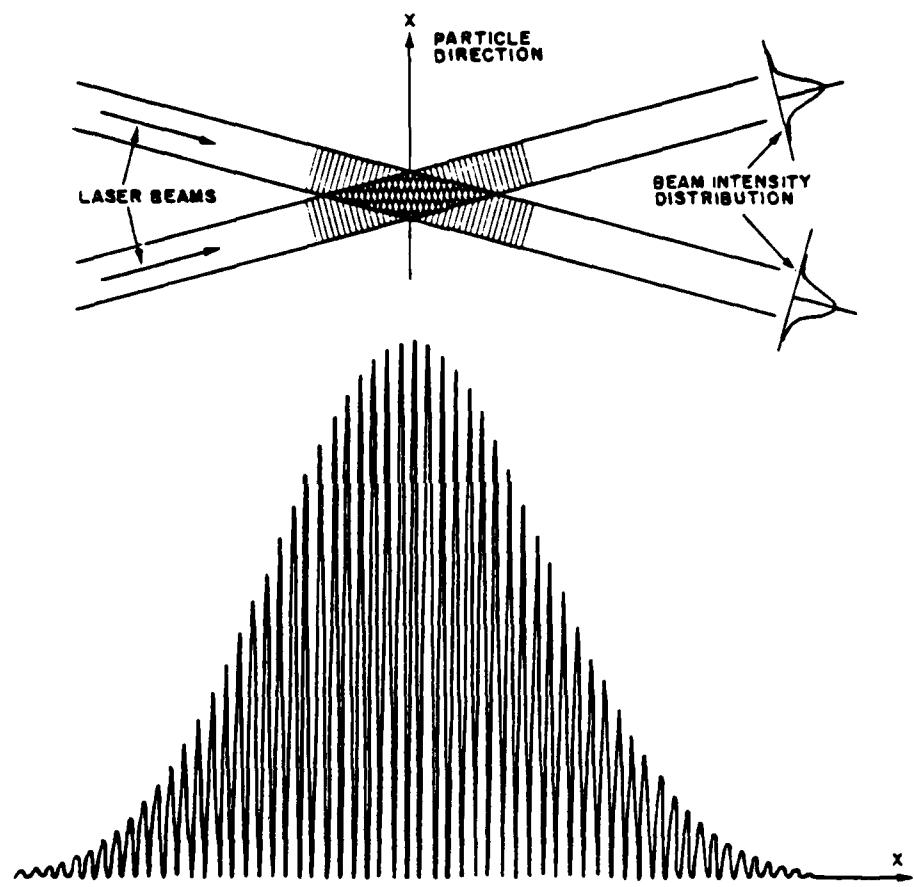
Figure 40. Photodetector output schematic

FRINGE PATTERN

$$d_f = \frac{\lambda}{2\sin\phi/2}$$

$$V = f d_f$$

λ = WAVE LENGTH OF INCIDENT LIGHT

f = FREQUENCY DETECTED AT PHOTODETECTOR

V = VELOCITY OF FLOW PERPENDICULAR TO FRINGES

d_f = FRINGE SPACING

ϕ = BEAM INTERSECTION ANGLE
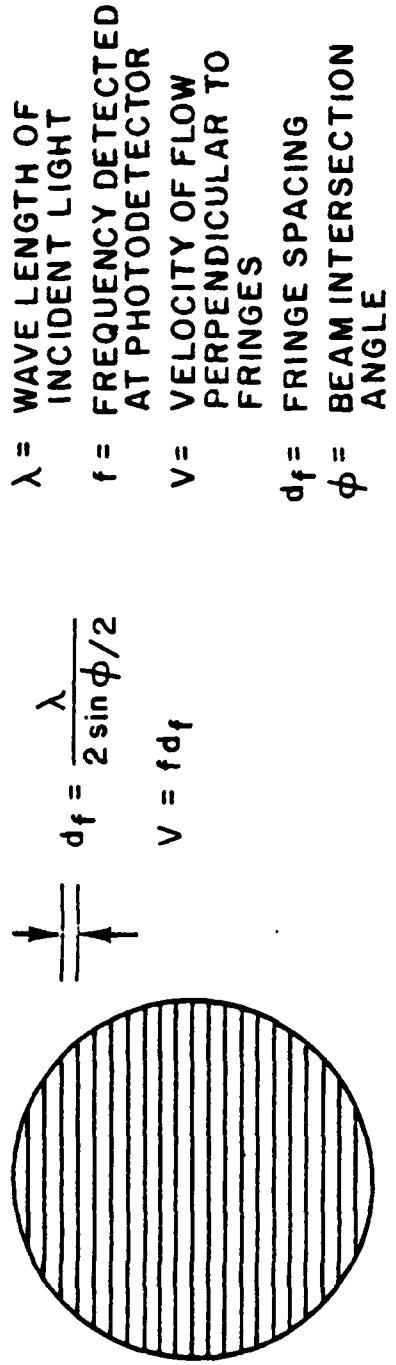
Figure 41. Scattered light frequency-velocity relationship

The fringes may be considered to be moving if one beam is frequency shifted relative to the other. If the difference in frequency between the two beams is 40 MHz, then a stationary particle in the measuring volume will yield a 40-MHz signal from the photodetector. A particle travelling in the same direction as the fringes will produce a lower frequency while a particle moving in the opposite direction will raise the frequency. Frequency shifting provides a zero offset or bias allowing bi-directional measurements. Techniques for frequency shifting include Bragg cells (40 MHz-shift) and rotating diffraction gratings (a few MHz shift).

The region of the beam crossover point called the measuring volume may be considered shaped as shown in figure 42. The diameter of the measuring volume, d_m, and the length of measuring volume, l_m, are expressed in terms of the diameter of the laser after focusing, d_ℓ, and the beam intersection angle $\emptyset$ as follows:

$$d_\ell = \frac{4\lambda F}{\pi\, D_e-2} \, , \tag{45}$$

$$d_m = \frac{d_\ell}{\sin \emptyset/2} \, , \tag{46}$$

$$l_m = \frac{d_\ell}{\cos \emptyset/2} \, . \tag{47}$$
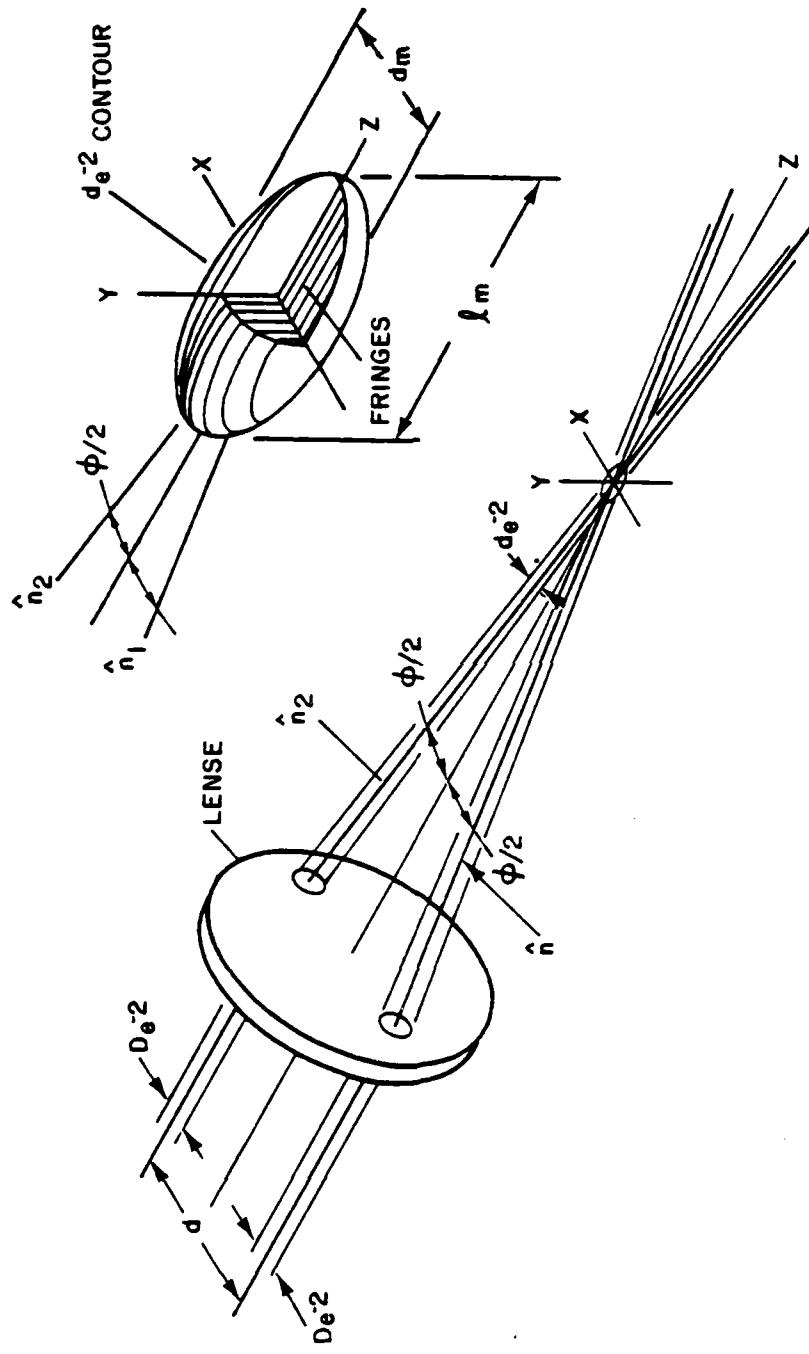
Figure 42. Measuring volume schematic

The volume of the measuring volume is given by equation (48)

$$V_m = \pi(D_e^{-2})^3 /6 \cos(\emptyset/2) \sin(\emptyset/2). \tag{48}$$

The laser beam diameter, D_e^{-2}, is defined as that region where the light intensity is greater than $1/_e 2$ (about 14%) of the center-line beam intensity. Table 5 shows typical values of these terms.

TABLE 5

MEASURING VOLUME PARAMETERS

Parameter	Values		
F (mm)	103	241	598
$\emptyset$ (^{o})	25.0	11.58	4.71
D_e-2 (mm)	.8	.8	.8
d_ℓ (mm)	.104	.243	.602
d_m (mm)	.106	.244	.603
l_m (mm)	.479	2.406	14.657
d_f (mm)	1.462×10^{-3}	3.136×10^{-3}	7.700×10^{-3}
N_{FR}	73.0	77.0	78.0
V_m (cm^3)	2.766×10^{-6}	74.600×10^{-6}	$2,785.900 \times 10^{-6}$

The position of the measuring volume, when making measurements in a liquid flow, will be different as compared to that in

air flow. Figure 43 shows the location of the measuring volume
relative to the wall in terms of the parameters of Snell's law
equation, such that

$$\lambda_1 n_1 = \lambda_2 n_2 \ . \tag{49}$$

The position of the measuring volume may be determined with
the use of equations (50) and (51) as follows:

$$n_1 \sin \frac{\phi_1}{2} = n_2 \sin \frac{\phi_2}{2} \ , \tag{50}$$

$$a = \frac{b}{2} \cot \frac{\phi_2}{2} \ . \tag{51}$$

Photodetector

The quantum efficiency, a measure of the photodetector's
efficiency to convert light energy to electrical current, is
typically higher for photodiodes than for photomultiplier tubes.
However, photomultipler tubes make up for this lack of efficiency
with internal gains of from 10^4 to 10^7. In general, the selection
of a photodetector depends on signal intensity, with low scattered
light levels requiring photomultiplier tubes; high light levels
require photodetectors. The photomultiplier also has higher
frequency response than photodiodes making them more applicable
to frequency shifted systems.

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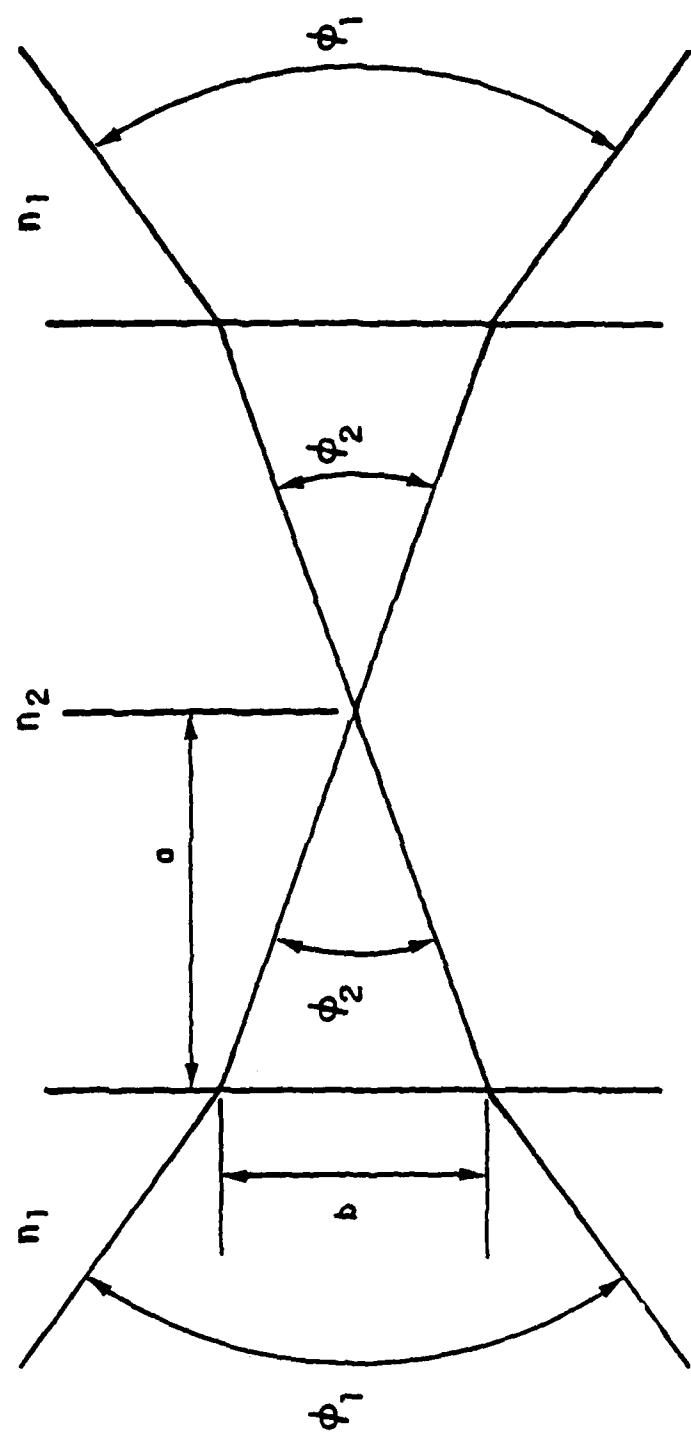
Figure 43. Location of measuring volume

Signal Processor

The signal processor used is of the tracker type. Basically, it consists of a combination of tracking filter and frequency-to-voltage converter, with provision for signal validation and retention of the last valid reading when no signal is present. The fundamental limitation of a tracker is the capture range. Should the change in velocity between one particle and the next exceed the capture range, the unit will not track the new value and any succeeding values that remain outside the capture range. However, the tracker system will return the last valid reading until a new reading occurs within the captive bandwidth. Table 6 indicates the range of tracking frequencies available with this system.

TABLE 6

TRACKING FREQUENCIES

Frequency Range	Tracking Range (MHz)	Output (FS=10V DC)	Capture Window
High frequency	0.5 - 50	0.2V/MHz	5 MHZ
Mid frequency	0.02 - 5	2.0V/MHz	0.5 MHz
Low frequency	0.002 - 0.5	20V/MHz	0.05 MHz

Particles

Particles in the flow form the basic requirement for the application of LDA techniques. The relative relationships between mode of operation and direction of maximum scattered light intensity have been discussed previously. The types of particles and their concentration are also related to the mode of LDA operation as well as the fluid medium to be measured.

For gas flows, the diameter of particles should be of the order of 0.1 to 1 micron while liquid flows generally required 1 to 10 micron particles. Excellent results have also been attained with natural contaminants in water flows. If possible the water used should be filtered to remove large particles providing a uniform dispersal of contaminants and lessening the chance of large particles saturating the photodetector output.

Melling (1971) has shown that 1-micron sized particles are able to follow flow fluctuation of up to 10 kHz in air with an accuracy of 1.0%. Table 7 presents a list of types of seeding and the medium in which they are used.

TABLE 7

TYPES OF SEEDING

Medium	Seed
Water	Polystyrene beads
	Milk, cream powder
	Natural pigment
Hydraulic oil	Paint pigment
Air	Ammonium chloride
	Magnesium oxide
	Silicon oil droplets
	Smoke
	Water vapor
	Dioctyl phthalate
	Aluminum oxide
Burning gas	Silicon dioxide
	Magnesium oxide
	Silicon oil droplets
	Dibutyl phthalate

Laser Power

A determination of laser power required to attain a desired signal-to-noise ratio (SNR) requires precise information on the configuration, characteristics of the fluid, properties, and concentration of the scattering centers as well as characteristics of the photodetector. This selection is quite complex and beyond the scope of this work. However, the following general remarks by Fingerson (1976) may be used as a general laser selection guide.

In forward scatter with focal lengths up to 250 mm for subsonic flows containing scattering centers above 0.1 micron in size, a 5-mW laser is adequate and generally of the He-Ne type.

In the backscatter mode with short focal distances of 120 mm or less and favorable conditions, a 15-mV laser is the minimum. For long focal lengths, especially in backscatter, 1-W lasers of the Argon-Ion type are generally used. For these applications, a variable power adjustment feature is desirable in matching laser power to actual conditions.

In general a 5-mW or 15-mW He-Ne laser is adequate for most laboratory experiments particularly where forward scatter can be used. For increased focal length, increased velocity, backscatter

or dual component velocity measurements, a larger laser such as Argon-Ion is desirable.

An oscilloscope photograph of the photodiode output due to the reflected light from a single particle travelling through the measuring volume is shown in figure 4. The total Doppler burst consists of approximately 100 fringes indicating a particle travelling close to the center of the measuring volume. Figure 45 represents good particle distribution, providing nearly continuous signal output due to sequential occupancy of the measuring volume by individual particles. Multiple particles in the measuring volume at the same time are to be avoided. This type of output is shown in figure 46. Although providing nearly continuous signal, the relatively high background noise level, caused by varying phase and amplitude, affects the accuracy of the apparent frequency measurement. The accepted term for this is Doppler ambiguity. A count rate distribution of 2.7×10^3 particles/ second corresponding to figure 45 is shown in figure 47. Each spike contains a Doppler burst envelope for a single particle.

Errors in Laser Doppler Average

Edwards et al. (1971), and George and Lumley (1973) treat laser anemometry measurement errors in detail. Limitations of a theoretical nature include:
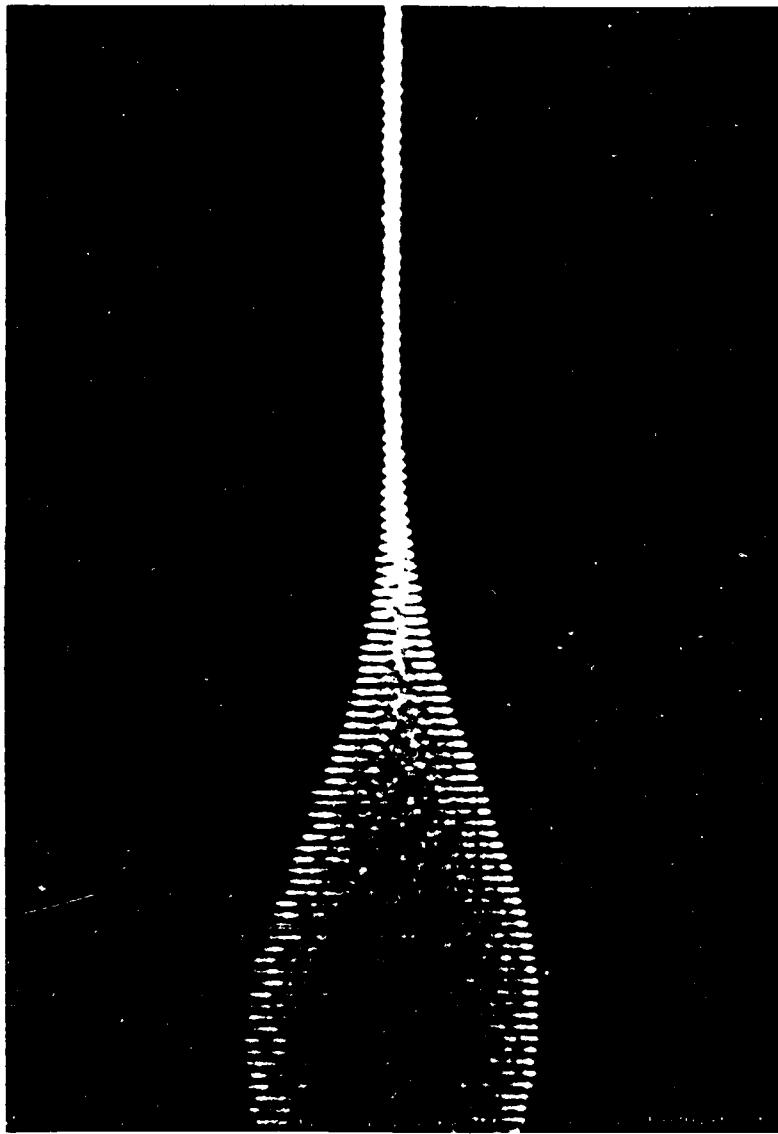
Figure 44. Doppler burst from single particle photograph
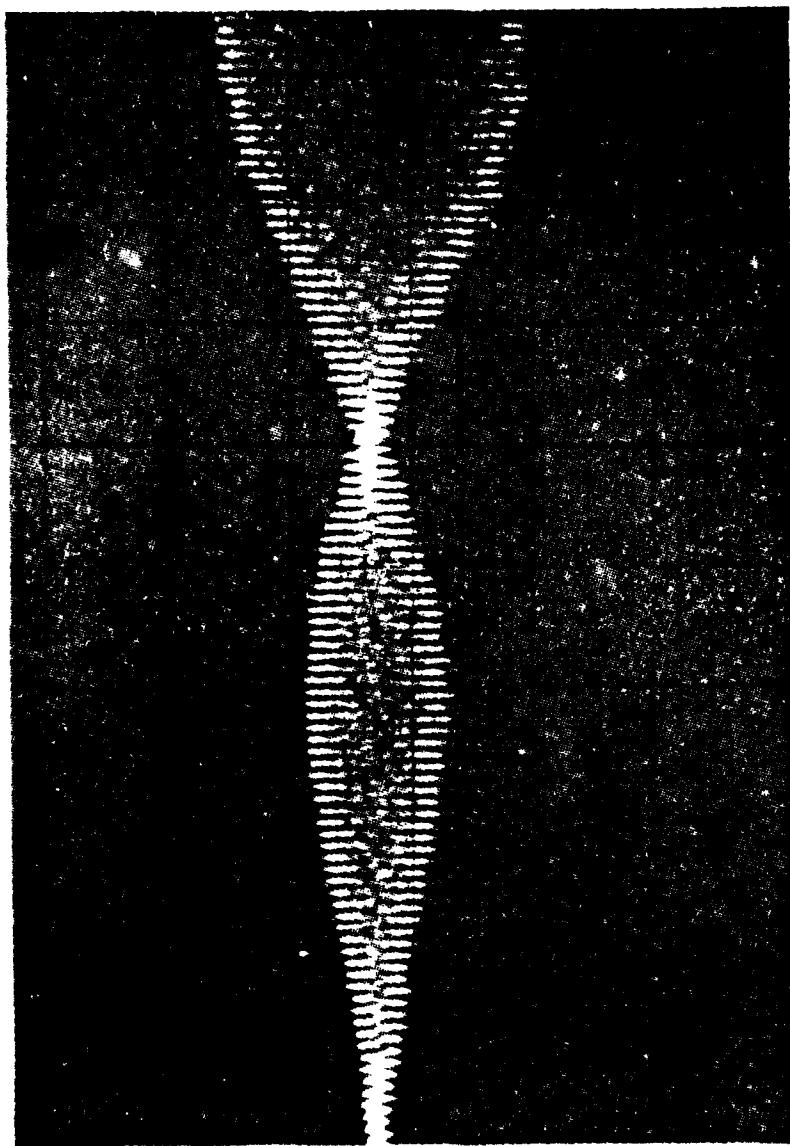
Figure 45. Doppler burst from good particle distribution photograph
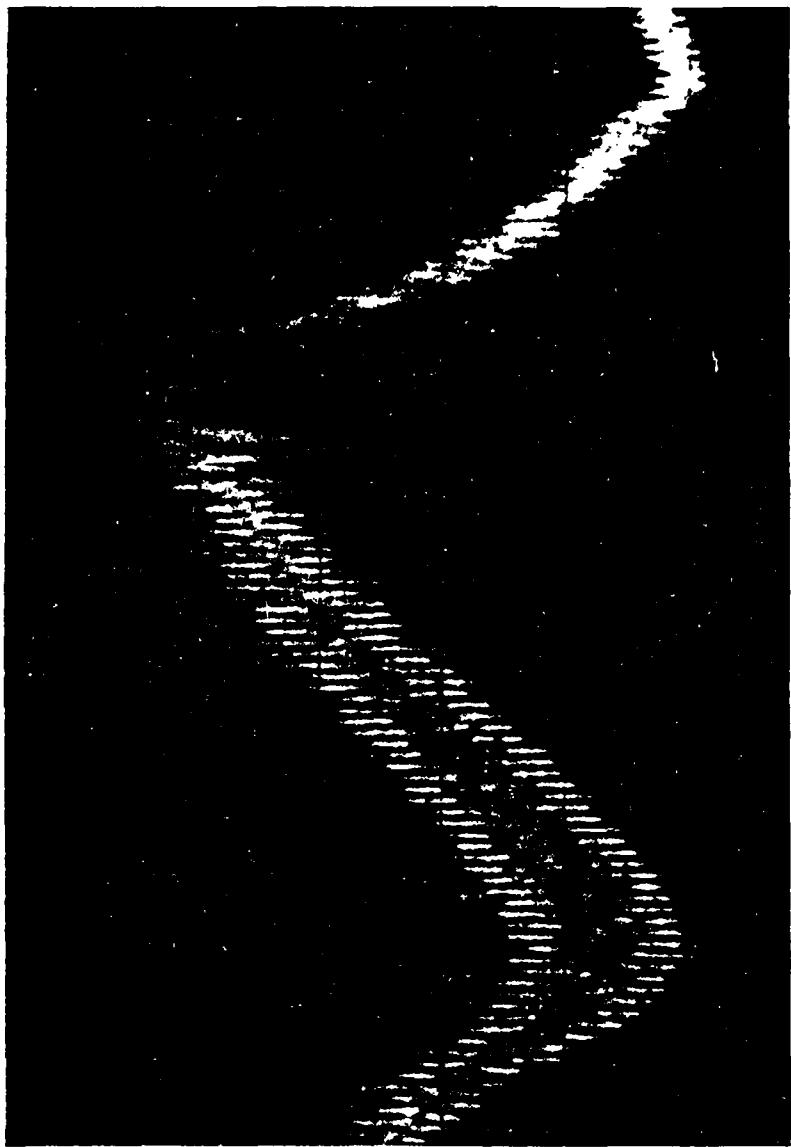
Figure 46. Doppler bursts due to multiple particles in measuring volume photograph
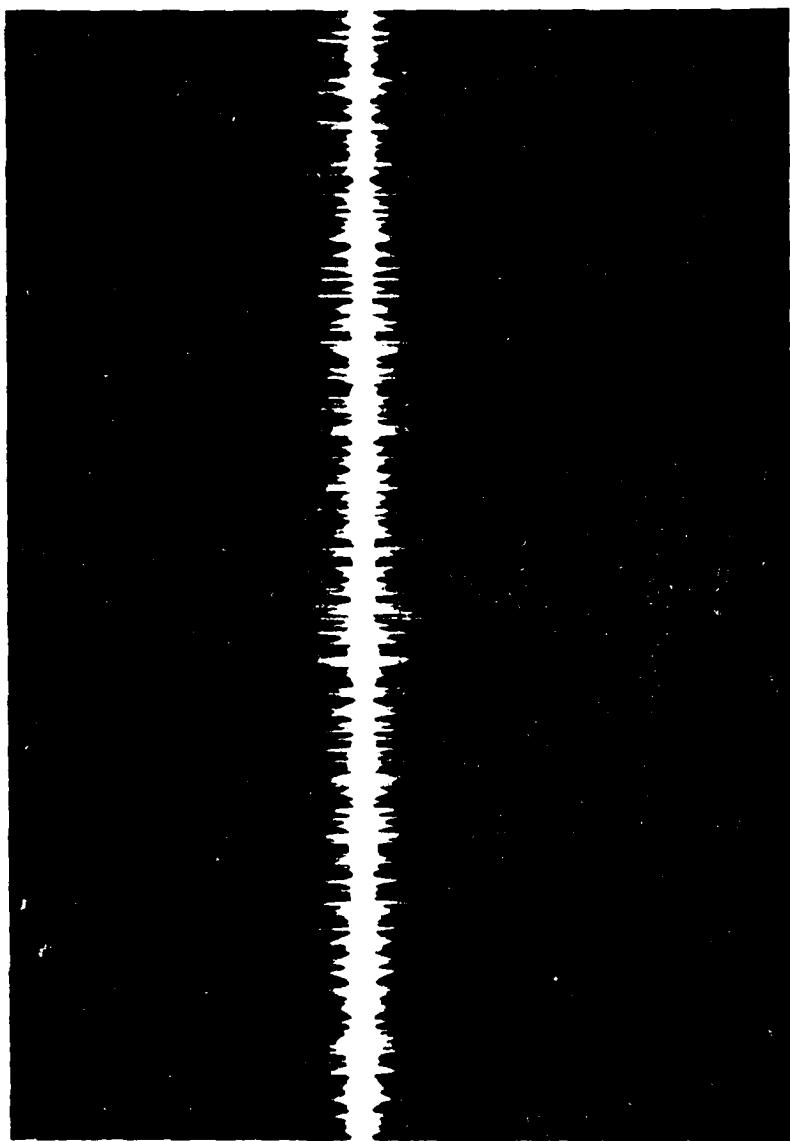
Figure 47. Count rate distribution photograph

1. Velocity fluctuations in the measuring volume

2. Mean velocity gradients across the volume

3. Effect of the finite transit time of the particles through the volume.

The finite measuring volume of the laser anemometry has an effect on mean flow measurements in steep velocity gradients. Turbulent particle movement within the measuring volume can also influence resulting turbulence data. Since the Doppler signal from a single particle has a beginning and an end, higher frequency variations are generated appearing as spectral broadening (even in laminar flow). This spectral broadening gives a minimum turbulence intensity which can be measured.

Doppler ambiguity caused by phase and amplitude modulation of the Doppler signal is a source of inherent noise. This noise is not a problem at low particle concentrations but becomes significant where the probability of multiple particle in the measuring volume is high. However, interpretation of spectral broadening and its effects on signal processor remains controversial.

Even under the most ideal conditions, there are certain errors associated with laser Doppler anemometry. These errors are frequently called "broadening errors" since, in the frequency

domain, they would cause the frequency to be broadened into a distribution with a finite width. In a tracker, this broadening translates into a voltage fluctuation in the output. The major sources of broadening are

1. σ_F - finite transit time broadening
2. σ_T - small scale turbulence fluctuations
3. σ_G - mean velocity gradient broadening
4. σ_I - instrument broadening

If it is assumed that each of the broadening components has a Gaussian distribution, the output variance may be written as

$$\left(\frac{\bar{e}}{E^2}\right)^2_b = \left(\frac{\bar{e}}{E^2}\right)^2_F + \left(\frac{\bar{e}}{E^2}\right)^2_T + \left(\frac{\bar{e}}{E^2}\right)^2_G + \left(\frac{\bar{e}}{E^2}\right)^2_I. \tag{52}$$

If a measurement in a laminar flow with no velocity gradient is considered, then the broadening reduces to

$$\left(\frac{\bar{e}}{E^2}\right)^2_{b,\ laminar} = \left(\frac{\bar{e}}{E^2}\right)^2_F + \left(\frac{\bar{e}}{E^2}\right)^2_I. \tag{53}$$

The second term on the right hand side, the instrument broadening, may be determined by entering a constant amplitude, constant frequency, voltage fluctuations as the tracker input, and observing the corresponding voltage fluctuation at the output. This gives

the level of the instrument broadening. Comparison with the
results of the laminar flow measurement allow the finite transit
time broadening to be determined.

The transit time broadening, σ_F, comes about as a result of
the signal from any one particle existing for only a finite time.
If there is more than one particle in the control volume at a
given time, then there will be voltage and phase variations due to
"old" particles leaving the control volume and "new" particles
entering it. The finite transit time broadening is a strong
function of control volume geometry and many discussions of its
cause and effects may be found in the bibliography.

An estimate of the velocity gradient broadening is given by
Durst (1976) as

$$\frac{\sigma_g^2}{W_D^2} \simeq \frac{\sigma_3^2}{U_0^2} \left(\frac{dU}{dx_3}\right)_0^2 \tag{54}$$

where the velocity in the vicinity of $X_3 = K_{30}$ has been approx-
imated by

$$U = U_0 + (X_3 - X_{30})\left(\frac{dU}{dx_3}\right)_0 + \frac{\left(X_3 - X_{30}\right)^2}{2}\left(\frac{d^2U}{dx_3}\right)_0 + \ldots, \tag{55}$$

and $\sigma_3 \equiv$ scattering volume dimension in the direction of the
velocity gradient, and $W_D \equiv$ Doppler frequency.

The previous approximation shows the desirability of keeping
the smallest dimension of the control volume in the direction
of the velocity gradient in order to minimize gradient effects.

The final source of broadening is usually the parameter of
interest. In some measurements the turbulence variation has been
taken as the difference between the variance measured in turbulent
flow and that measured in laminar flow under the same conditions.

Data Processing

A modest real-time computer data system was used in con-
junction with the laser Doppler anemometer. The data system
components shown in figure 48 are listed in table 8.

TABLE 8

DATA SYSTEM COMPONENTS

Laser Doppler Anemometer	TSI Model 1090-1
Display Keyboard	Wang Model 2220
Central Processing Unit (CPU)	Wang Model 2200-72
Plotter	Wang Model 2272-2
Printer	Wang Model 2231W
Data Interface	NUSC

The data system provides a printout of seven variables and
functions of variables, a plot of normalized velocity ratio versus
position, and a cassette recording of four data functions. Table
9 lists printer output.
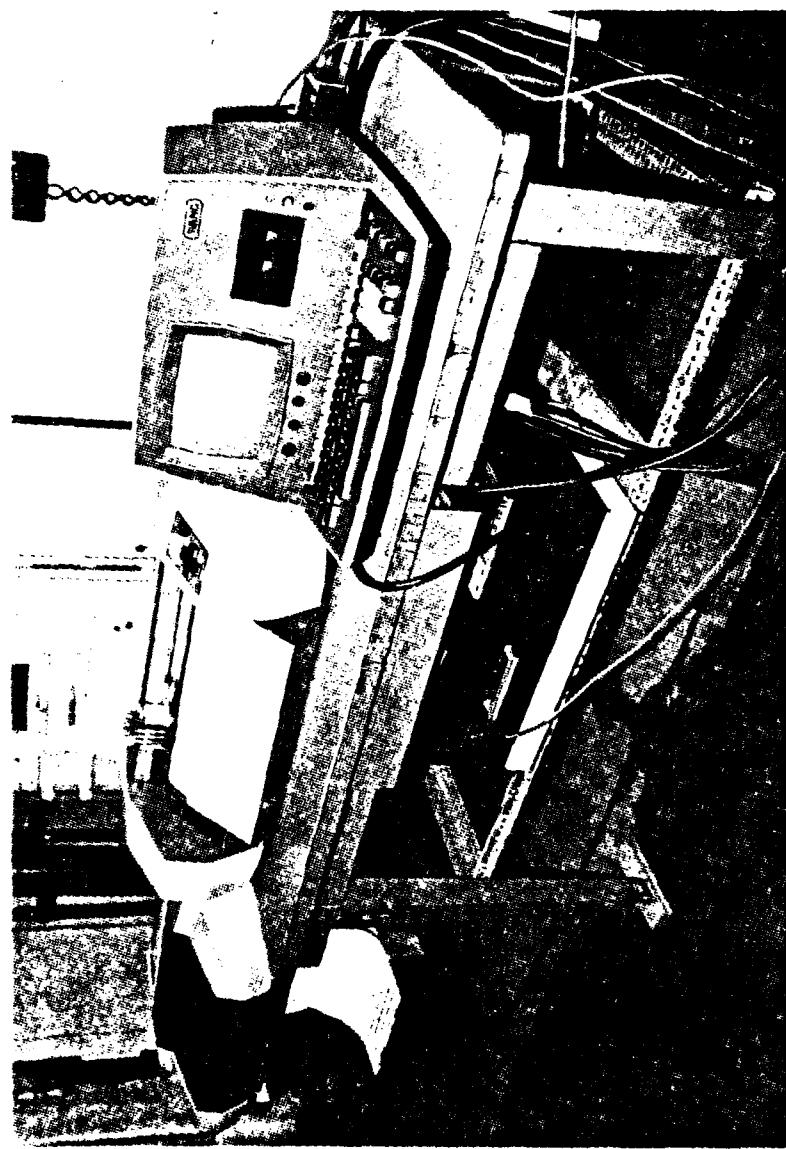
Figure 48. Computer data system photograph

TABLE 9

PRINTER OUTPUT LIST

Variables	Variable Functions
y (inches)[*]	Height above plate
u (ft/sec)[*]	Velocity
u^1 (ft/sec)[*]	Velocity fluctuation
$u/_{UFS}$[*]	Normalized velocity ratio
u^1/u	Local turbulence level
u^1/UFS	Relative turbulence level
volts	Tracker output

[*] stored on tape cassette.

The program "DATA" selects 61 data points from each channel
of input and computes their average for computational use. The
four channels of input include velocity and rms of the velocity
fluctuation from the LDA system output, position y from the
digital position indicator, and a counter channel.

From listings and laboratory data, x station and velocity
profile data were key punched as input to a comprehensive data
processing program called "DATAPAC." This program calculated
hydrodynamic parameters, listed data, plotted data, and theoret-
ical curves using a high-speed Calcomp plotter. Appendix B

presents a copy of program DATAPAC and a sample output listing.
Typical output curves from one velocity profile are shown in
appendix C.

IV. EXPERIMENTAL PROCEDURES

Three types of tests were performed in this research. These were: (1) tunnel characterization tests, (2) water injection boundary layer sampling tests, and (3) polymer injection boundary layer sampling tests. The preparations for these experiments will be discussed first.

Polymer Preparation

The polymer used in this experiment was polyethylene oxide (Polyox WSR-301) of approximate molecular weight 6×10^6. This polymer was selected due to its availability and widespread use among researchers, thereby minimizing data correlation problems due to use of polymers of different molecular weight.

As the experimental program called for solutions of 1000 WPPM or less to be tested, a master solution was prepared at 2000 WPPM and diluted to the required concentrations.

The master solution was prepared using the same procedure as Sirmalis (1974). Premeasured Polyox powder was sifted onto the surface of the carefully weighed water, which was slowly stirred by a magnetic mixing bar. After a clear solution resulted (1 hour),

the solution was left to stand for approximately 40 hours to
minimize the high viscoelastic effects of freshly mixed solution
and to assure homogeneity. The master solution was kept in a
dark, cool place after initial mixing to reduce the auto-oxidation
problem with polyethers. When required, the proper amount was
pipetted out of the master solution and diluted to the required
concentration. These dilute solutions were used within 24 hours
of the makeup time.

The Uranine master solution was prepared in a similar manner.
When a test solution was required, the proper amount of each
constituent was added to enough water to make one liter of
solution. This solution was poured back and forth into the con-
tainer ten times to assure complete mixing and then let to stand
for 1 hour to achieve homogeneity. A series of calibration tests
with the fluorometer indicated that this procedure provides
accurate results.

Injection System Calibration

The polymer injection system was designed to minimize
mechanical degradation of the polymer concentration and avoid
polymer effects on the flow measuring device. This was accomp-
lished by using water, bypassed from a recirculatory loop to

displace polymer from a reservoir. The reservoir contained a free-sliding piston with O-ring seals and was thoroughly cleaned after each test to prevent alteration of the next concentration to be tested. The water flow passed through a Fisher Scientific Company variable area flowmeter with tube number 448-225. The float diameter was 1/8 inch and made of stainless steel.

The calibration curve for the flow range of 20, 40, and 80 cm^3 per minute is shown in figure 49. The accuracy of the curve applied to the delivery system was confirmed by the capture of polymer solution in a graduated cylinder. There were no oscillations of the float in the variable area tube indicating the steadiness of delivery.

Boundary Layer Sampling

Five boundary layer sampling stations were used in these experiments, located at axial distances of 3.75, 8.5, 12.5, 16.5, and 20.5 inches from the leading edge of the plate. All probe rakes were retracted up into the ceiling of the tunnel out of the flow at the start of the test. The rake furthest downstream was lowered to the plate first and a set of samples captured in the cuvettes. The wall sample was not taken through the hole in the plate at this time to prevent probe interferences. When
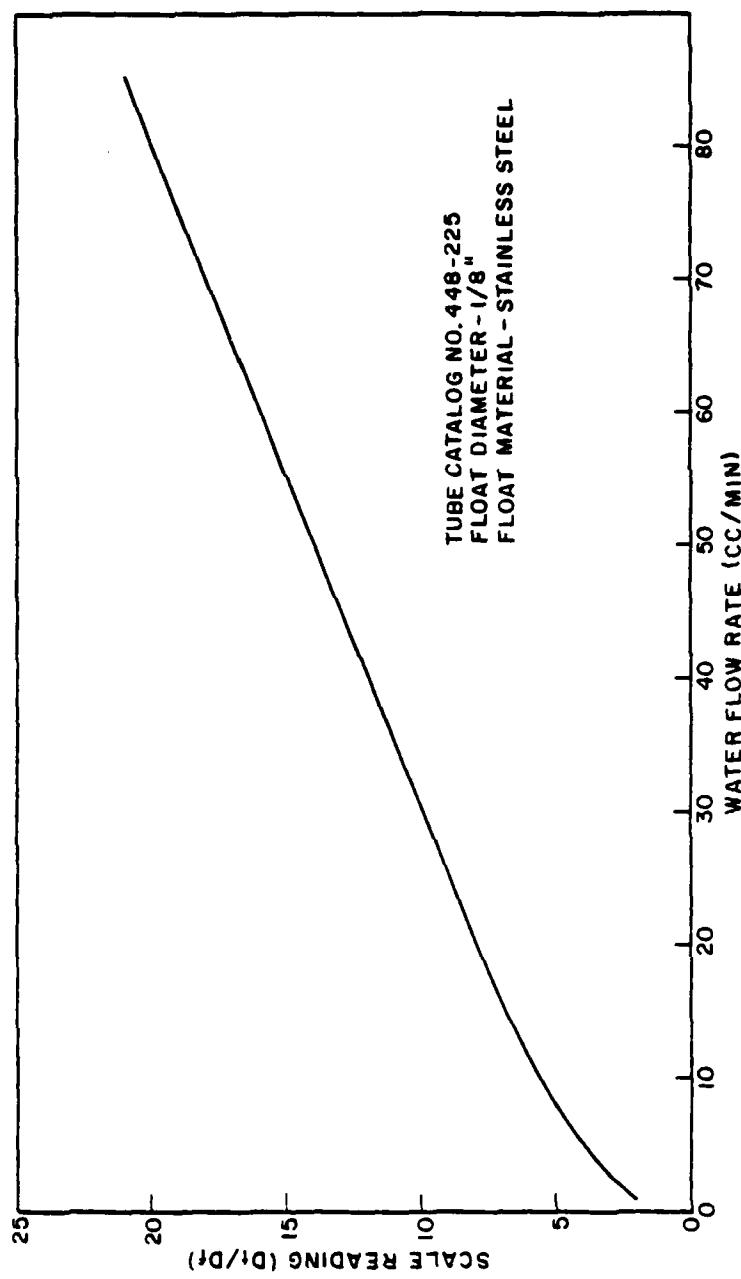
Figure 49. Polymer injection flowmeter calibration curve

a sufficient sample volume was captured, at approximately 8 to
10 minutes, the sample lines were transferred to a reservoir
and the probe rake raised to position number 2. The new position
was determined by a gage block inserted between a collar on the
rake body and the top of the tunnel fitting. Using the reservoir
and placing the probes in the next position allowed the lines
to purge themselves while the next set of cuvettes was being
readied. When the probe was raised to position number 2, the
wall sample was taken. After all the samples were taken at the
station, the rake was retracted up into the ceiling of the tunnel,
out of the flow. The upstream probe stations were used, in order,
until all the samples were taken. All samples were expelled
under internal tunnel pressure. No suction was used to avoid
interference effects between probes during sampling. Volume
rates of sampling were based on the slowest velocity in the
boundary layer that allowed all the samples to be taken within
the capability of the delivery system. Table 10 presents the
non-dimensional locations of each of the stations at which con-
centration measurements were taken.

Fluorometer Calibration

The fluorometer used in these tests is a G.K. Turner Model
111 Fluorometer. Uranine-B dye was selected since it was readily
adaptable to laboratory use, compatible with polymer flows, and

gave reproducible readings to about 1 part in 10^9 on a weight

basis. This range was more than sufficient for the experiment to

TABLE 10

NON-DIMENSIONAL LOCATIONS OF CONCENTRATION MEASUREMENTS STATIONS

Station	X (Inches)	X/L (L=22.5 Inches)	X/S (S=.020 Inches)	X/S (S=.005 Inches)
0	1.75	.078	87.5	350.0
1	3.75	.167	187.5	750.0
2	8.50	.378	425.0	1700.0
3	12.50	.556	625.0	2500.0
4	16.50	.773	825.0	3300.0
5	20.50	.911	1025.0	4100.0

be performed as the dye concentration in the solution to be

ejected could be adjusted, thus producing samples along the plate

sufficiently above background levels to give reproducible results.

The Model 111 Fluorometer has four sensitivity settings allowing

calibration curves to be drawn over a range of about 1 part per 10^6 to 1 part per 10^9, on a weight basis. For concentrations greater than the lower value, dilution prior to measurement was performed. The calibration charts for this experiment are shown in figures 33 and 34. Approximately 3.5 to 4.5 ml of sample is required for measurement. The sample, contained in a cuvette (a small test tube), is placed in the instrument and the measurement is taken. Cleanliness was found to be a key factor in the technique as fingerprints, body oils, and trace contamination could affect readings. All cuvettes were carefully wiped with soft laboratory paper towels and handled only by the upper edge. All samples were allowed to achieve room temperature prior to reading.

Wall Location

The location of the plate surface was determined with use of the laser beam. The beam of diameter d_m = .0096 inches was lowered until two visual effects were observed. First, surface imperfections were observed, highlighted by the laser beam as it approached the plate. Secondly, with the aid of a viewing telescope, observation of the measuring volume at the beam crossover point showed the reflection of the intersected beams from the plate surface as a second image. When the second image blended

with the original image, the beams were considered on the surface.
The center of the measuring volume was then about 0.0048 inch off
the surface of the plate. The beams were lowered to that dis-
tance and the vertical height digital indicator set at zero. The
measurements were all considered to be made at a height correspond-
ing to the center of the measuring volume above the surface of
the place.

Tunnel Characterization

To determine the suitability of the water tunnel for the
experiment, a velocity field map of the tunnel was made using the
LDA system. (The two-dimensional character of the tunnel is
shown in figure 50.) Additional tests downstream and at higher
flow rates also showed that corner flow effects, although present,
were removed from the center of the tunnel.

Velocity profiles and turbulence intensity distributions were
taken and are presented in comparison with the work of Laufer
(1951) as figures 51 and 52. These curves are in excellent
agreement.

A critical requirement for this experiment was that transi-
tion be effected on the plate, preferably at its mid-length to
allow observation of laminar, transitional, and turbulent flow

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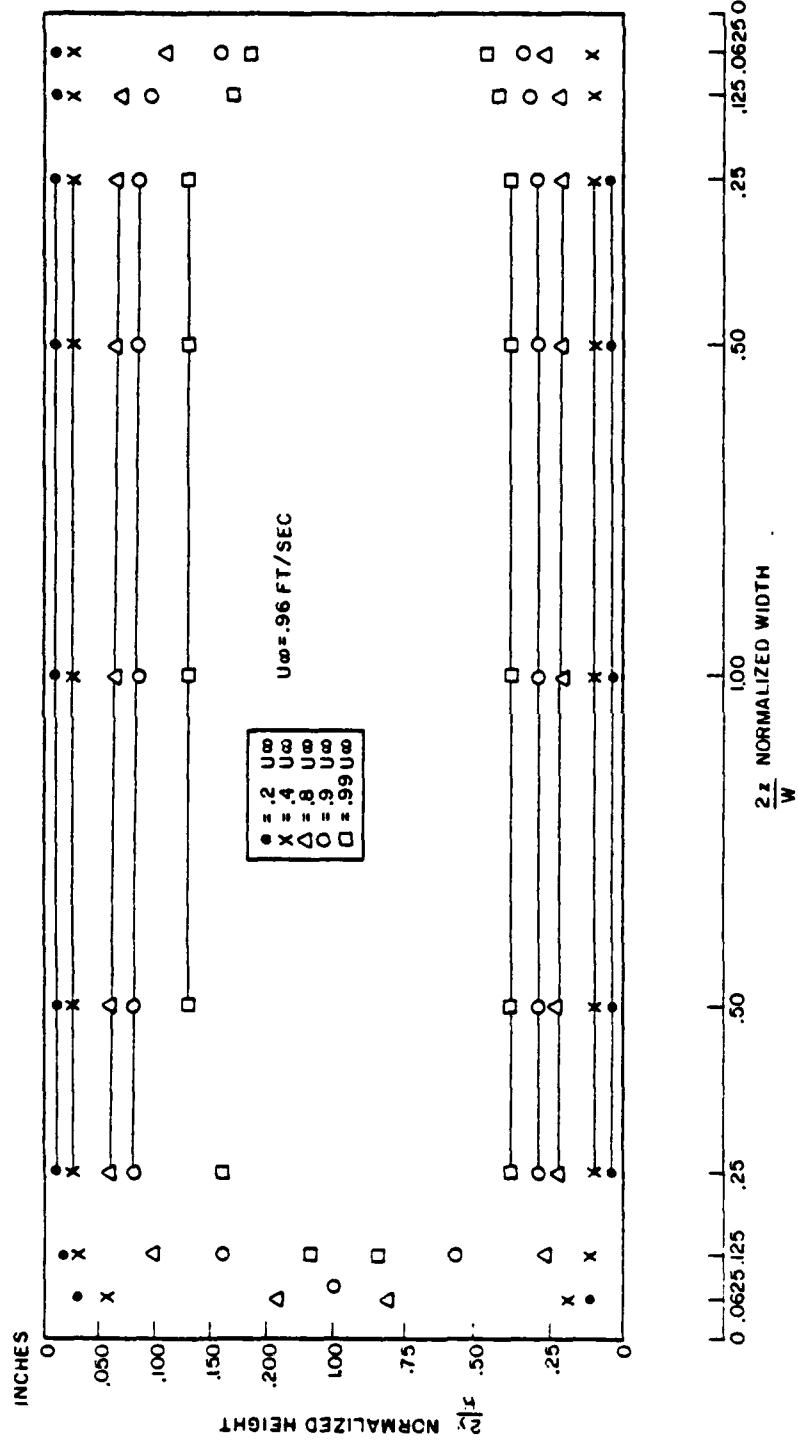
Figure 50. Tunnel velocity field map
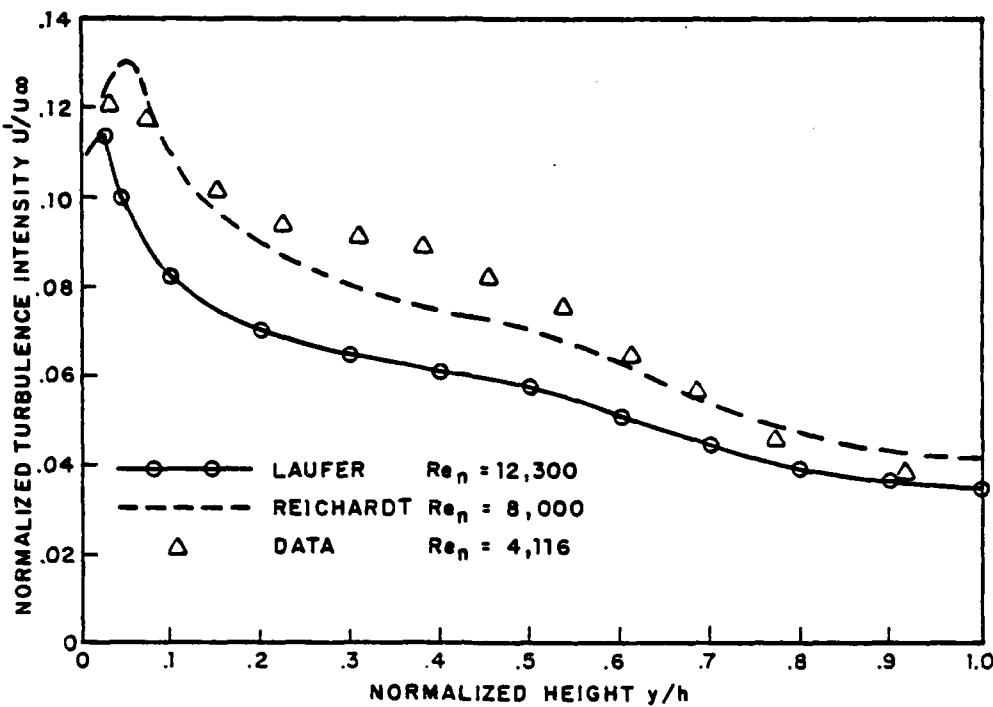
Figure 51. Normalized turbulence intensity distribution
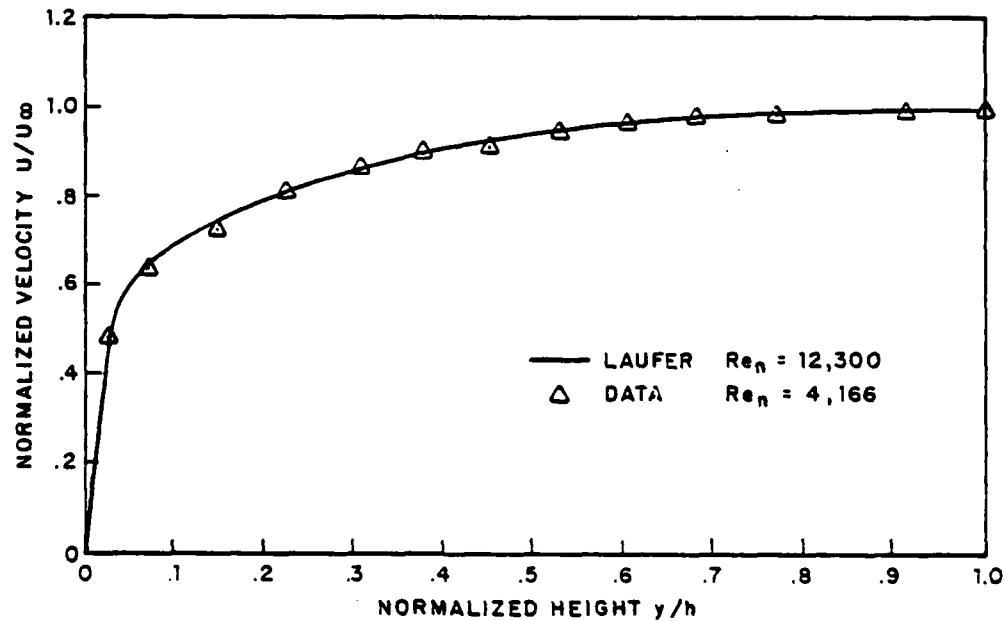
Figure 52. Non-dimensional velocity profiles

boundary layers. The incoming velocity profile is shown in figure 53. Velocity profiles at axial distances of 3.75, 14, and 18 inches, presented in figure 58, are also laminar, transitional, and turbulent. Ejection of a dye onto the surface of the plate showed turbulent bursting occurring at X = 6.00 inches.

Application of a laser Doppler technique to locate transition on a flat plate as developed by P. E. Gibson of the Naval Underwater Systems Center, Newport, Rhode Island, is shown in figure 54. From laminar and turbulent velocity profiles, the height above the plate which yielded the greatest corresponding velocity difference was determined. The laser measuring volume was then set at that height and transversed the length of the tunnel, producing a plot of axial velocity distribution. This technique confirmed that transition was occurring on the plate.

Test Series

The test program consisted of two parts: (1) water ejection tests, and (2) polymer ejection tests. All tests were run at a constant tunnel flow of 8 gal/min and an average velocity, V_{avg}, of 2.2 ft/sec. Volumetric flow rates, Q_i, of 20, 40, and 80 cm^3/min were selected to be injected through two slot openings, s, of 0.020 and 0.005 inch. Table 11 lists the injection parameter
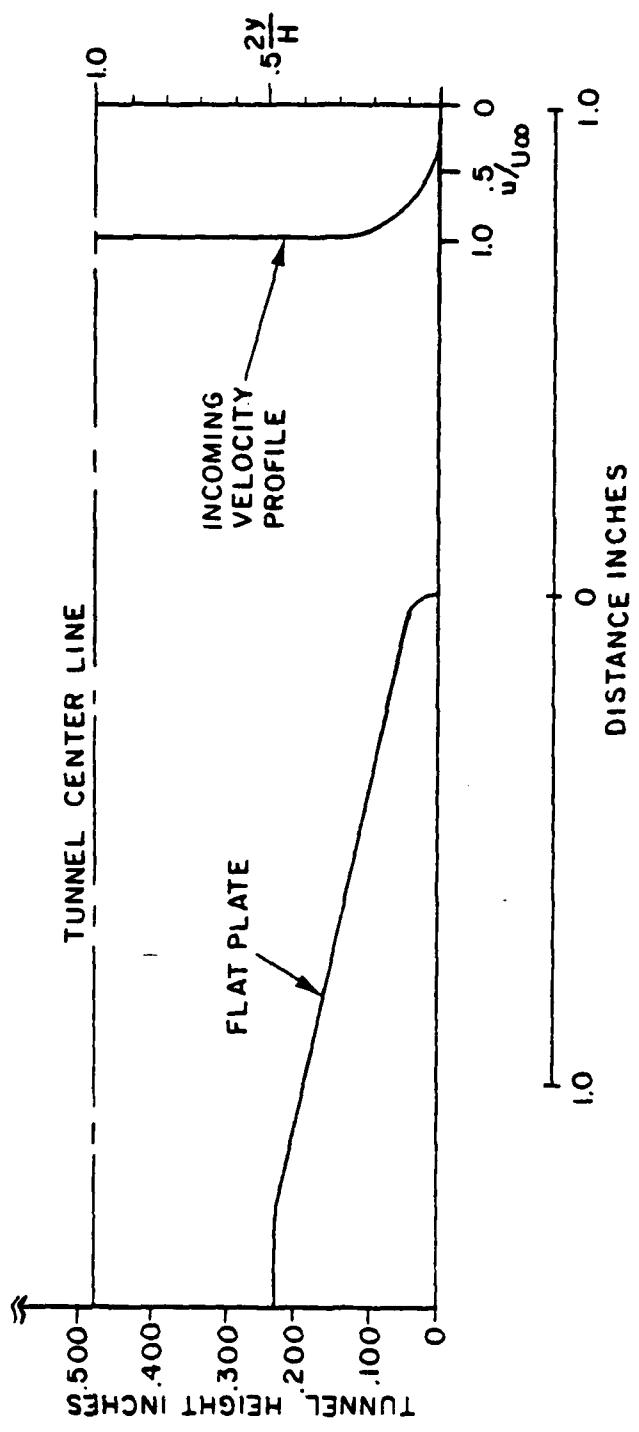
Figure 53. Incoming velocity profile
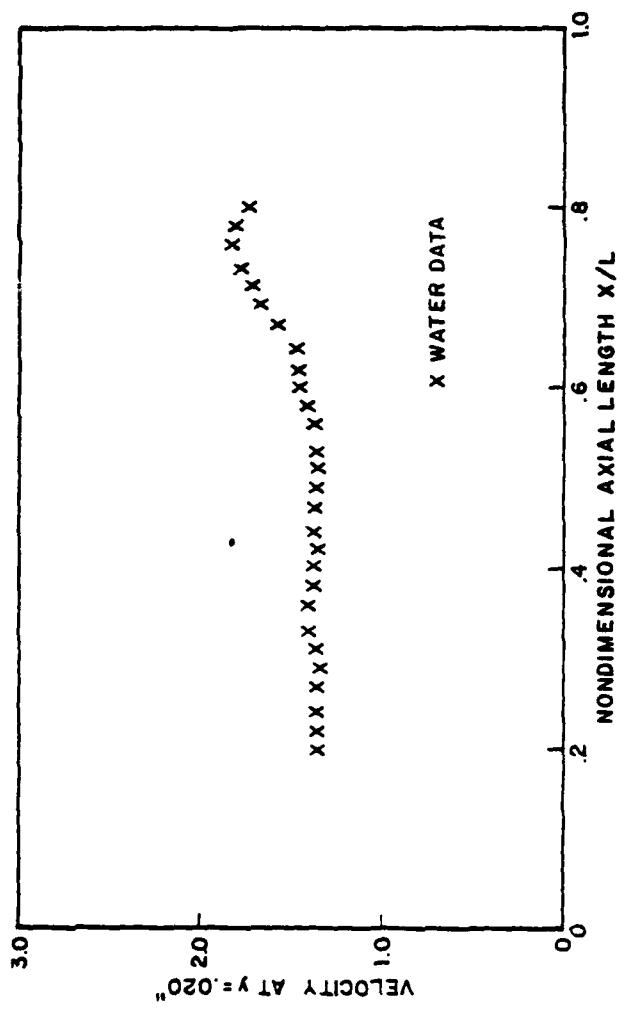
Figure 54. Axial velocity profile at y = .020 inches through transition

used. At a given velocity of ejection determined by the combi-
nation of slot area and volumetric ejection flow rate, various
concentrations of polymer were ejected. Table 12 presents the
concentration flux matrix tested.

TABLE 11

INJECTION PARAMETERS

Slot Height S(in)	Slot Area $A_s(in)^2$	Injection Flow Rate Q_1(cc/min)	Injection Velocity V_1(ft/sec)	Normal Component of Injection Velocity V_{1y}(ft/sec)	Velocity Ratio (V_1/V_{avg})	Normal Velocity Ratio (V_{1y}/V_{avg})
.020	.04424	20	.0383	.0099	.0174	.00451
.020	.04424	40	.0766	.0198	.0348	.00901
.020	.04424	80	.1532	.0396	.0696	.01802
.005	.01106	20	.1537	.0398	.0699	.01809
.005	.01106	40	.3074	.0795	.1398	.03614
.005	.01106	80	.6148	.1591	.2795	.07233

TABLE 12

CONCENTRATION FLUX MATRIX

Slot	.020 Inch	.020 Inch	.020 Inch	.005 Inch
Q_i	20 cc/min	40 cc/min	80 cc/min	80 cc/min
		$C_i Q_i$		
100	2,000	4,000	8,000	-
200	4,000	8,000	16,000	-
400	8,000	16,000	32,000	32,000
500	10,000	20,000	40,000	40,000
800	16,000	32,000	64,000	64,000
2000	-	-	-	160,000

V. ANALYTICAL CONSIDERATIONS

Boundary Layer Model

This section presents an analytical approach for developing turbulent flow over a submerged flat plate with polymer injection. A model is developed for predicting growth of the boundary layer, diffusion of polymers, and prediction of skin friction coefficients and transition as a function of local wall polymer concentrations.

In the analysis that follows, the velocity profile, continuity equation, momentum equation, and an expression for polymer wall concentration from a regression analysis which, in turn, was obtained from experimental data are combined with an effective Reynolds number analogy. This combination yields expressions for the growth of the boundary layer and the skin friction coefficient as a function of the local polymer wall concentration.

Velocity Profile Relation

Meyer (1966) provided a ΔB correction for the law of the wall accounting for polymer effects

$$u^+ = \frac{1}{K} \ln y^+ + B + \Delta B. \tag{56}$$

An alternate method of accounting for the polymer effect would be by adjustment of the mixing length constant, according to Virk (1971). In his work on maximum drag reduction along the ultimate asymptote line, he noted that a change in K by a factor of up to 5 is possible and in full agreement with data. However, from his studies, Meyer concluded that drag reduction in pipes is due to thickening of the laminar sublayer. The mixing length constant, K, which equals approximately 0.4, is apparently unaffected by dilute concentrations of additive. Meyer further correlated the constant B and showed it to remain constant at the Newtonian value of 5.5 until the friction velocity reached a threshold value, V_o^*, after which B increased logarithmically with V^*,

$$B = Z \ln(V^*/V_o^*). \tag{57}$$

The term Z is a dimensionless constant dependent on the type of polymer additive and the concentration. A constitutive relation for Z is suggested by White (1968) from simple curve fit expressions as

$$Z = \alpha(C_w)^\gamma, \tag{58}$$

where $\alpha = 2.3$ and $\gamma = 0.5$. (59)

Skin Friction Relations

The boundary layer continuity, and momentum equations for flow over a submerged flat plate with zero pressure gradient as applied to the polymer flow case studied are given as

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{60}$$

and

$$\rho u \left(\frac{\partial u}{\partial x}\right) + \rho v \left(\frac{\partial u}{\partial y}\right) = \frac{\partial \tau}{\partial y}. \tag{61}$$

Upon changing independent variables x and y to x, y^+, and Cw, the x derivatives must be handled by the chain rule, since the parameters y^+ and Cw are functions of x in the law of the wall. Thus, the substitute is

$$\frac{\partial}{\partial x} = \frac{\partial y^+}{\partial x}\left(\frac{\partial}{\partial y^+}\right) + \frac{\partial C_w}{\partial x}\left(\frac{\partial}{\partial C_w}\right) . \tag{62}$$

As may be noted in equation (62), concentration derivatives are added. The fluid properties of density, ρ, and viscosity, μ, are assumed constant and equal to those of the solvent water in this analysis.

The resultant boundary layer equation, after considerable algebraic manipulation, is

$$
\int_{X_o}^{X} \frac{U}{v}\, dx = \int_{\lambda o}^{\lambda} \left[\delta^+ \left(\left\{ \mathcal{B}\ln\left(\frac{v^*}{v_o^*}\right) \left[\frac{2}{K}(\ln\delta^+ - 1) + 2B + \mathcal{B}\ln\left(\frac{v^*}{v_o^*}\right) \right] \right\} \right. \right.
$$

$$
\left. \left. + \left\{ \frac{2}{K}(\delta^+ - 1)(B - \frac{1}{K}) + \frac{1}{K^2}(\ln\delta^+)^2 + (B)^2 \right\} \right) \right] d\lambda
$$

$$
- \int_{C_{wo}}^{C_w} \left[\delta^+ \lambda \left\{ \frac{\mathcal{B}}{C_w}\ln\left(\frac{v^*}{v_o^*}\right) \left[1 + \frac{1}{K^2}(\ln\delta^+ - 1) + B - \frac{1}{K} \right] \right\} \right] dC_w \quad (63)
$$

where

$$
\lambda = \frac{U}{v^*} = \sqrt{\frac{2}{C_f}} \tag{64}
$$

and

$$
\delta^+ = e^{K(\lambda - B)} \left[\left(\frac{v^*}{v_o^*}\right)^{-K\mathcal{B}} \right]. \tag{65}
$$

Appendix A presents a complete development of the equations related to this method.

With elimination of the polymer terms, equation (63) compares favorably with the classical skin friction relations of Prandtl-

Schlichting and Schultz-Grunow, as well as the simplified relation
of F. White. This comparison is presented in figure 55 .

Prior to the solution of equation (63) and (65), it is neces-
sary to determine a means of calculating C_w. A regression analysis
was performed on all of the data for the slowest injection rate of
20 cc/min and injection concentrations of c = 100, 200, 400, 500,
and 800 WPPM. Equation (66) represents the results of that
analysis which successfully correlates 88% of the data obtained,
so that

$$\frac{C_w}{C_i} = .8343 + 2.3033\left(\frac{X}{L}\right) - 7.385\left(\frac{X}{L}\right)^2 + 4.2855\left(\frac{X}{L}\right)^3. \quad (66)$$

Equation (66) is valid for X/L > .27. For X/L < .27, equation
(67) holds true. It follows then that

$$\frac{C_w}{C_i} = 1.0. \quad (67)$$

Effective Reynolds Number Analogy

White (1968) rearranged the friction factor relation derived
by Meyer (1966) and evolved the following effective Reynolds
number analogy. The friction factor in the presence of polymer
may be considered equal to the Newtonian friction factor evaluated
at an effective Reynolds number given by
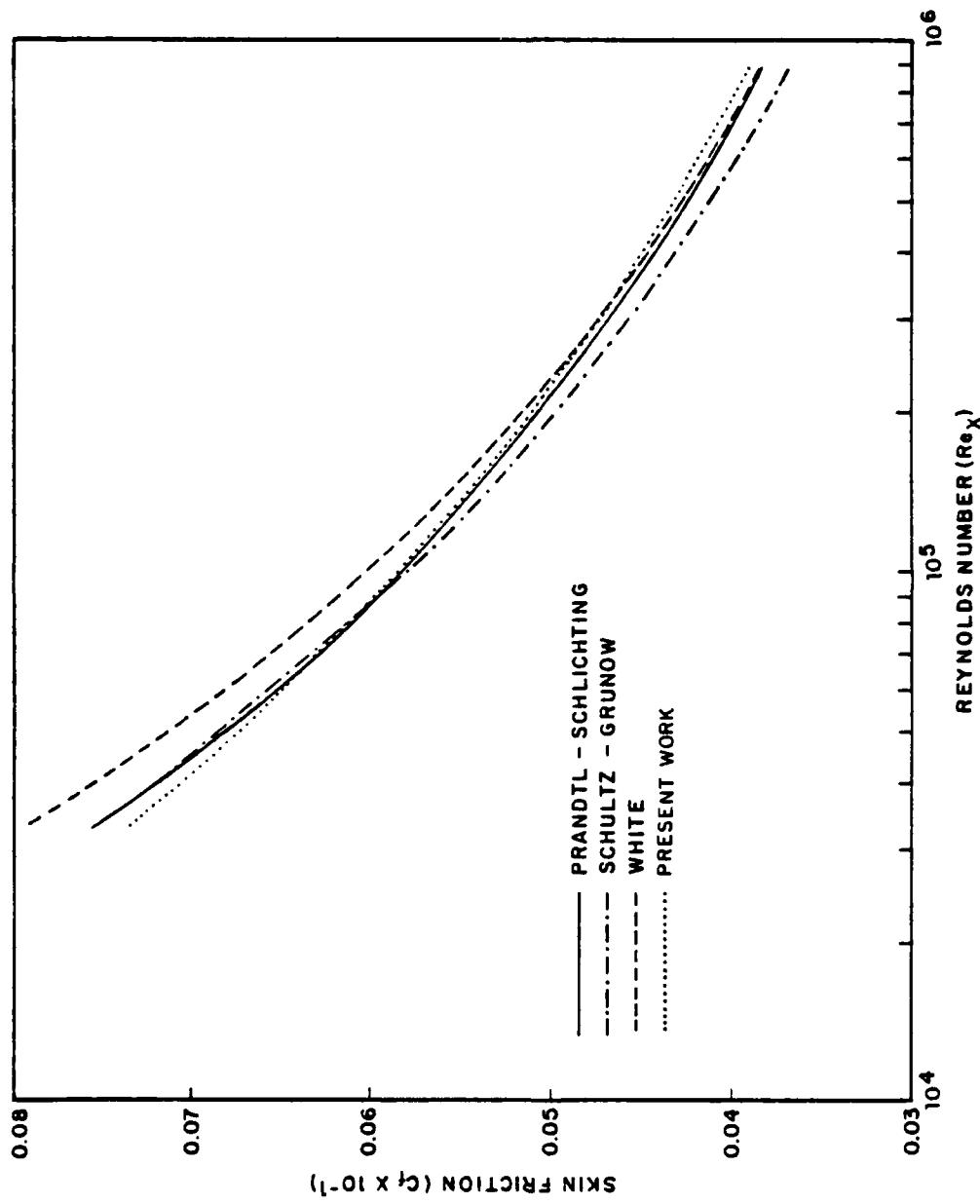
Figure 55. Flat plate skin friction coefficient curve

$$Re_p = Re\left(\frac{v^*}{v_o^*}\right)^{KZ}.$$ (68)

Thus, the polymer has the effect of increasing the Newtonian Reynolds number, Re, and thereby yielding the lower polymer solution friction factor. For constant freestream velocity and assuming constant polymer ocean flow, skin friction versus Reynolds number curves may be obtained as follows:

1. Calculate Reynolds number at given X,

$$Re_x = \frac{Ux}{\nu}.$$ (69)

2. Calculate initial skin friction value using White's simplified relation,

$$C_f = \frac{.455}{[\ln(.06\ Re_x)]^2}.$$ (70)

3. Calculate initial shear velocity,

$$v^* = U_\infty \sqrt{\frac{C_f}{2}}.$$ (71)

4. Calculate effective polymer Reynolds number,

$$Re_p = Re_x \left(\frac{v^*}{v_o^*}\right)^{KZ},$$ (72)

where

$$z = \alpha C_w^\gamma \quad \text{for } C_w \leq C_w^* \tag{73}$$

$$z = \alpha(C_w^*)^\gamma \quad \text{for } C_w > C_w^* $$

5. Calculate polymer skin friction,

$$C_{f,p} = \frac{.455}{[\ln(.06 \, Re_p)]^2} . \tag{74}$$

6. Calculate shear velocity,

$$v^* = U\infty \sqrt{\frac{C_f}{2}} . \tag{75}$$

Equations (72) through (75) should be repeated until the shear velocity does not change by more than 0.5%. This iterative procedure yields skin friction values as a function of Reynolds numbers. A new value of x along the plate should be selected and the procedure should be repeated. Figure 56 presents curves of skin friction versus Reynolds number for concentrations of 5, 10, 15, 20, and 25 WPPM as well as turbulent and laminar flow skin friction curves given by equations (70) and (76) for water flow only,

$$C_f = \frac{.664}{\sqrt{Re_x}} . \tag{76}$$

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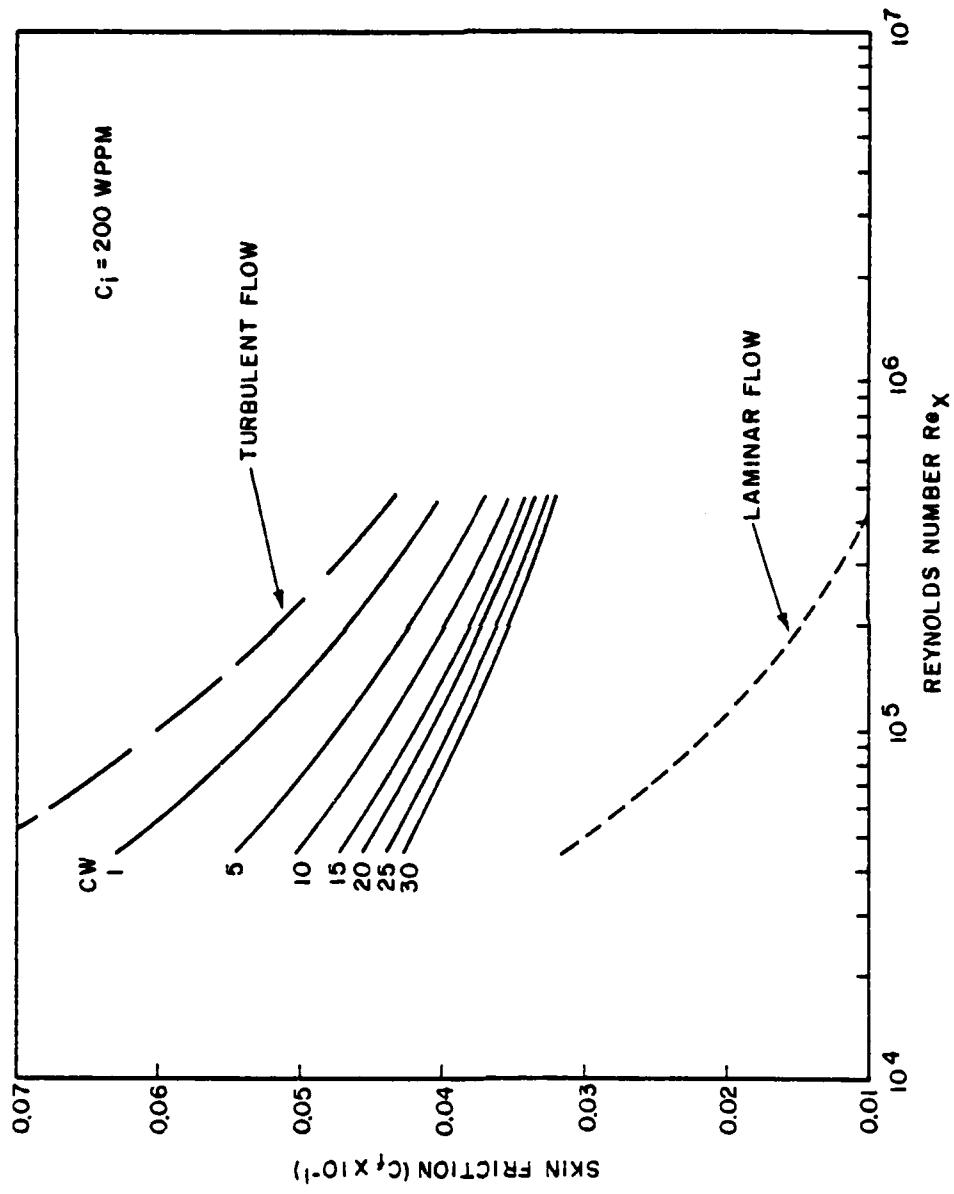
Figure 56. Skin friction versus reynolds number as a function of polymer concentration

Using the concept of the effective Reynolds number analogy and the variation in the wall concentration of polymer determined from the regression analysis performed on the experimental data, it is possible to numerically integrate the right hand side of equation (63) and compare it with the resulting Reynolds number obtained from integration of the left hand side of equation (63).

The computational procedure is presented in appendix F as a listing of computer program SKINFRIC. The model allows for an initial laminar flow over the plate, changing to turbulent flow at $x = x_t$. The integrations are performed from initial values of x where injection occurs, an initial wall concentration equal to the injected concentration of polymer and an initial value of λ determined from laminar flow conditions given by equation (76). This is based on the assumption that small concentrations of polymers do not affect laminar flow. The work of Hoyt and Fabula (1964) showed that non-Newtonian is a misnomer applied to low concentrations of polymers. They found that these solutions were of constant viscosity and greater magnitude than that of the solvent. White (1968) determined the limiting value of Z to be approximately 11.0 by single curve fitting of data. Application of the maximum drag reduction concentration for Polyox of $C_w^* = 30$ WPPM as determined by Hoyt and Fabula (1964), and shown in figure 57, allowed the limiting value of Z to be taken as 12.6.
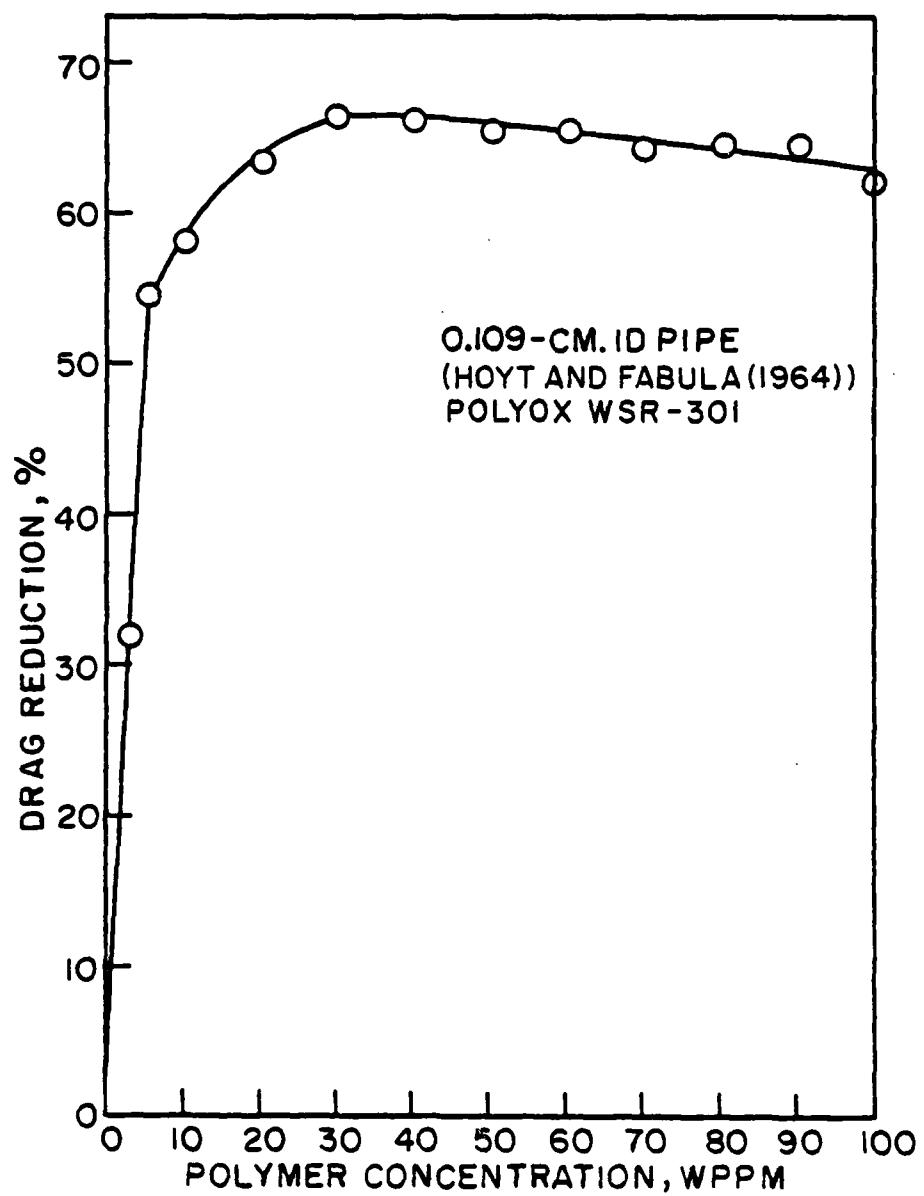
Figure 57.   Percent drag reduction as a function
of polymer concentration

It is further assumed that the effective Reynolds number
analogy is a solution to equation (63). Values of λ are then
used in the numerical integration of the right hand side of equa-
tion (63). Values of the polymer wall concentration, equation
(66), from the regression analysis are also used. The integration
proceeds from $X_o = 0$ at the point of injection of the polymer
and neglects leading edge effects. Laminar flow equations give
the flow over the initial region of flat plate until the preset
transition point is reached at $x = x_t$, after which turbulent
effective Reynolds analogy parameters apply. Comparison of the
difference between integrand 1 and integrand 2 on the right
hand side of equation (63) with the left hand side Reynolds number
is made at the end of the plate. If they are not within 1%, the
location of the point of transition is adjusted and the procedure
repeated until the results are within the set tolerance limits.
The transition region is defined as starting at the transition
point and ending at the distance where the Reynolds number equals
350,000. In this zone a linear intermittency factor is assumed
and an appropriately proportioned friction factor calculated.

Typical plots of skin friction coefficient versus Reynolds
number for various polymer concentrations injected at $Q_i = 20$ cc/min
are shown in figures 92 through 96. Boundary layer growth over
the length of the plate is presented in figures 85 through 88 and
compared with experimental data.

VI. EXPERIMENTAL RESULTS AND DISCUSSION

General Remarks

The experimental program was conducted at a constant tunnel
flow of 8 gallons per minute. Injection into the boundary
layer was performed through two different size slots of width,
s = .020 inch, and s = .005 inch, at injection rates of 20, 40,
and 80 cc/min. These six injection velocity ratios were tested
with dyed water injection. Polymer concentrations of 100, 200,
400, 500, and 800 WPPM were injected at 20, 40, and 80 cc/min
through the s = .020 inch slot and at 80 cc/min. through the s =
.005-inch slot, producing injection velocity to average freestream
velocity ratios of 0.0174, 0.0348, 0.0696 and 0.2795, respectively.

Velocity profiles were taken at nine stations along the plate
corresponding to axial distances of x = 3.75, 6, 8, 10, 14, 15,
16, 17, and 18 inches. Concentration measurements were taken at
five stations along the plate corresponding to axial distances of
x = 3.75, 8.5, 12.5, 16.5, and 20.5 inches. At each concentration
measuring station, nine samples were taken throughout the boundary
layer including a wall sample. Table 2 indicates the relative
heights in the boundary layer at which the concentration samples

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were captured according to which spacer was used in conjunction with the sampling probe.

In total, 243 velocity profiles and 130 concentration profiles were measured. Space limitations prohibit the complete listing of the data file, however, copies of the data file are available from the author at the Naval Underwater Systems Center, Newport, Rhode Island. The focus of the discussion will concentrate on the 20 cc/min injection rate through the .020-inch slot which yielded an injection velocity ratio of 0.0174. This slowest injection velocity was selected as providing minimum boundary layer disturbance at the point of injection. Previous investigations such as Sirmalis (1976), Fruman and Tulin (1976), Wu (1968), and Wells (1968) have recommended minimization of the injection velocity. The hydrodynamic parameters for the tests discussed are presented in appendix D with velocity profiles presented in appendix E.

Water Flow Boundary Layer Characterization

Velocity profiles for the laminar flow region are presented
in figure 58 and compared with the classical Blasius profile.
The slight variation from the classical form may be attributed
to the effects of the small pressure gradient at the leading edge
of the plate and due to its shape and roughness. Displacements among
the data themselves are due to the effects of injection velocity,
with minimum injection velocity approaching the theoretical profile.

Downstream, at a station $x = 18$ inches, the velocity profiles
follow a 1/6th power law turbulent flow shape as shown in figure
59. Patel (1968) has shown that in developing flows the velocity
profiles go through various power law shaped profiles until a
1/7th power law profile is achieved for fully-developed turbulent
flow. The measured profiles are in agreement with Patel's findings
as fully-developed turbulent flow was not totally achieved due to
the length of the plate used.

The transitional nature of the velocity profiles from laminar
to turbulent flow for the case of 20 cc/min injection at a
velocity of injection, $V_i = .0383$ ft/sec, is shown in figure 60.
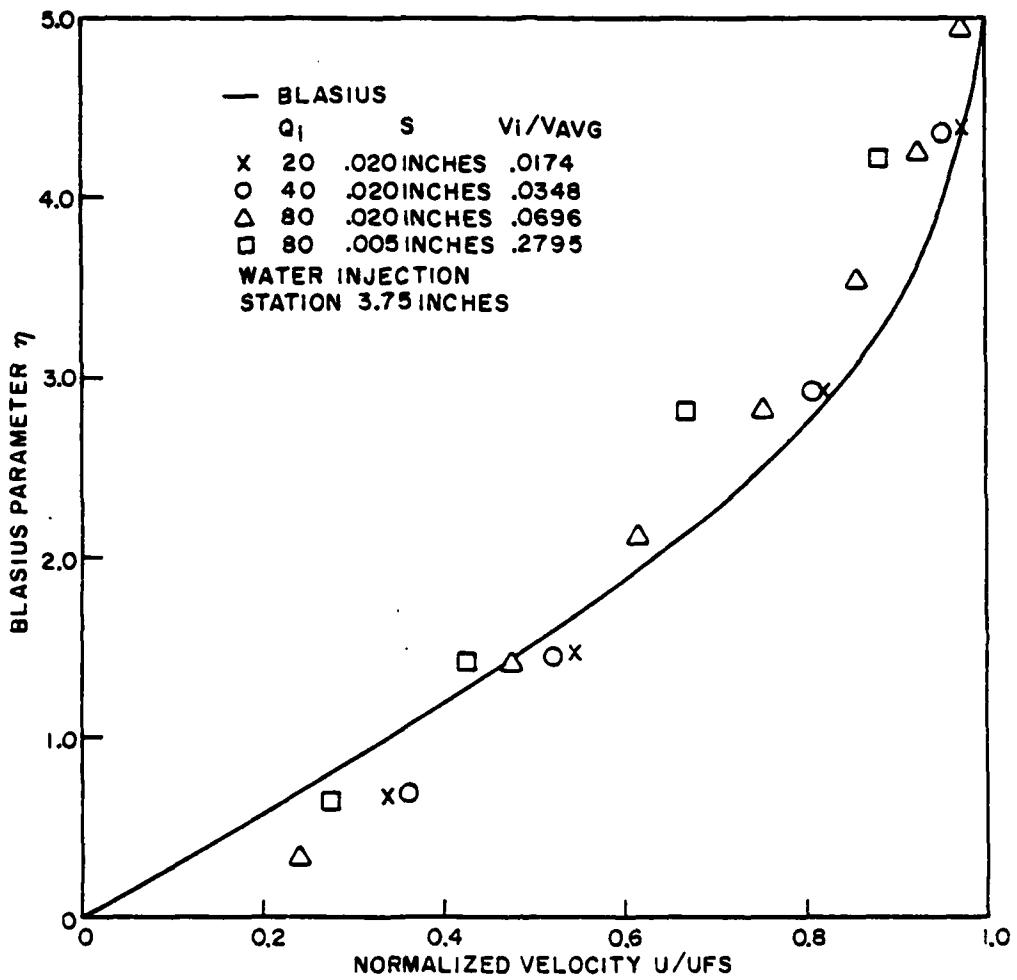
Figure 58. Normalized laminar velocity profiles
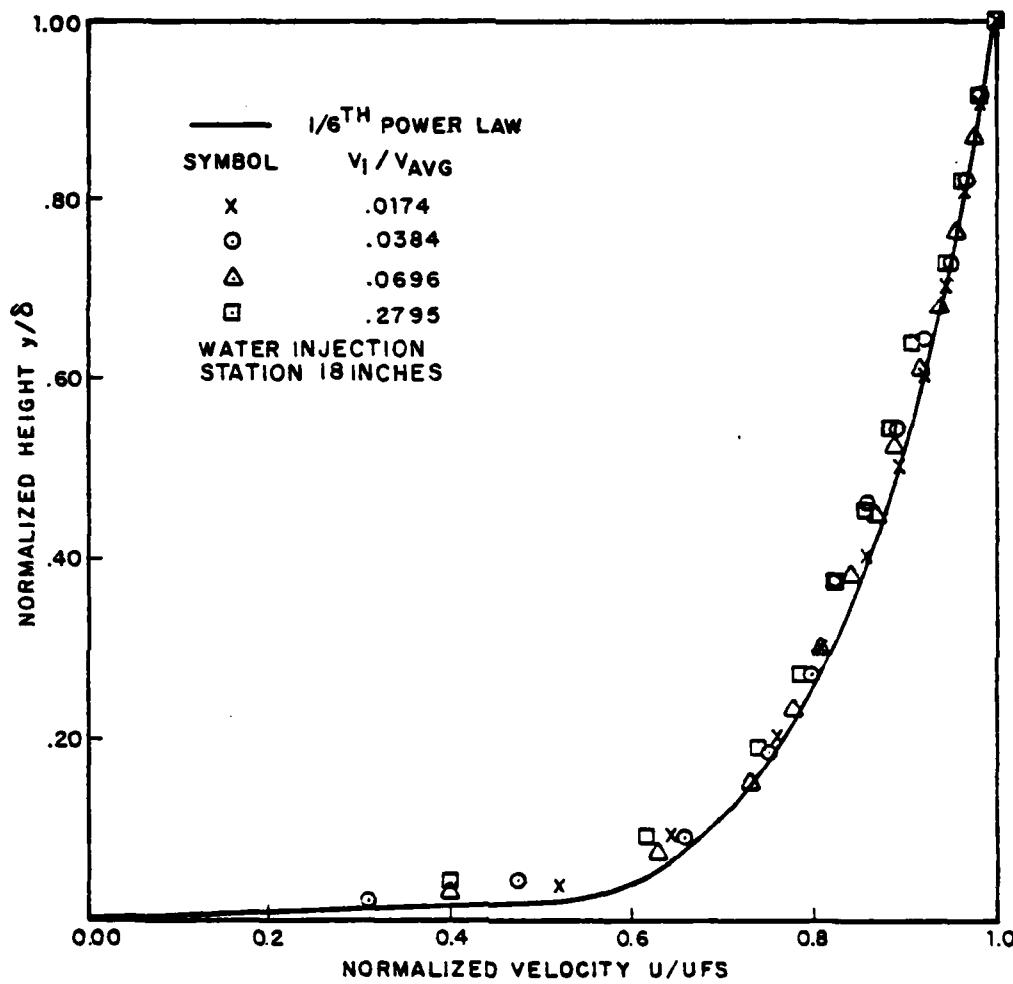
Figure 59.   Non-dimensional velocity profiles at station
x = 18.0 inches
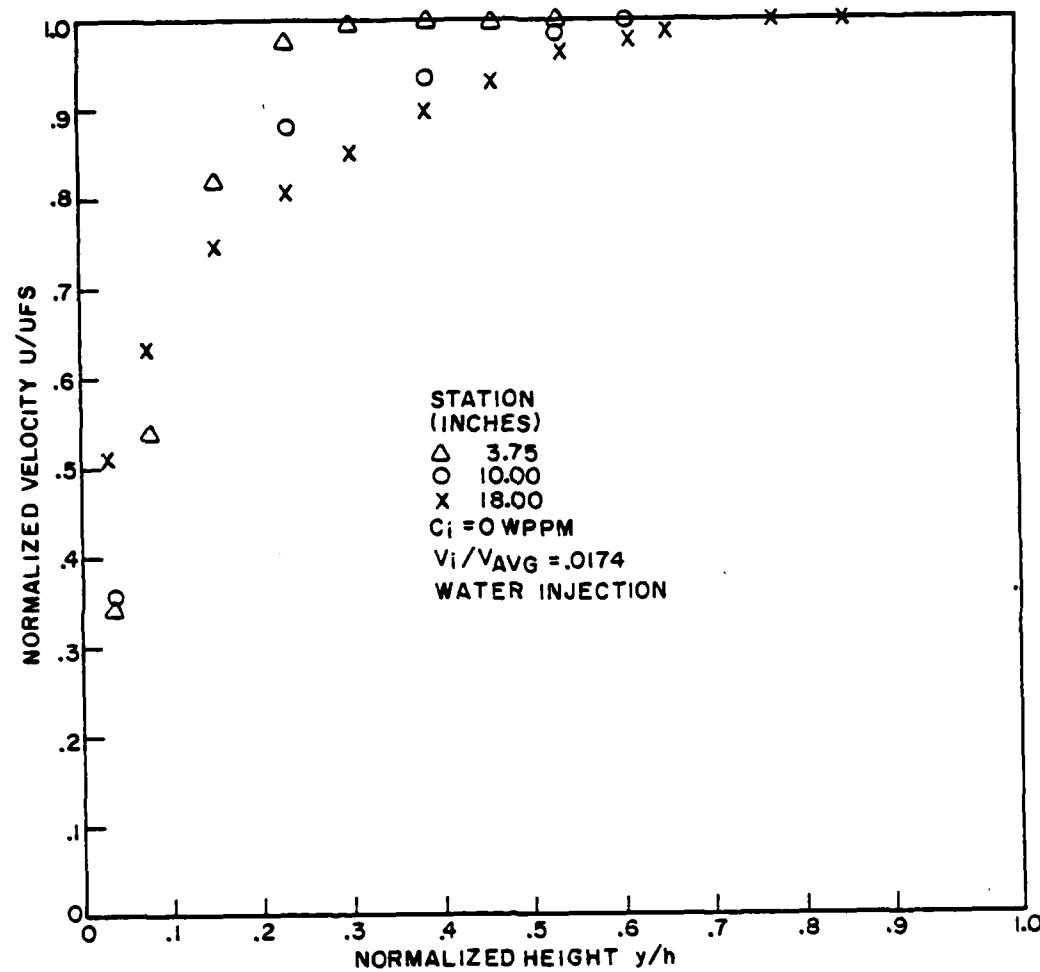
Figure 60. Transitional nature of the velocity profiles for water flow

The turbulent intensity distributions for these stations are presented in figure 61. As shown by Schubauer and Klebanoff (1955), the velocity becomes greater closer to the wall as the flow becomes turbulent. Transition to turbulent flow is generally observed by a sudden, rapid growth in the height of the boundary layer. The growth of the boundary layer for water injection is shown in figure 62 and reveals this characteristic.

An additional criterion to characterize turbulent flows is that the velocity distribution in the wall region should follow the well known law of the wall, such that

$$u^+ = A \ln y^+ + B,$$

where A and B are universal constants having the value of 2.5 and 5.5, respectively. Patel (1968) indicates that, for developing flow, the value of B decreases until the value of 5.5 is achieved for fully-developed turbulent flow. Application of this concept to the channel flows under discussion revealed the variation in the value of B as shown in figure 63 as a function of length Reynolds number. A representative value of B = 7.2 was used in this study for the turbulent flow region which is not fully developed. Figure 64 presents a plot of the law of the wall in inner variables
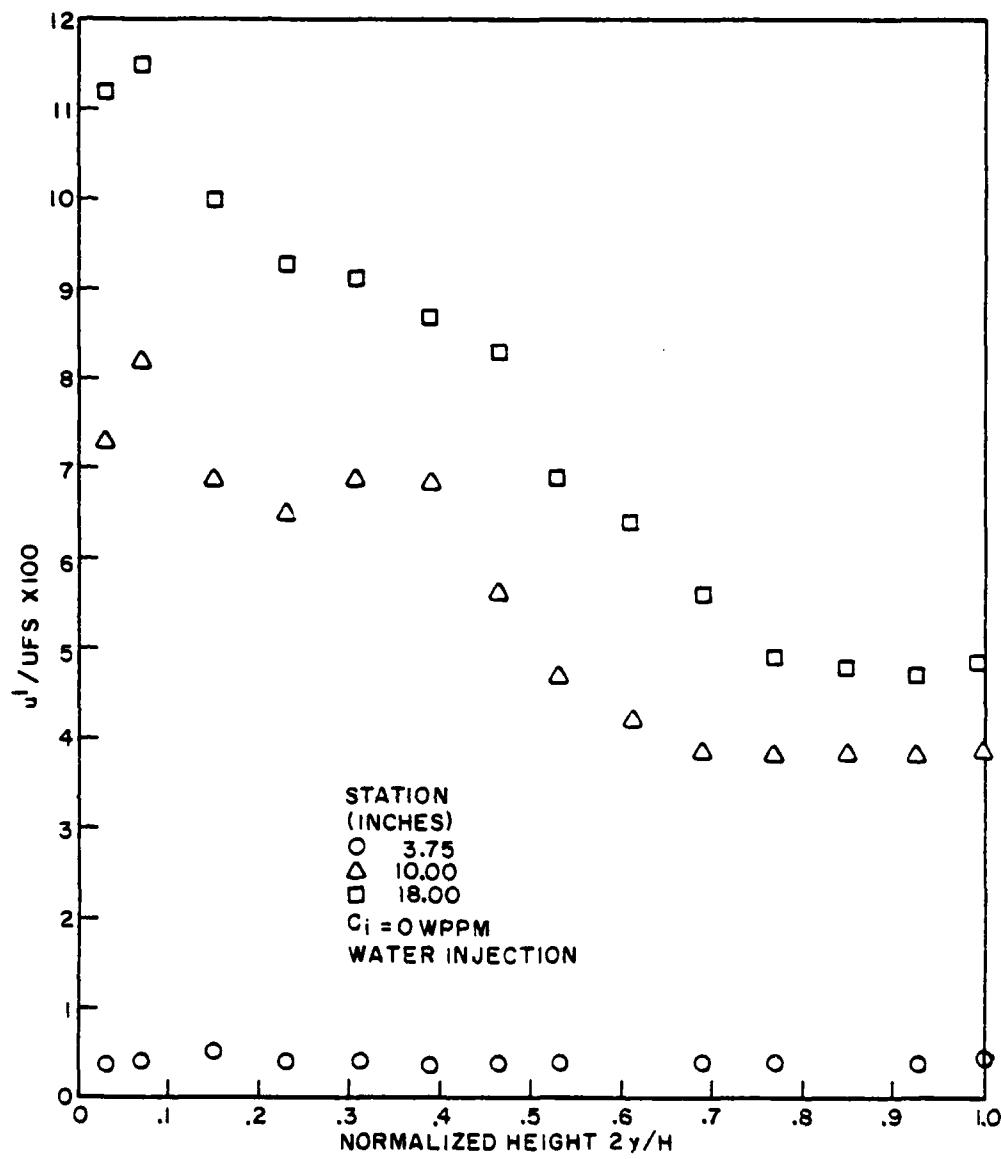
Figure 61.   Turbulence intensity distributions for
transitional velocity profiles
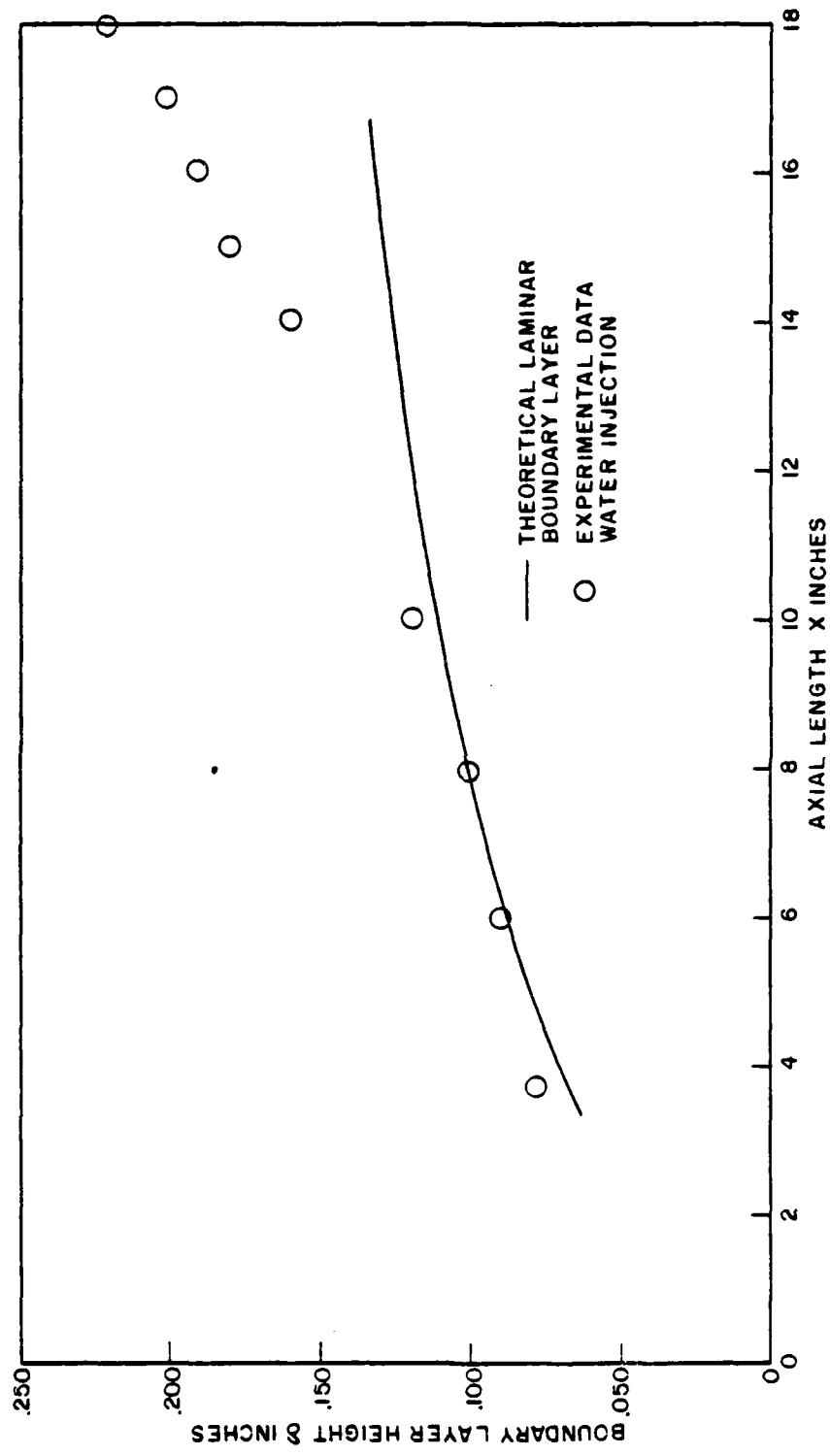
Figure 62. Growth of boundary layer for water flow
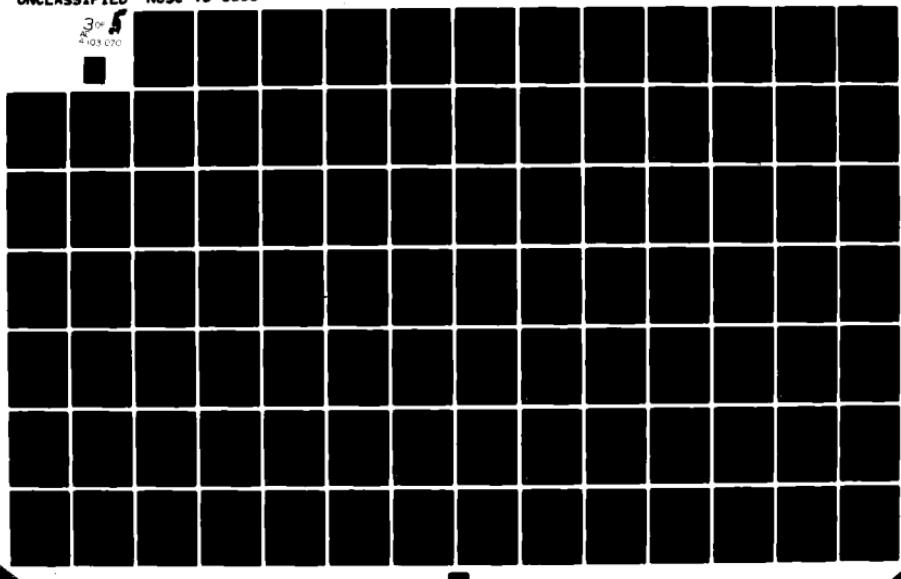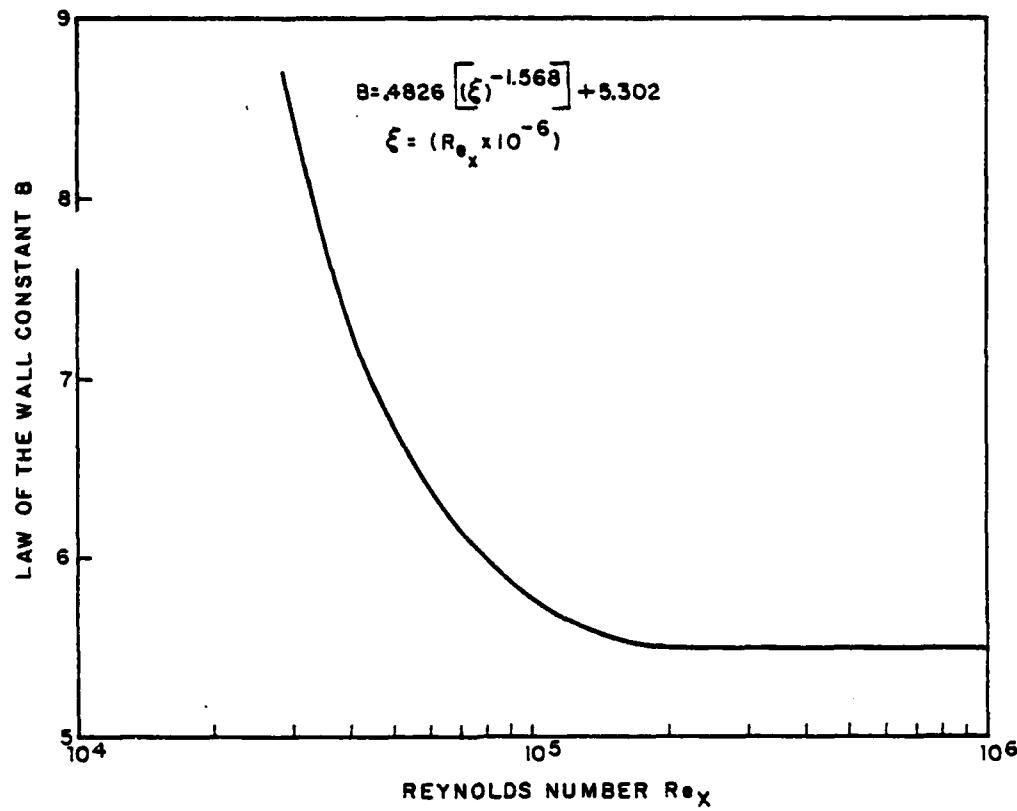
Figure 63. Variation in value of B as a function of
Reynolds number for developing channel flow
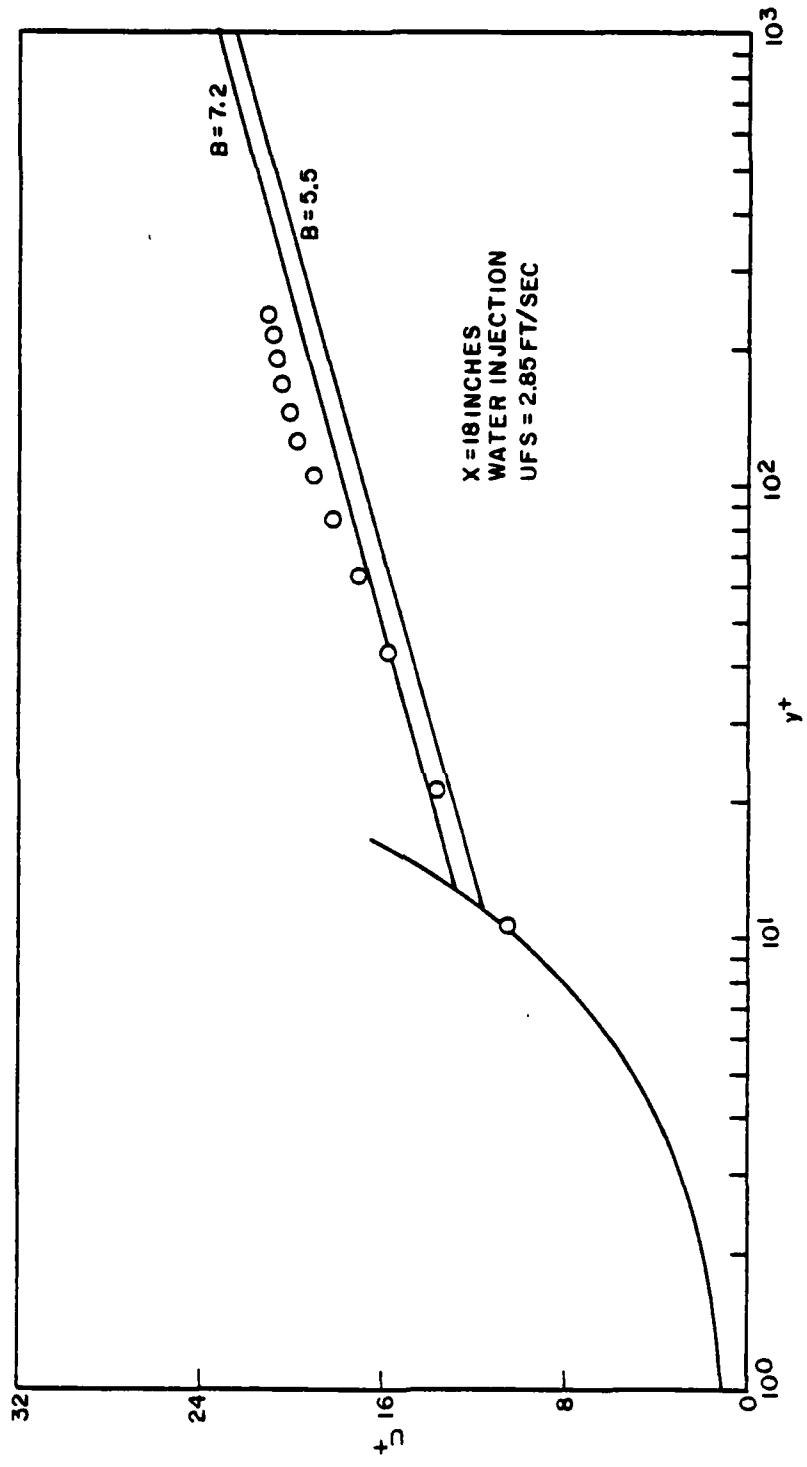
Figure 64. Law of the wall for developing flow

y^+ and u^+ for B = 5.5 and 7.2. A comparison is also made with experimental data showing excellent agreement with the value of B = 7.2. The value of B = 7.2 is therefore used in the expression for the law of the wall to account for the developing velocity profile aspect of the flow.

Polymer Flow Boundary Layer Characterization

Developing velocity profiles for injected polymer concentrations of C_i = 100, 200, 400, 500, and 800 WPPM are presented in figures 65 through 69. The profiles are from stations x = 3.75, 10.0, and 18.0 inches representing laminar, transitional, and turbulent flow regions. The characteristic increasing fullness of the velocity profiles over the length of the plate is evident in each test sequence.

Turbulence intensity distributions for these profiles are plotted in figures 70 through 74. The freestream turbulence intensity was .3% for the tests where water and polymer concentrations equal to 100, 200, and 400 WPPM were injected. For the injection of polymer at 500 and 800 WPPM, the freestream turbulence level was .4% and .5%, respectively.
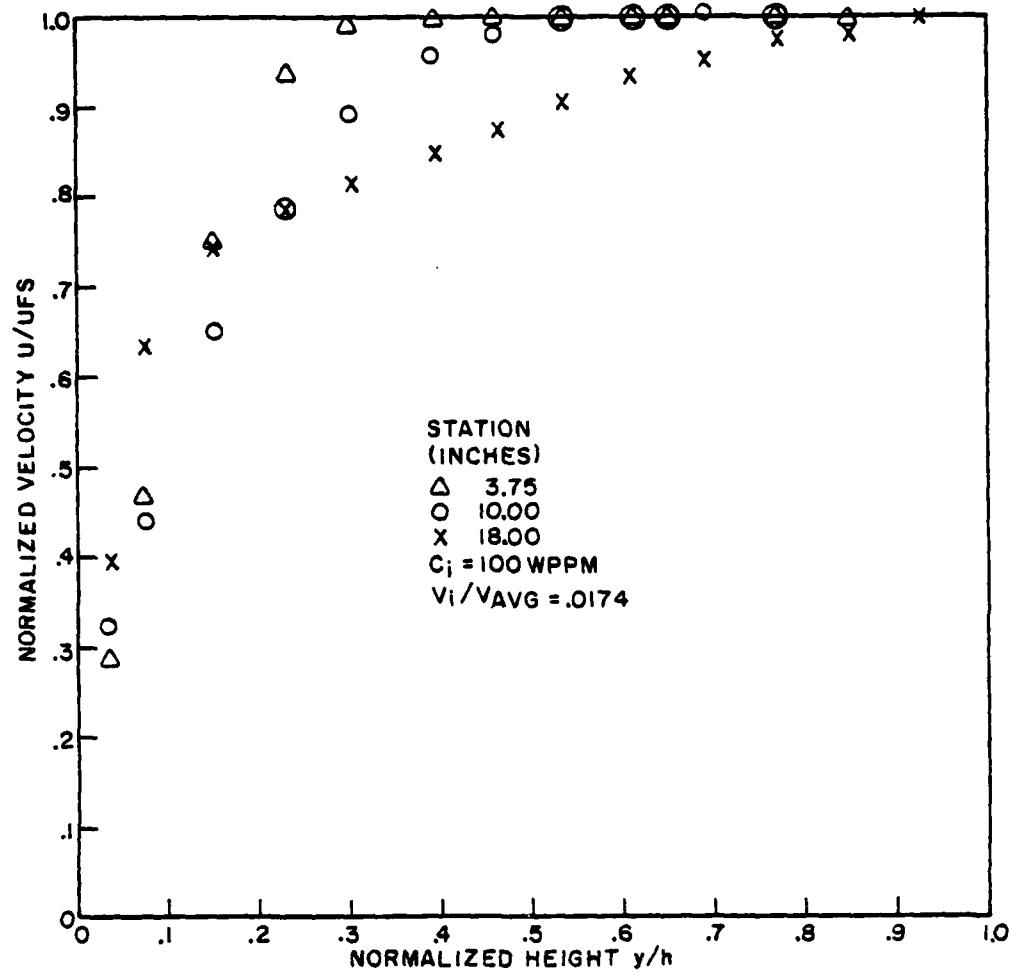
Figure 65. Developing velocity profiles for injected polymer concentration C_i = 100 WPPM
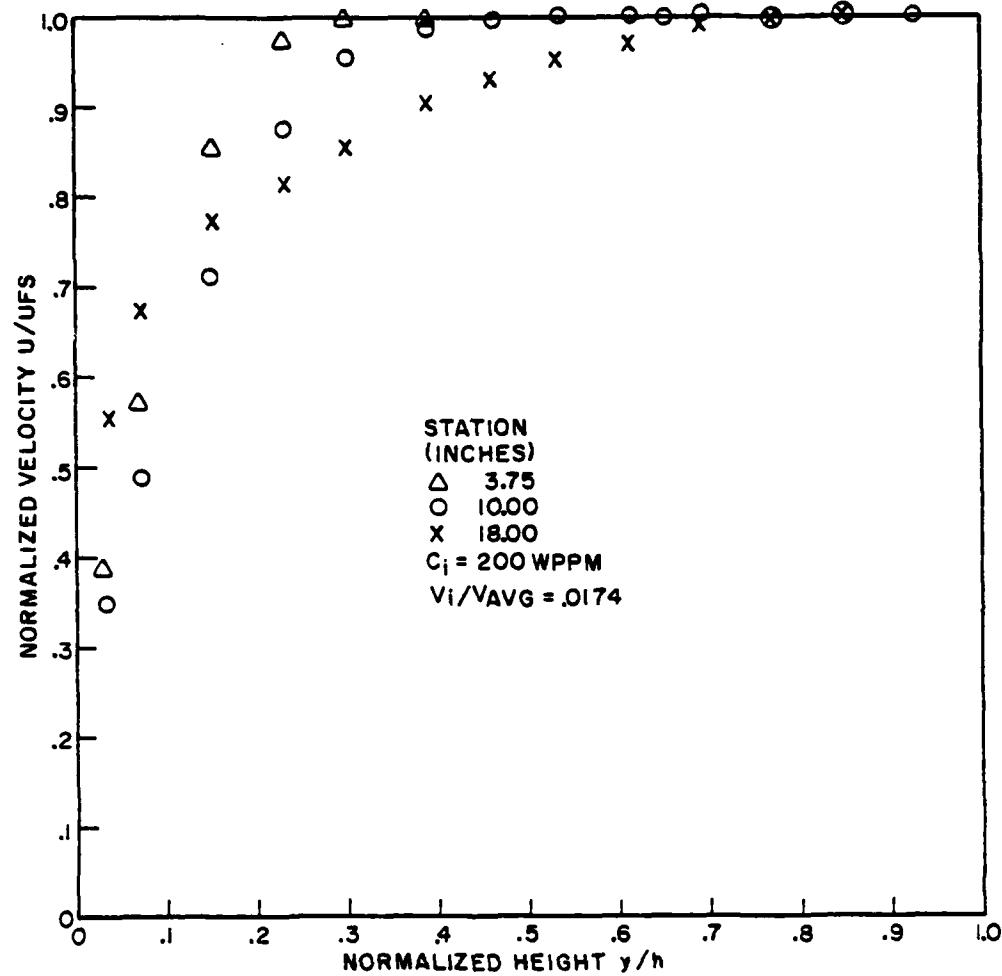
Figure 66. Developing velocity profiles for injected polymer
concentration C_i = 200 WPPM
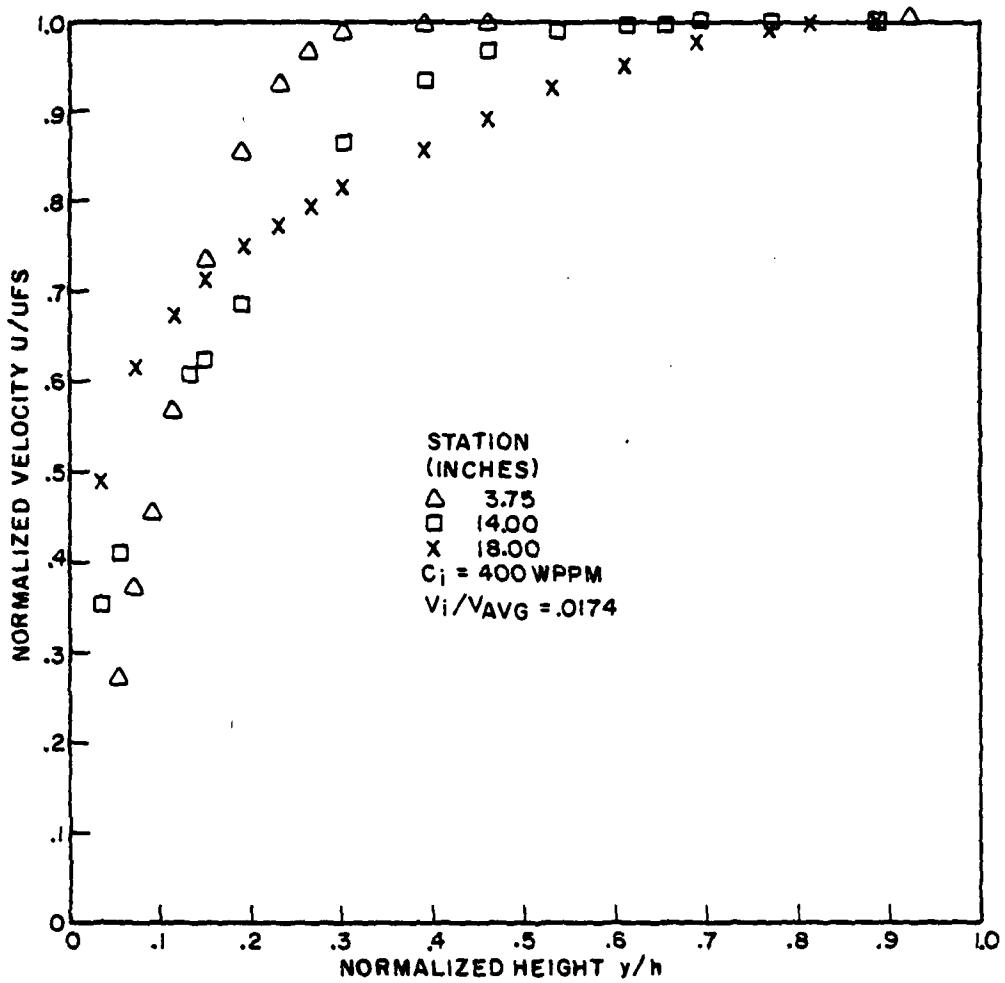
Figure 67. Developing velocity profiles for injected polymer concentration C_i = 400 WPPM
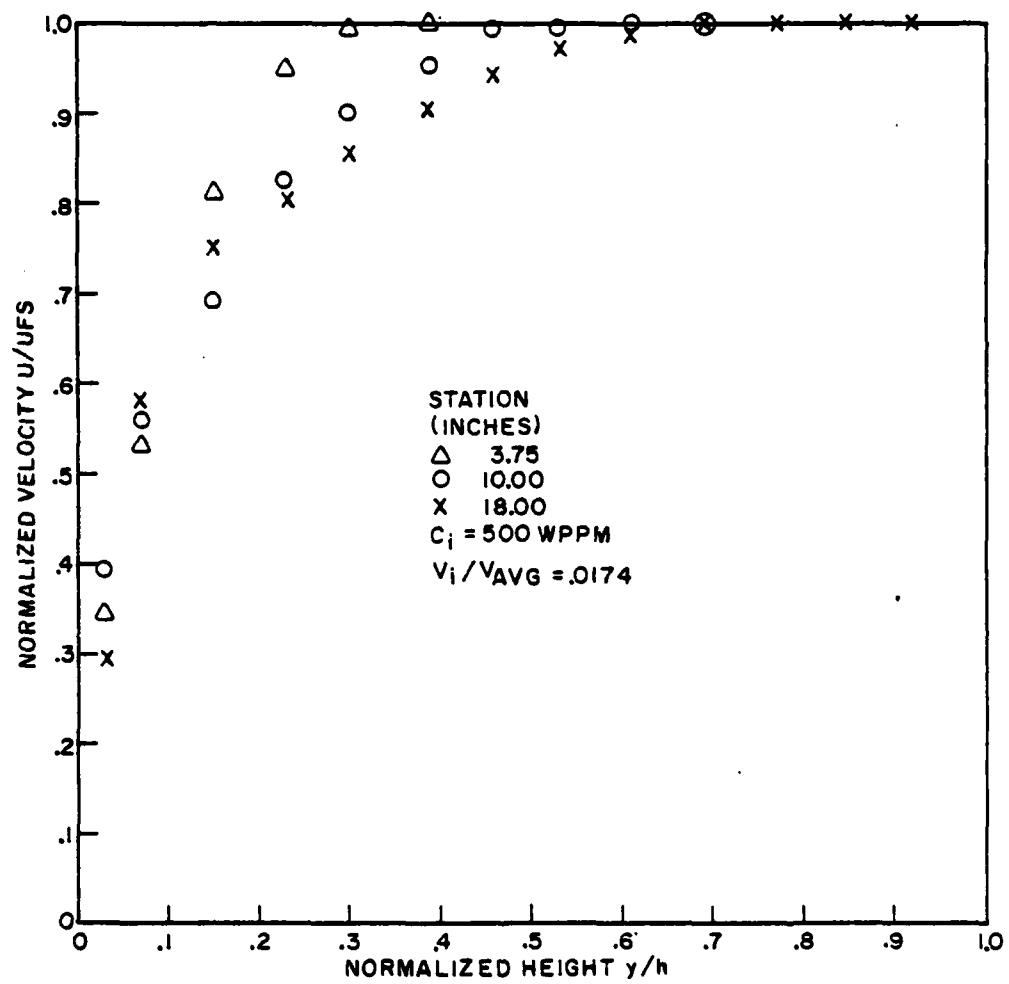
Figure 68. Developing velocity profiles for injected polymer concentration C_i = 500 WPPM
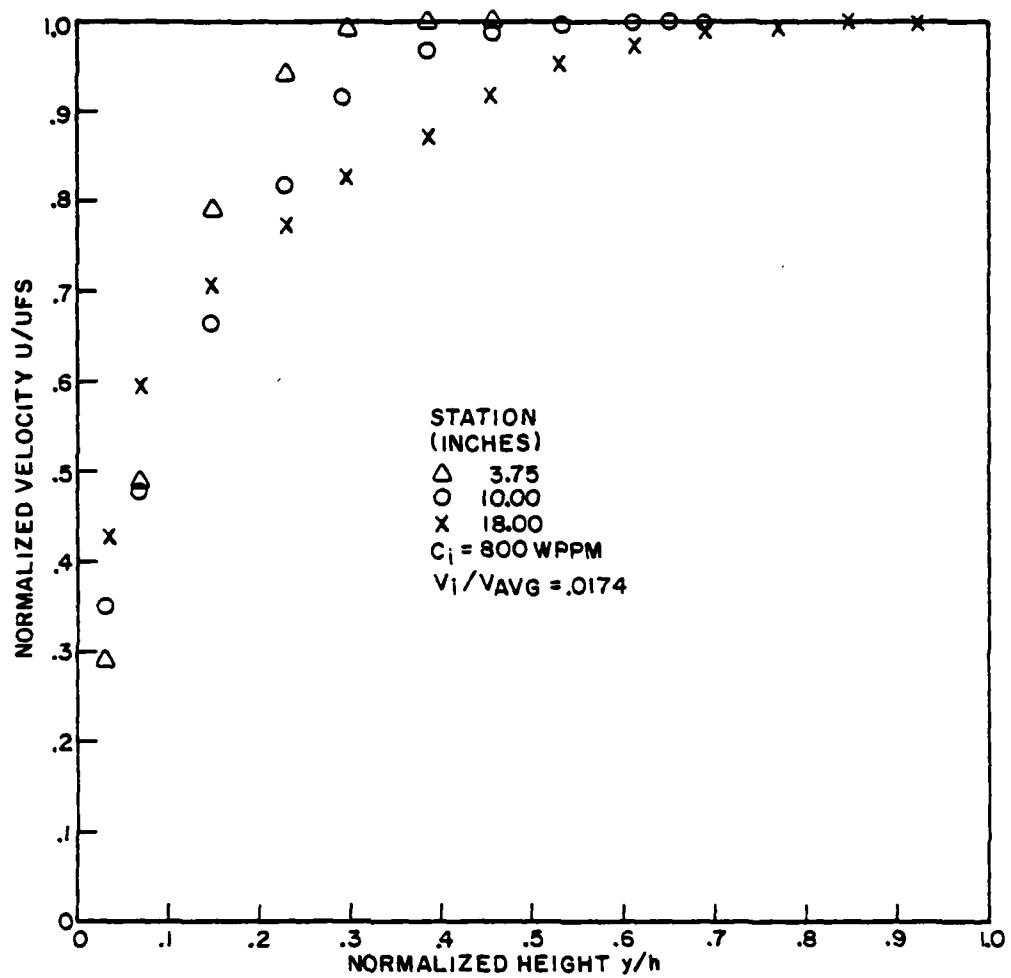
Figure 69. Developing velocity profiles for injected polymer
concentration C_i = 800 WPPM
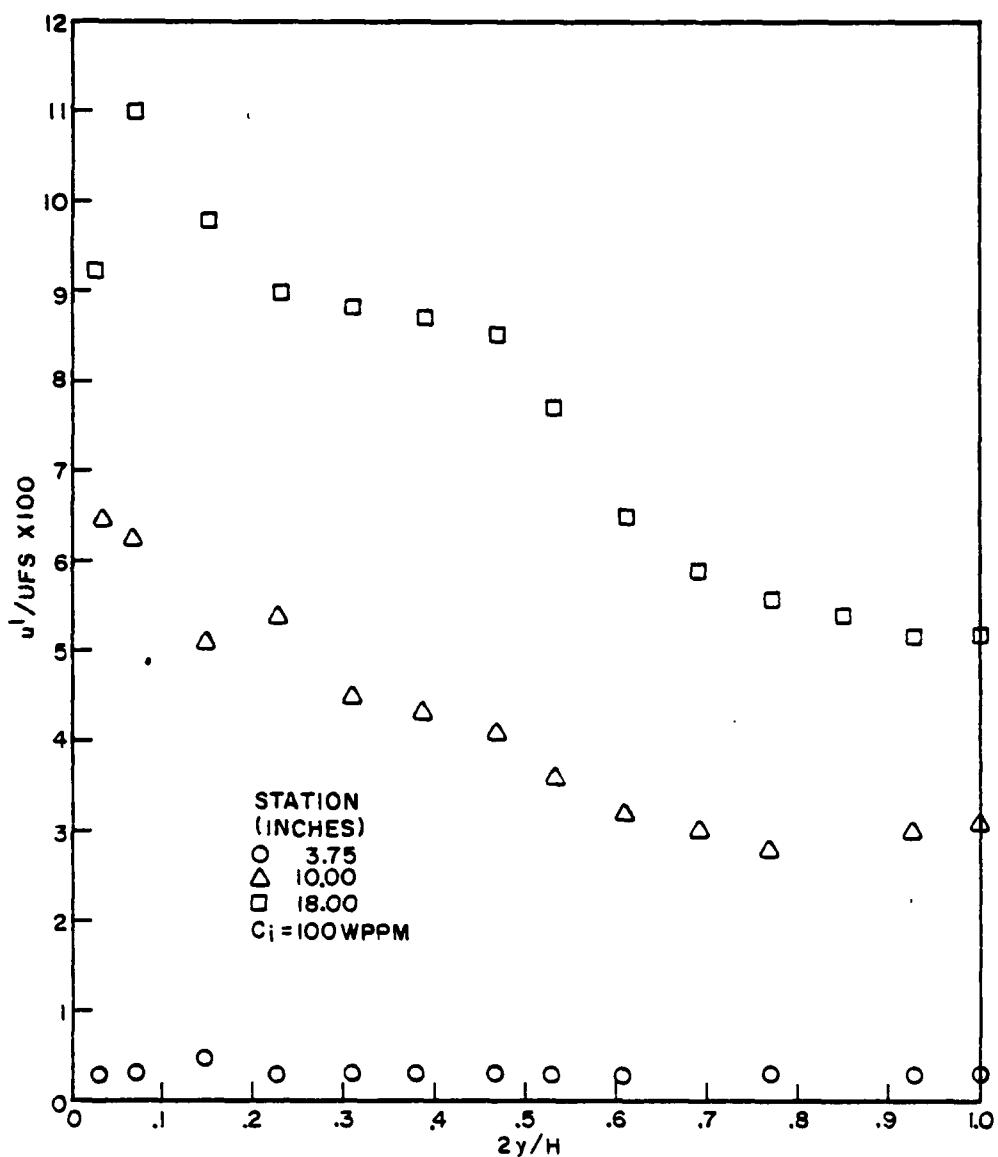
Figure 70.   Turbulence intensity distribution for injected polymer
concentration C_i = 100 WPPM
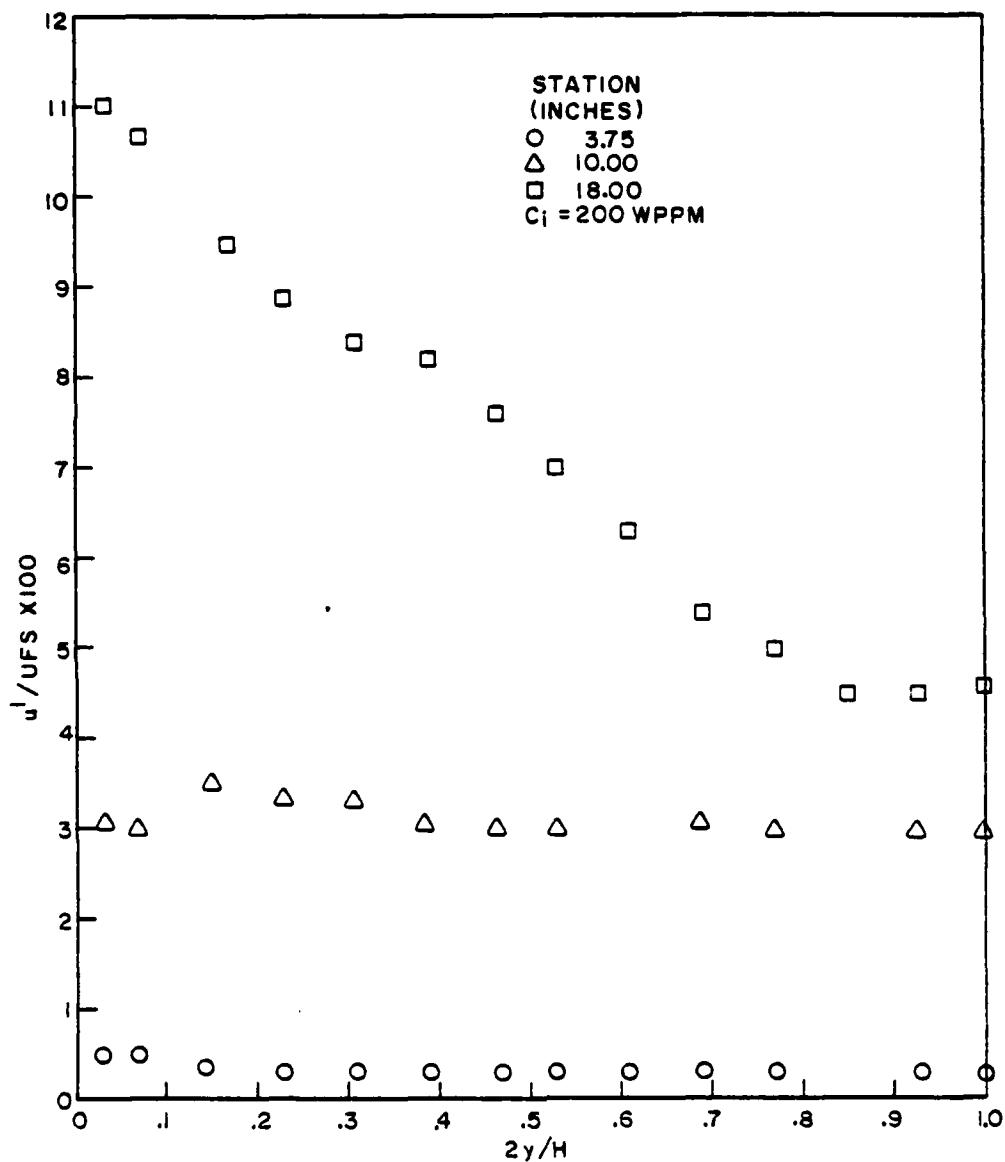
Figure 71. Turbulence intensity distribution for injected polymer concentration C_i = 200 WPPM
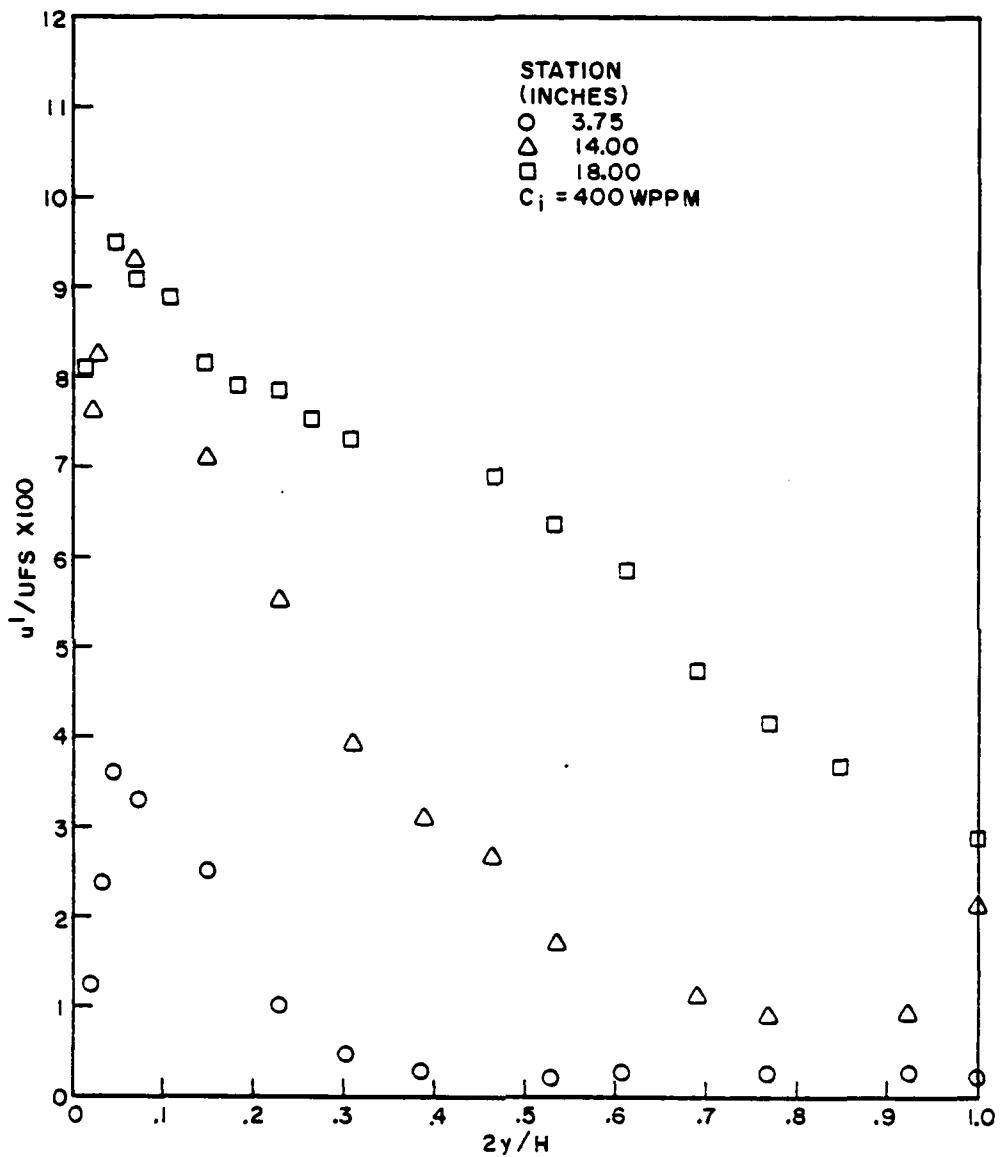
Figure 72. Turbulence intensity distribution for injected polymer concentration C_i = 400 WPPM
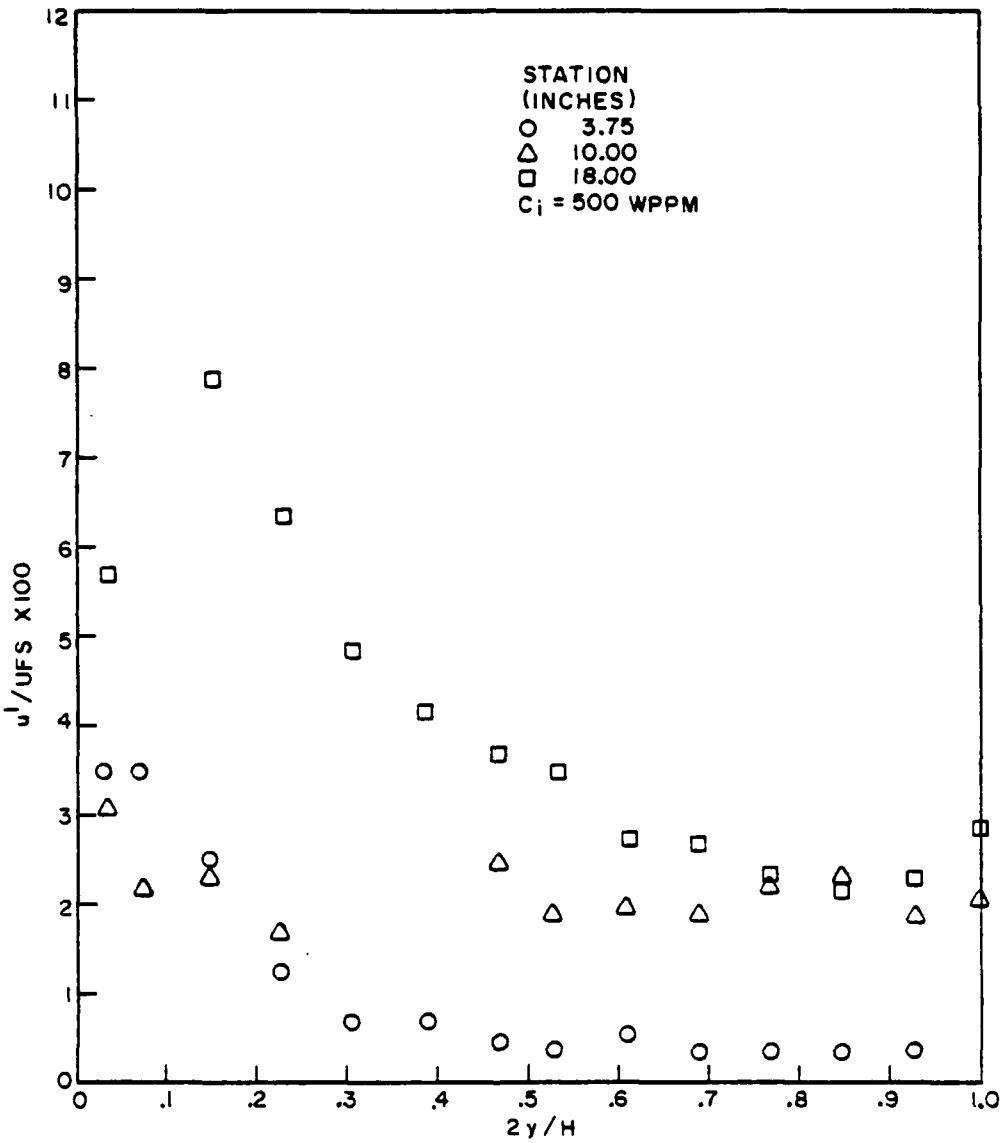
Figure 73.   Turbulence intensity distribution for injected polymer
concentration C_i = 500 WPPM
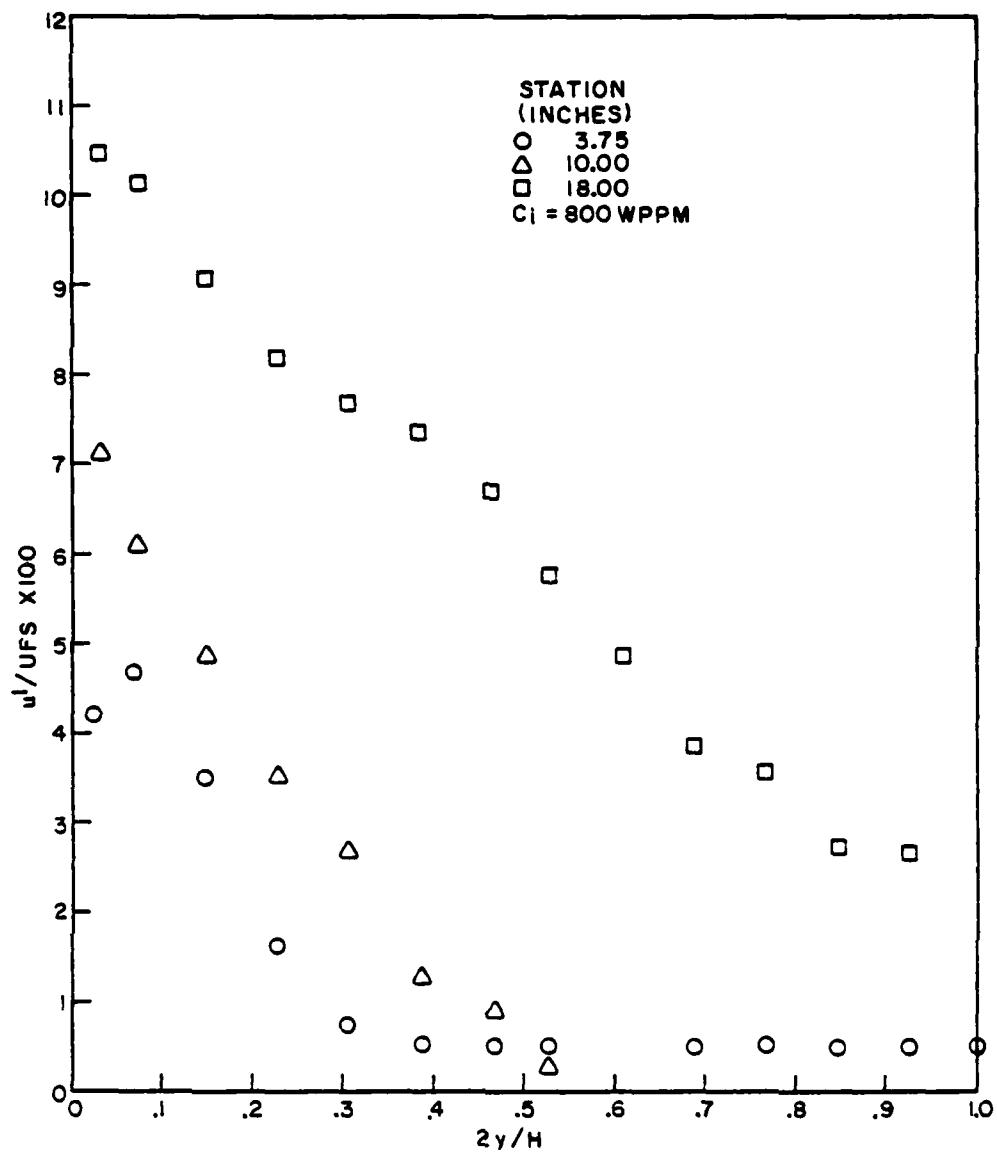
Figure 74. Turbulence intensity distribution for injected polymer concentration C_i = 800 WPPM

The velocity profiles at $x = 3.75$ inches are all laminar in shape. At $x = 10$ inches, the velocity profiles appear less turbulent than the water case. This characteristic is also shown in the lower turbulence intensity distributions at this station for all concentrations injected. The polymer causes a reduction in the peak value of turbulence intensity and shifts the position further away from the wall. The entanglement or "hand holding" of the long chain polymers may act as a woven blanket to dampen or suppress turbulence.

A comparison of velocity profiles at $x = 18$ inches, normalized with respect to boundary layer thickness and freestream velocity compared with a 1/5th power law velocity profile in figure 75 shows good agreement for the $C_i = 400$ WPPM injection case. The $C_i = 500$ WPPM injection data also plotted on this figure indicate a lesser developed profile and a possible thicker sublayer.

When 2000 WPPM was injected through the .005-inch wide slot at a velocity ratio $U/U_{avg} = .280$, the freestream turbulence intensity was 3.0%. Figure 76 presents a comparison of the velocity profile development for the 2000 WPPM injection case with the comparable water injection case. These data, taken at $x = 3.75$ inches (2 inches downstream from the injection slot), indicate a disturbance effect due to the higher injection velocity for
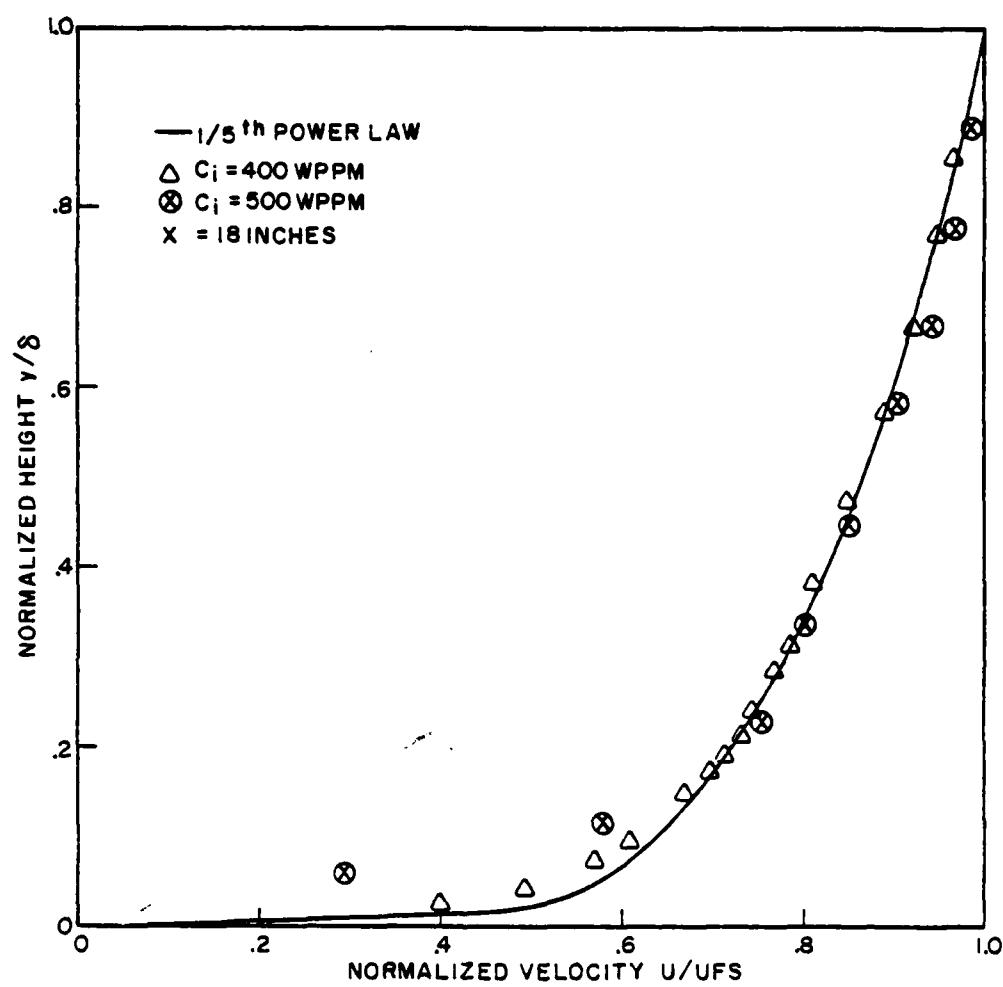
Figure 75. Comparison of non-dimensionalized velocity profiles
at station x = 18 inches with theoretical 1/5th
power law profile
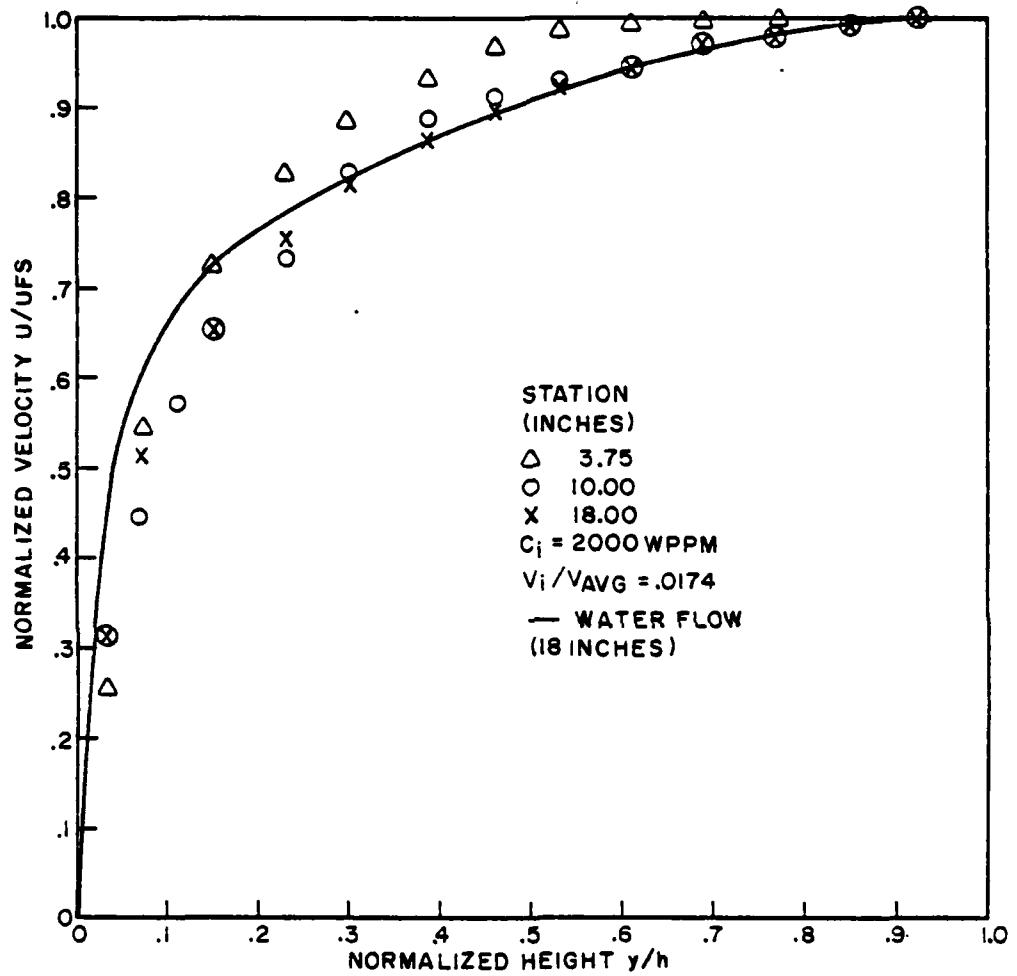
Figure 76. Developing velocity profiles for injected polymer
concentration C_i = 2000 WPPM

the narrow slot. The tendency of the thick 2000 WPPM injection concentration to stay together, coupled with the high injection velocity, contribute to the disturbance of the boundary layer.

Here again, at $x = 18$ inches, the polymer velocity profile compares well with the water turbulent velocity profile in the outer region of the boundary layer. Closer to the wall, the polymer velocity profile departs from the water profile and is less full. This anomaly in the near wall region may be due to a thickening of the laminar sublayer.

Figure 77 presents a comparison of the turbulence intensity distributions at $x = 18$ inches for the 2000 WPPM polymer injection case and a similar water case at the same injection velocity ratio ((V_i/Vavg $= .280$) and slot width ($s = .005$ inches). The characteristic lowering of the peak value of the turbulence intensity and shifting of its location away from the wall are also shown.

The lowering and redistribution of the primary turbulent activity into a broader band found in this study is supported by the work of Reischman (1973). However, due to experimental differences only qualitative comparisons can be made with the work of previous researchers. Rudd's data, typical of Logan's and Kumor and Sylvester's measurements, are presented in figure
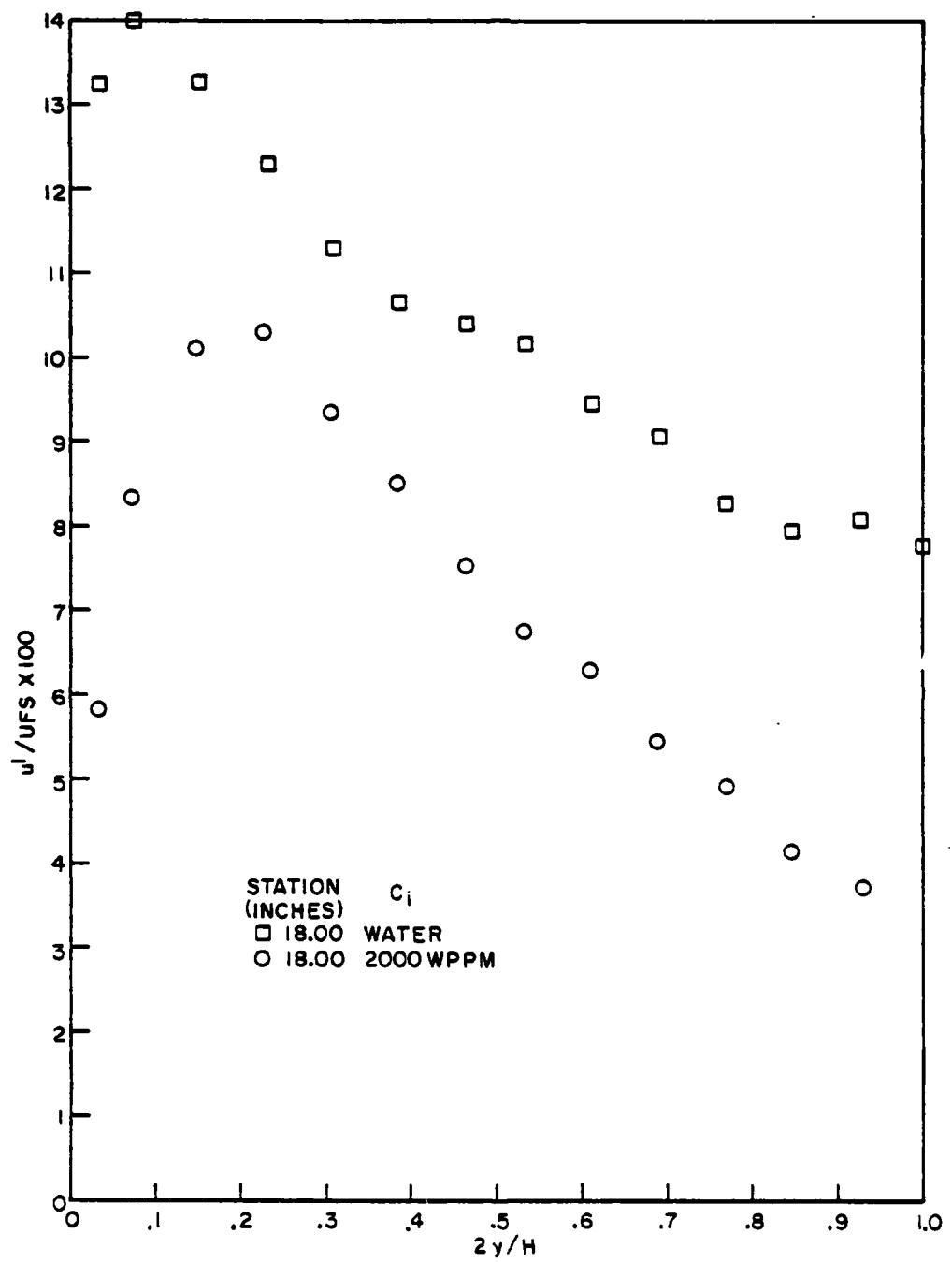
Figure 77. Turbulence intensity distribution for injected polymer
concentration C_1 = 2000 WPPM

78 taken from Reischman (1973). However, Reischman used a rectangular channel while the others conducted their experiments in a square duct.

A controversy exists concerning the so-called "sublayer thickening" which is represented by the velocity profile remaining linear to a y^+ value of 15 to 20 instead of the solvent value of 8 to 9. There have been five investigations (including this one) that have made velocity measurements at $y^+ < 10$. Table 13 from Reischman (1973) indicates the extent of the linear curve and the lowest y^+ location at which velocity measurements were made in each prior investigation. Data from this investigation have also been included. Reischman alone concluded that the sublayer does not thicken based on the maximum value of y^+ for which $u^+ = y^+$. Previous results achieving larger values of y^+ which indicated a thicker sublayer were held in serious question due to experimental facilities and technique. This problem was discussed in section II which reviewed experimental facilities. It has been found that in the rectangular channel used in this research linear values of y^+ up to 15 have been achieved with Polyox WSR-301. Figures 79 through 83 present velocity profiles plotted in terms of inner variables and compared to the law of the wall. It is recommended that future investigations concentrate their efforts in the region of y^+ from 7 to 50 to provide additional data at the interface of the sublayer and the buffer region or elastic

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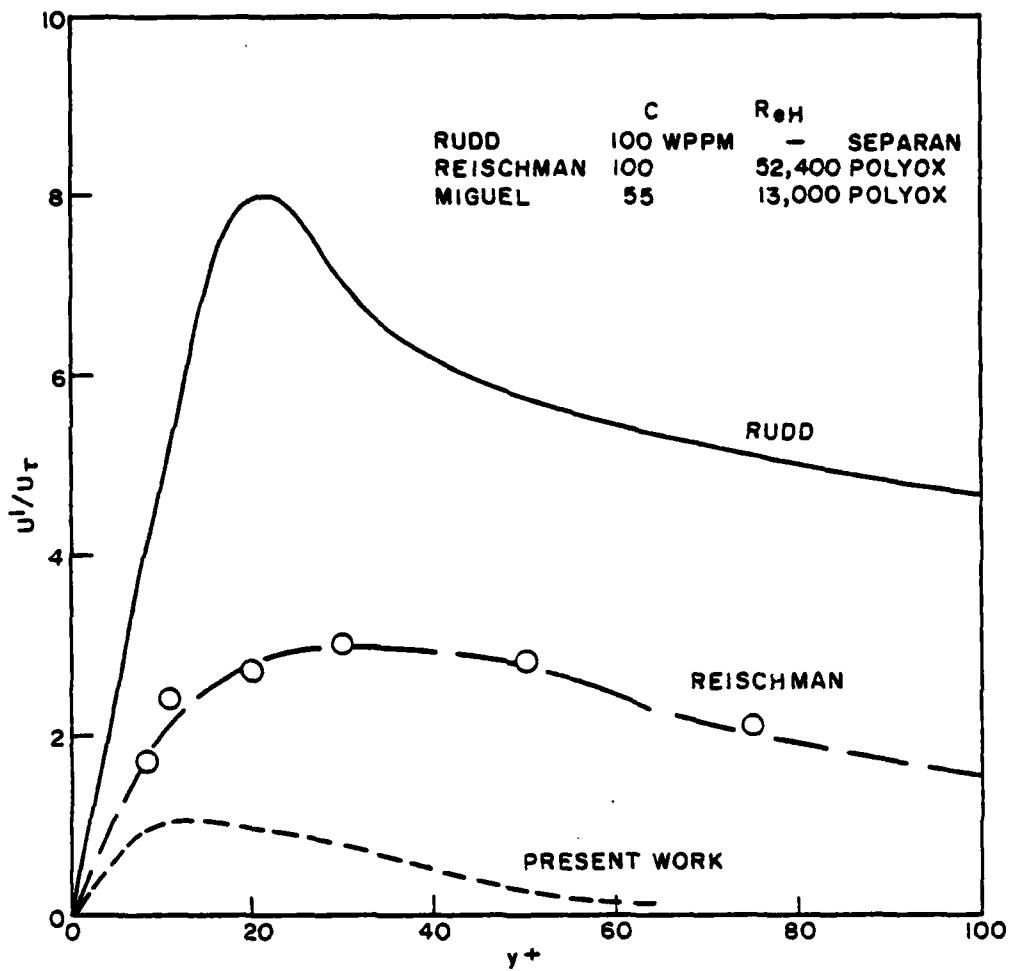
Figure 78.   Turbulence intensity distribution
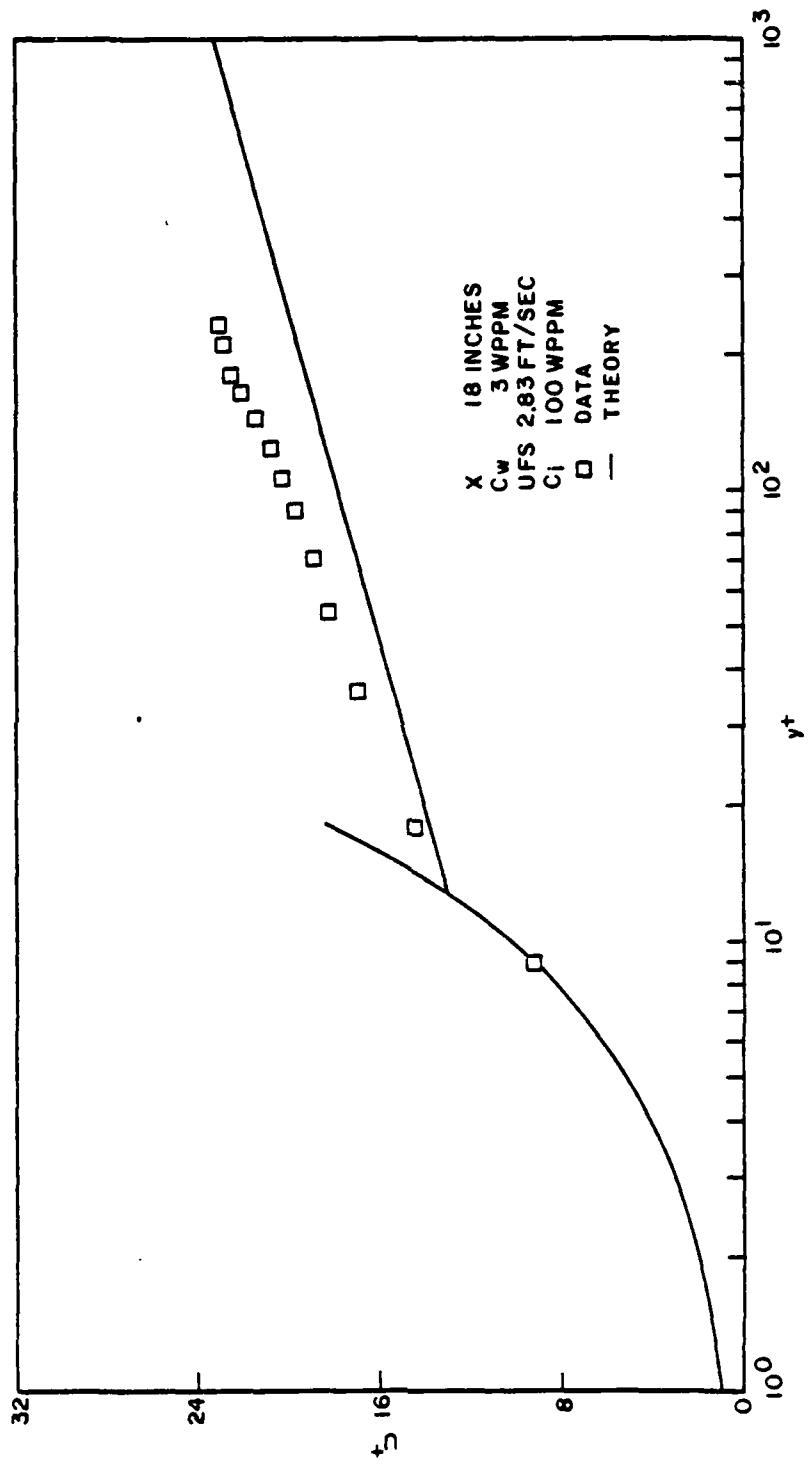
Figure 79. Comparison of the law of the wall with experimental data for injected polymer concentration $C_1 = 100$ WPPM
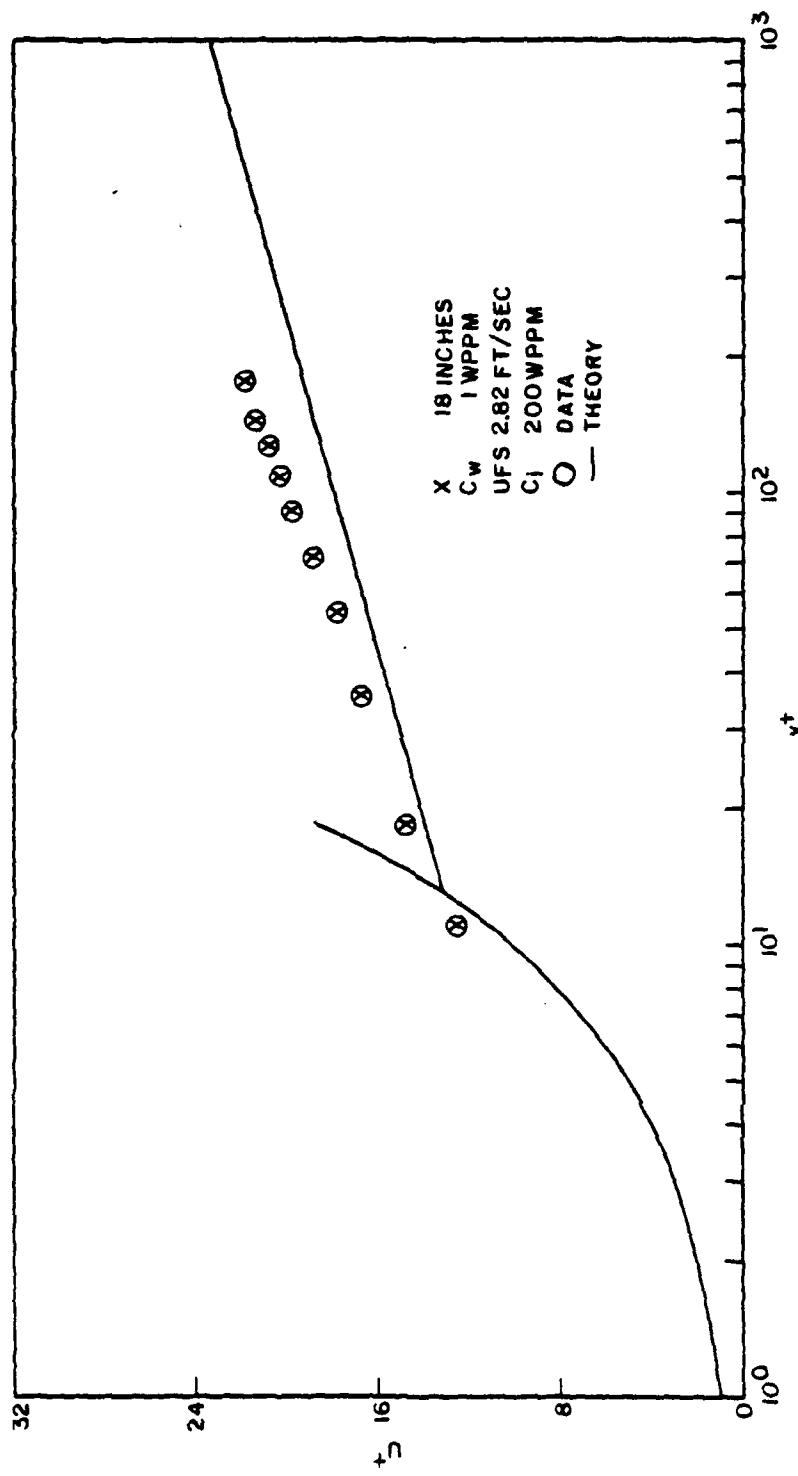
Figure 80. Comparison of the law of the wall with experimental data
for injected polymer concentration C_i = 200 WPPM
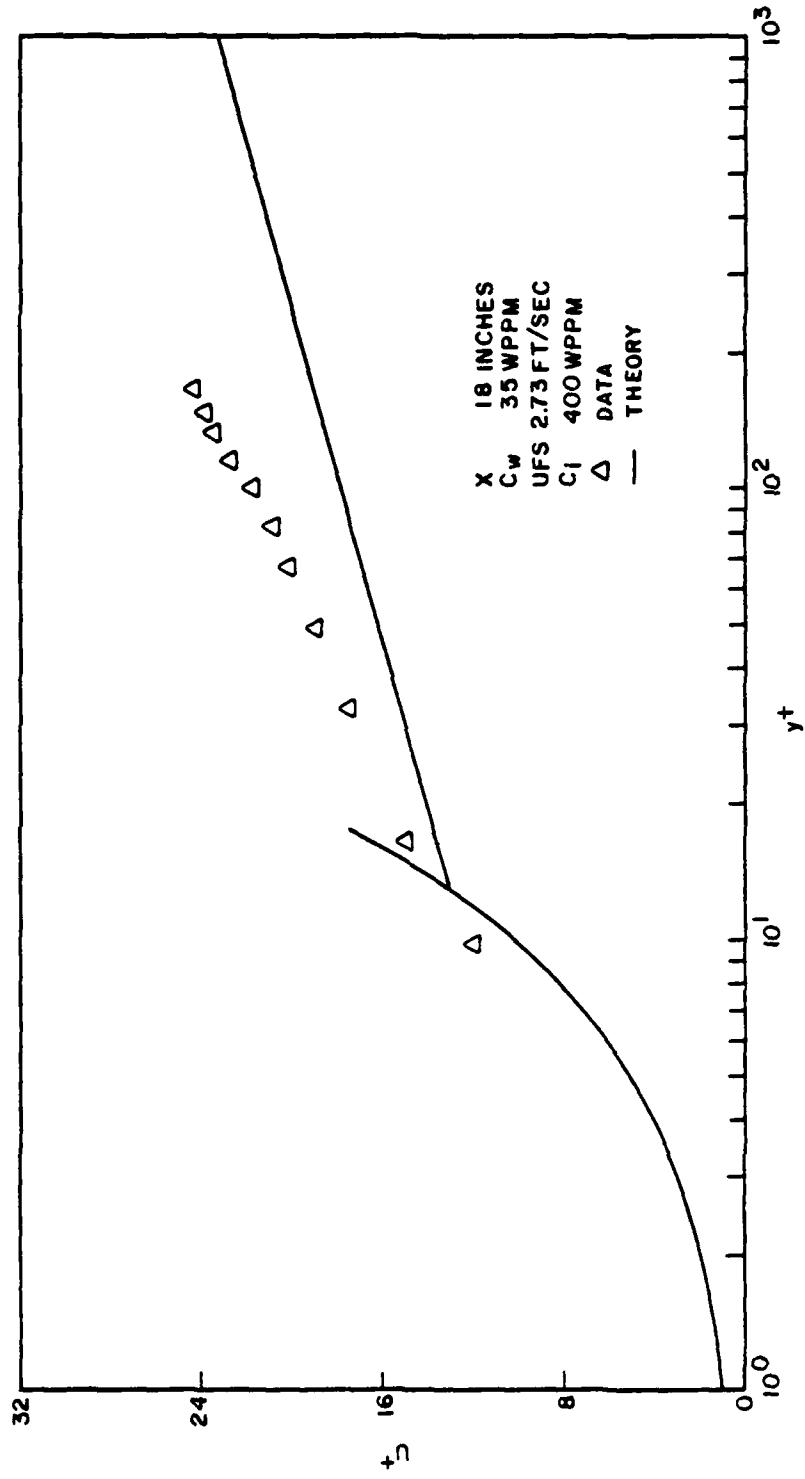
Figure 81. Comparison of the law of the wall with experimental data for injected polymer concentration C_i = 400 WPPM
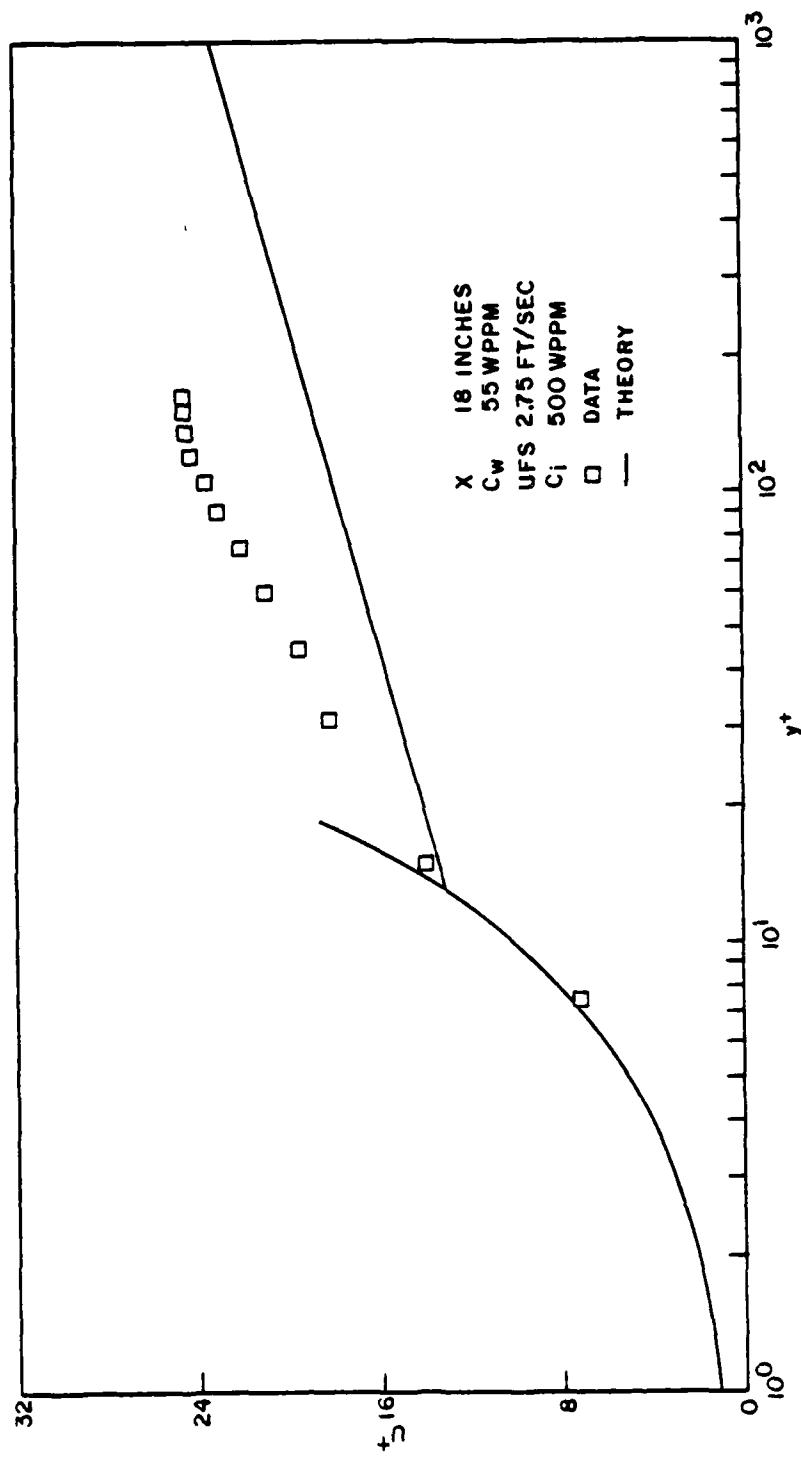
Figure 82. Comparison of the law of the wall with experimental data for injected polymer concentration $C_i = 500$ WPPM
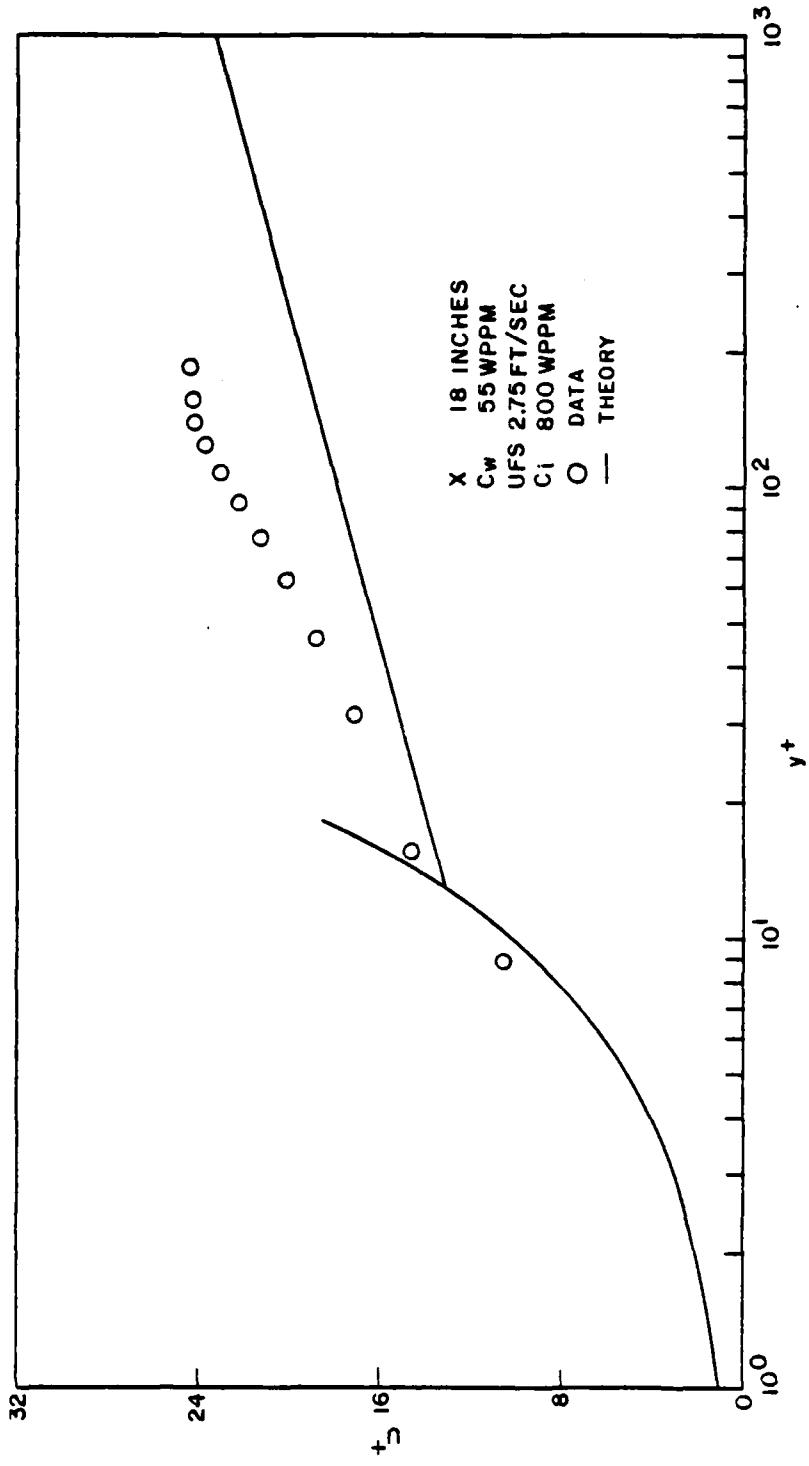
Figure 83. Comparison of the law of the wall with experimental data for injected polymer concentration C_i = 800 WPPM

TABLE 13

COMPARISON OF NEAR WALL LASER ANEMOMETER
MEASUREMENTS IN DRAG REDUCING FLOWS

Author	Maximum y^+ for Which $u^+=y^+$	Lowest y^+ Measurements
Lazar	10	10
Rudd	19	8
Kumor & Sylvester	16	3
Reischman	9	3
Present Work	15	7

sublayer which joins the viscous sublayer and the elevated log region. Figure 84 indicates the large variation in the velocity profile in this region. Profiles in the outer region tend toward the turbulent while becoming more laminar close to the wall.

Comparison of Analytical Predictions With Experimental Results

Typical boundary layer growth patterns are presented in figure 85 through 88 for various concentrations injected at a constant rate of $V_i/Vavg = .0174$. There is excellent agreement between the experimental data and the theoretical profile predicted by the mathematical flow model utilizing the effective Reynolds number analogy. The gradual increase in the boundary layer height for the 500 WPPM injection case may be explained in terms of the water temperature and associated solvent viscosity. For this case, the water temperature was the coldest at $57^{\circ}F$. The resulting higher kinematic viscosity resulted in lower axial Reynolds number affecting transition and the growth of the boundary layer.

Skin friction calculations as a function of local polymer wall concentration were based on three techniques: (1) slope of the velocity profile, (2) Clauser technique, and (3) flow model. The results, indicating good agreement, are presented in tables 14 through 19. Skin friction computation from the slope of the velocity profile at the wall is relatively straightforward and will not be discussed in detail. This method is sensitive, however, to the value of the velocity near the wall. Velocity measurements must be made in the vicinity of the laminar sublayer and are susceptible to velocity gradient biasing in the laser measuring volume.
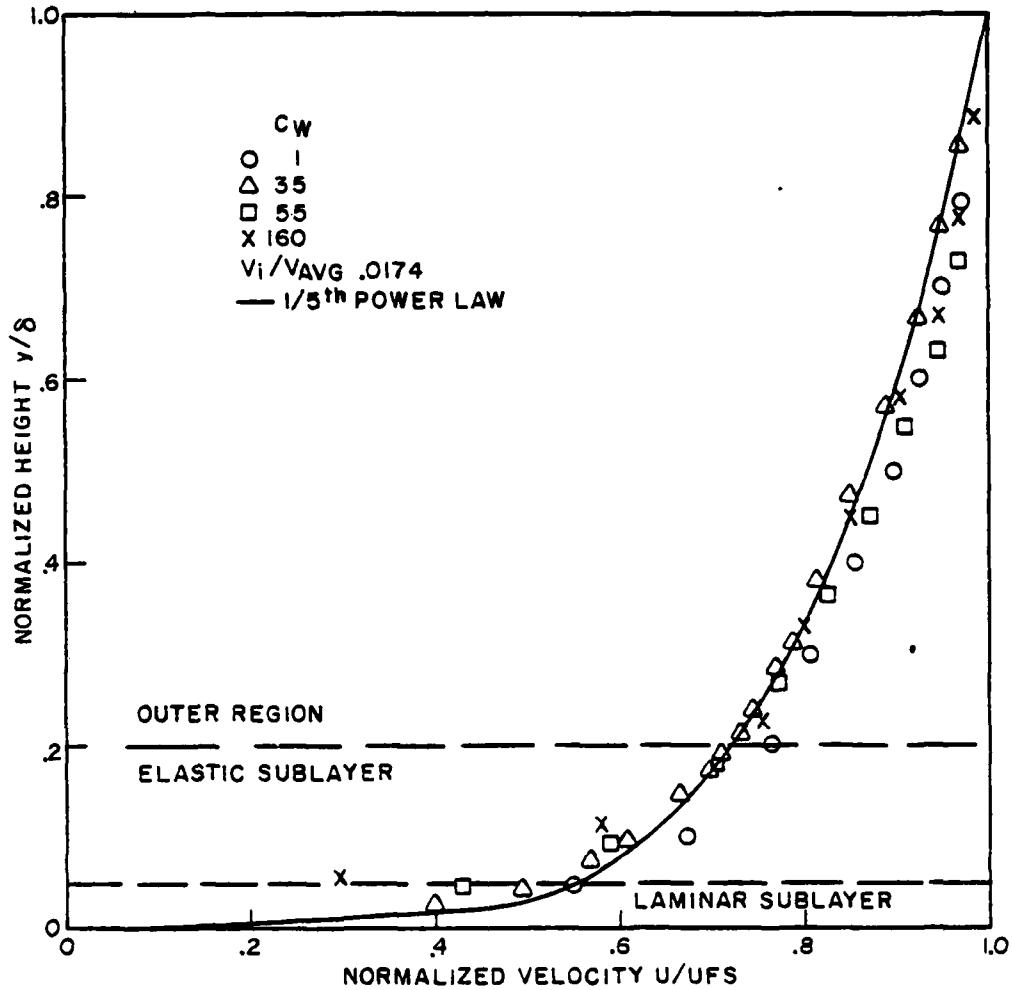
Figure 84. Comparison of normalized velocity profiles with
1/5th power law at station x = 18 inches
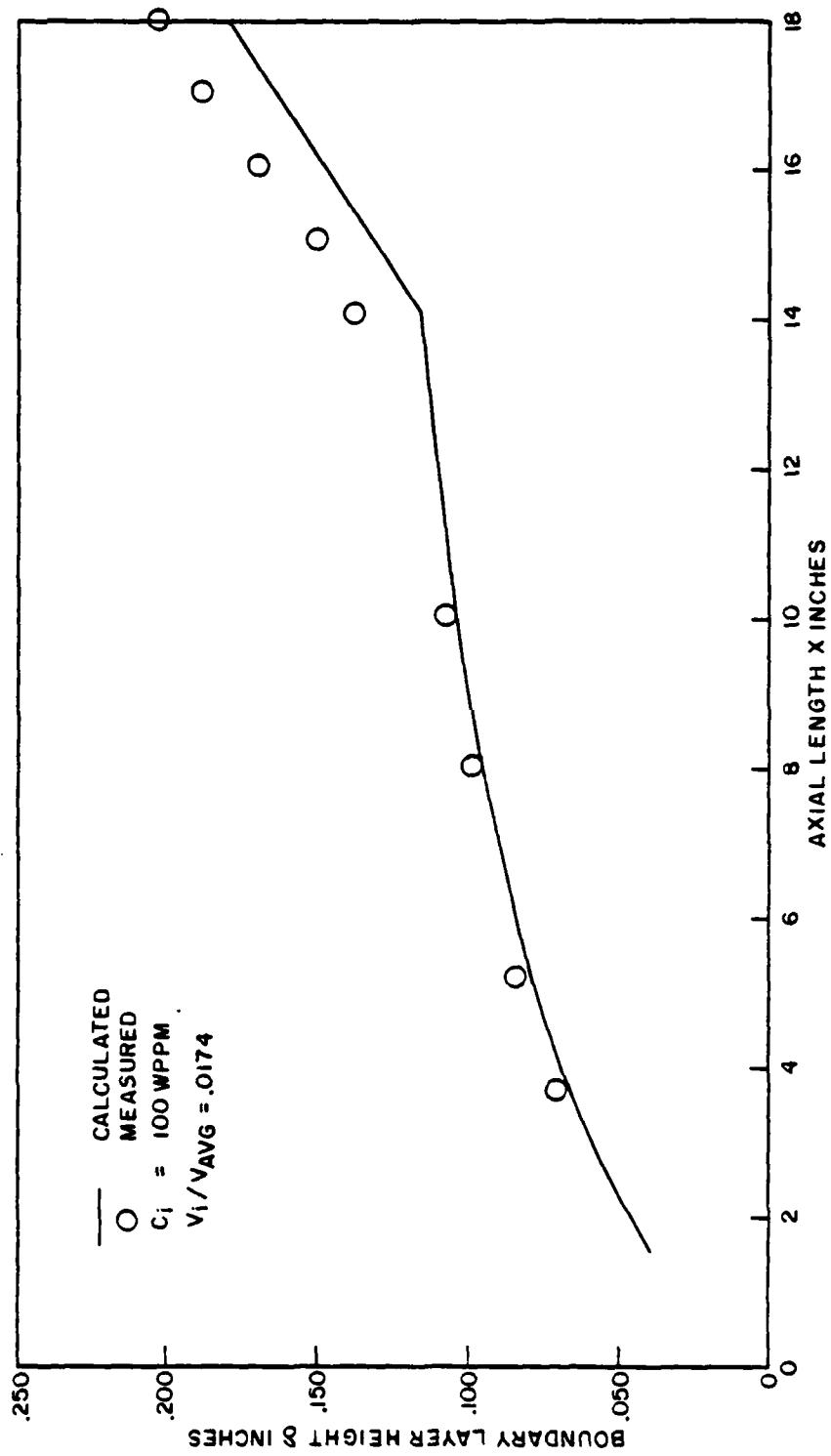
Figure 85. Comparison of experimental boundary layer growth with analytically predicted values for injected polymer concentration $C_1 = 100$ WPPM

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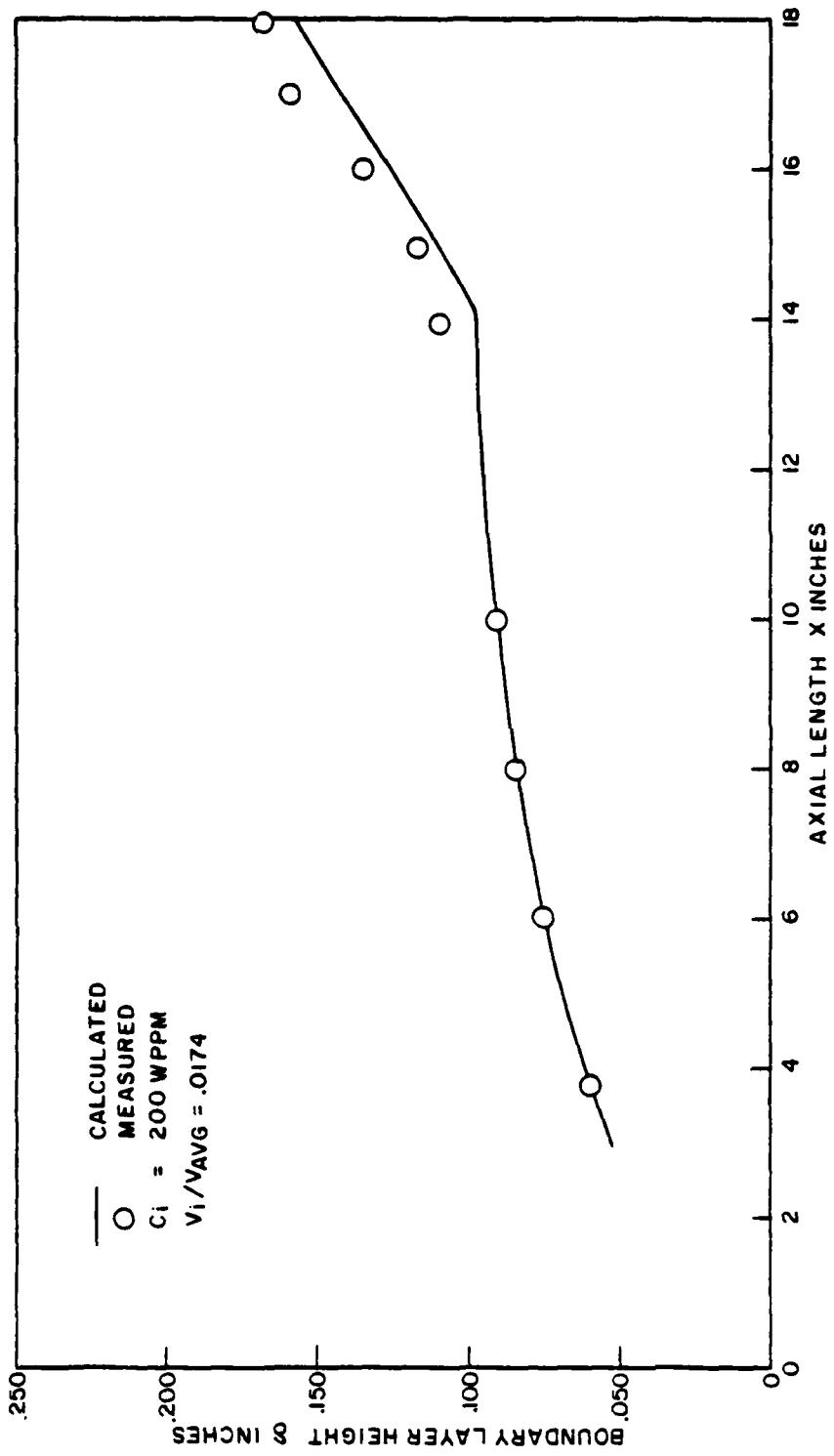
Figure 86. Comparison of experimental boundary layer growth with analytically predicted values for injected polymer concentration C_i = 200 WPPM
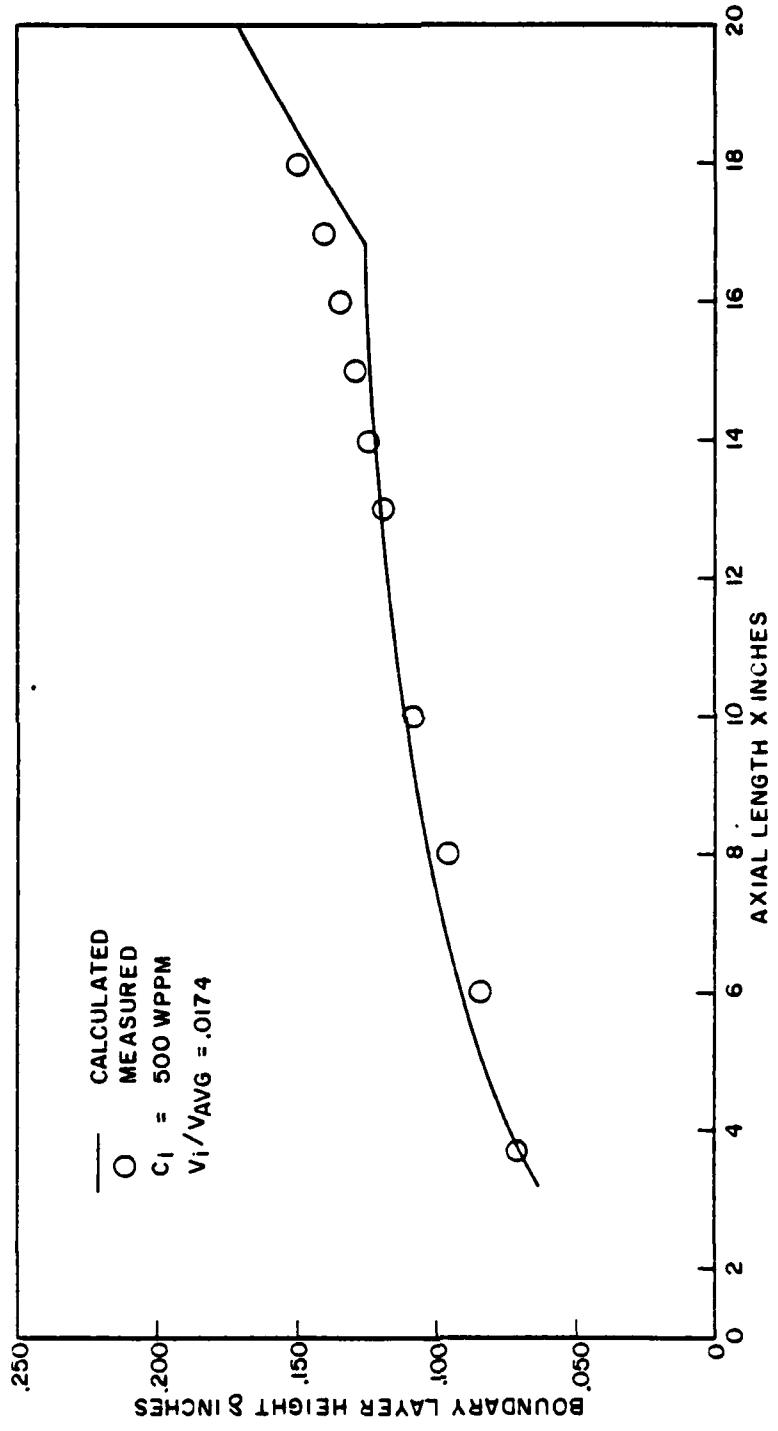
Figure 87. Comparison of experimental boundary layer growth with analytically predicted values for injected polymer concentration C_1 = 500 WPPM
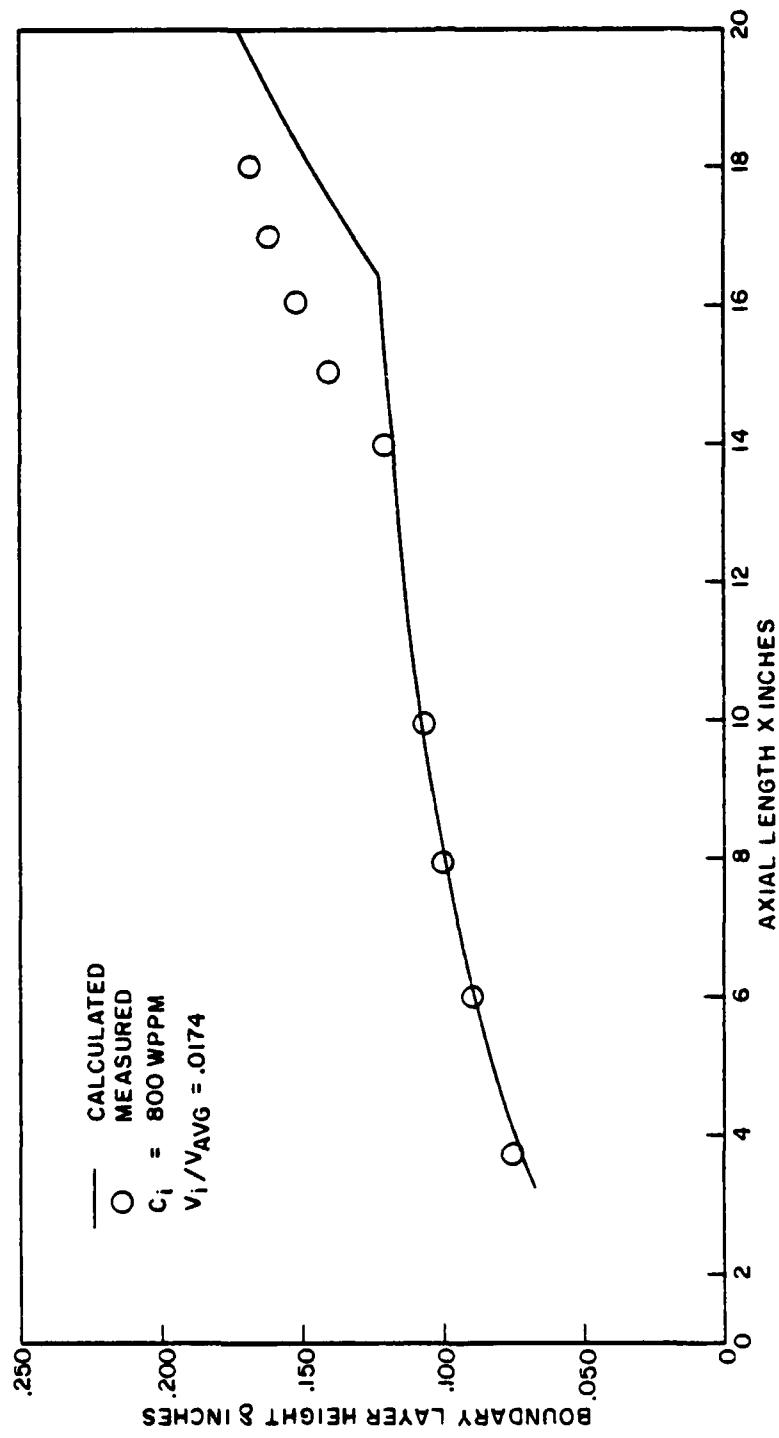
Figure 88. Comparison of experimental boundary layer growth with analytically predicted values for injected polymer concentration $C_i = 800$ WPPM

TABLE 14

COMPARISON OF SKIN FRICTION COEFFICIENTS FOR WATER CASE

x(inches)	16	17	18
R_{ex}	354252	376793	402064
C_f (Model)	.00458	.00453	.00447
C_f (Clauser)	.00450	.00440	.00425
C_f (Velocity Profile)	.00497	.00478	.00461
C_w	0	0	0

TABLE 15

COMPARISON OF SKIN FRICTION COEFFICIENTS FOR C_i = 100 WPPM OF POLYMER

x (inches)	16	17	18
R_{ex}	330,812	352,991	375,215
C_f (Model)	.00347	.00380	.00426
C_f (Clauser)	.00325	.00335	.00350
C_f (Velocity Profile)	.00405	.00406	.00424
C_w	10	4.6	3

TABLE 16

COMPARISON OF SKIN FRICTION COEFFICIENTS FOR
C_i = 200 WPPM OF POLYMER

x (inches)	16	17	18
R_{ex}	345601	346215	365801
C_f (Model)	.00327	.00345	.00359
C_f (Clauser)	.00300	.00340	.0040
C_f (Velocity Profile)	.00356	.00395	.00420
C_w	15	5	1

TABLE 17

COMPARISON OF SKIN FRICTION COEFFICIENTS FOR
C_i = 400 WPPM OF POLYMER

x (inches)	16	17	18
R_{ex}	322,476	343262	365,325
C_f (Model)	.00337	.00354	.00350
C_f (Clauser)	.00220*	.00265*	.00280*
C_f (Velocity Profile)	.00345	.00354	.00374
C_w	80	60	35

* Clauser skin friction values only valid for values of $C_i \leq 200$ WPPM polymer.

TABLE 18

COMPARISON OF SKIN FRICTION COEFFICIENTS FOR
C_i = 500 WPPM OF POLYMER

x (inches)	16	17	18
R_{ex}	291,816	311,736	331,261
C_f (Model)	.00302	.00318	.00336
C_f (Clauser)	.00280*	.00290*	.00310*
C_f (Velocity Profile)	.00265	.00279	.00294
C_w	250	210	160

TABLE 19

COMPARISON OF SKIN FRICTION COEFFICIENT FOR
C_i = 800 WPPM OF POLYMER

x (inches)	16	17	18
R_{ex}	301,196	321,545	341,454
C_f (Model)	.00314	.00332	.00351
C_f (Clauser)	.00250*	.00260*	.00290*
C_f (Velocity Profile)	.00307	.00332	.00347
C_w	110	80	55

* Clauser skin friction values only valid for values of $C_i \leq 200$ WPPM polymer.

Clauser Skin Friction Techniques

The Clauser technique, presented in 1954, and discussed by C.C. Lin (1959), bears some discussion. Utilizing the law of the wall and expressions (77), (78), and (79), Clauser obtained a family of curves of U/UFS versus log (Re$_y$) with the skin friction, C_f, as a parameter, so that

$$U\tau = \frac{U_e}{\sqrt{\frac{2}{C_f}}} \tag{77}$$

$$\frac{U}{U_\tau} \equiv \frac{U}{U_e} \sqrt{\frac{2}{C_f}} \tag{78}$$

and

$$y^+ = \frac{U_\tau y}{\nu} \equiv \frac{U_e y}{\nu} \sqrt{\frac{C_f}{2}} \; . \tag{79}$$

This family of curves for water flow utilizing the value of B = 7.2 is shown in figure 89. Application of this figure to a determination of C_f merely requires the placing of a measured velocity distribution thereon and reading off the value of C_f, interpolating where necessary. Adapting this technique to polymer flows requires that the appropriate form of the law of the wall be used, resulting in

$$\frac{U}{UFS} = \sqrt{\frac{C_f}{2}} \left[5.6 \left(\log\sqrt{\frac{C_f}{2}} + \log \frac{(UFS)y}{\nu} \right) \right.$$
$$\left. +7.2 + 2\log \frac{UFS \sqrt{\frac{C_f}{2}}}{U_\tau^*} \right] \qquad . \tag{80}$$
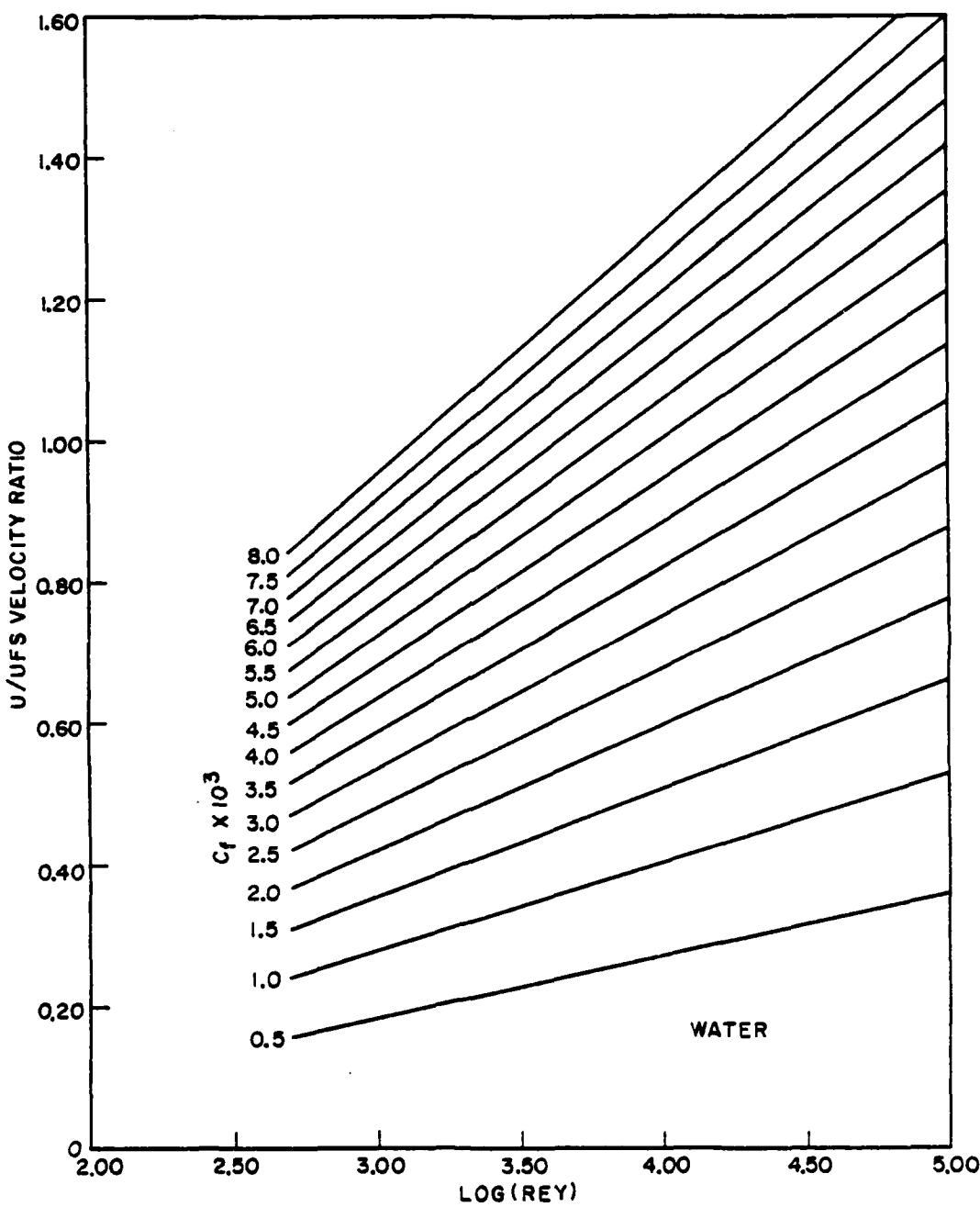
Figure 89. Plot of Clauser family of curves for water flow

where $\mathbf{z} = \alpha \ C_w^\gamma$. (81)

Figure 90 demonstrates the application of this technique to polymer flows. A new family of curves must be obtained for each polymer concentration considered.

Designed for fully-developed turbulent flow, application here has been attempted for developing flow. Additionally the ΔB shift for polymer flow is assumed to be of the form given by equation (81). For this reason, the skin friction values are presented for comparison purposes only. Additional experimentation with fully-turbulent flows over a range of polymer concentrations should be performed to further develop this technique since the present data indicate much promise.

Figures 91 through 96 present variations in the value of the local skin friction along the plate for various injection polymer concentrations as predicted by the flow model. Also plotted on these figures are values of skin friction determined from data taken at the plate. The comparative agreement is excellent. The lower values of C_f determined by the modified Clauser technique may be attributed to the developing aspect of the velocity profiles and their near-wall anomaly due to the polymer. The flow model prediction compares very well with the velocity profile data in all cases. During transition, a linear intermittency function
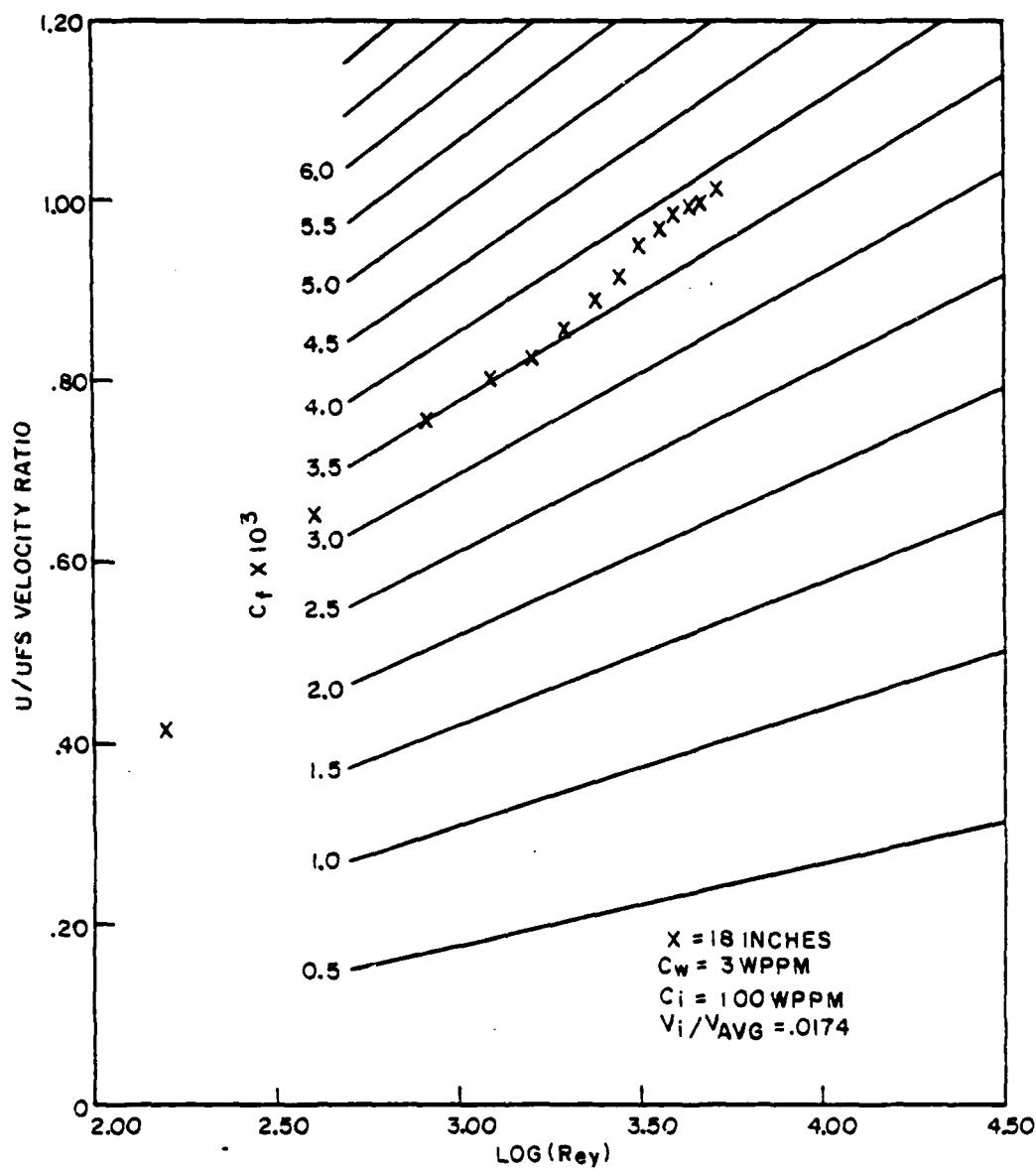
Figure 90. Plot of Clauser family of curves for injected polymer concentration C_i = 100 WPPM
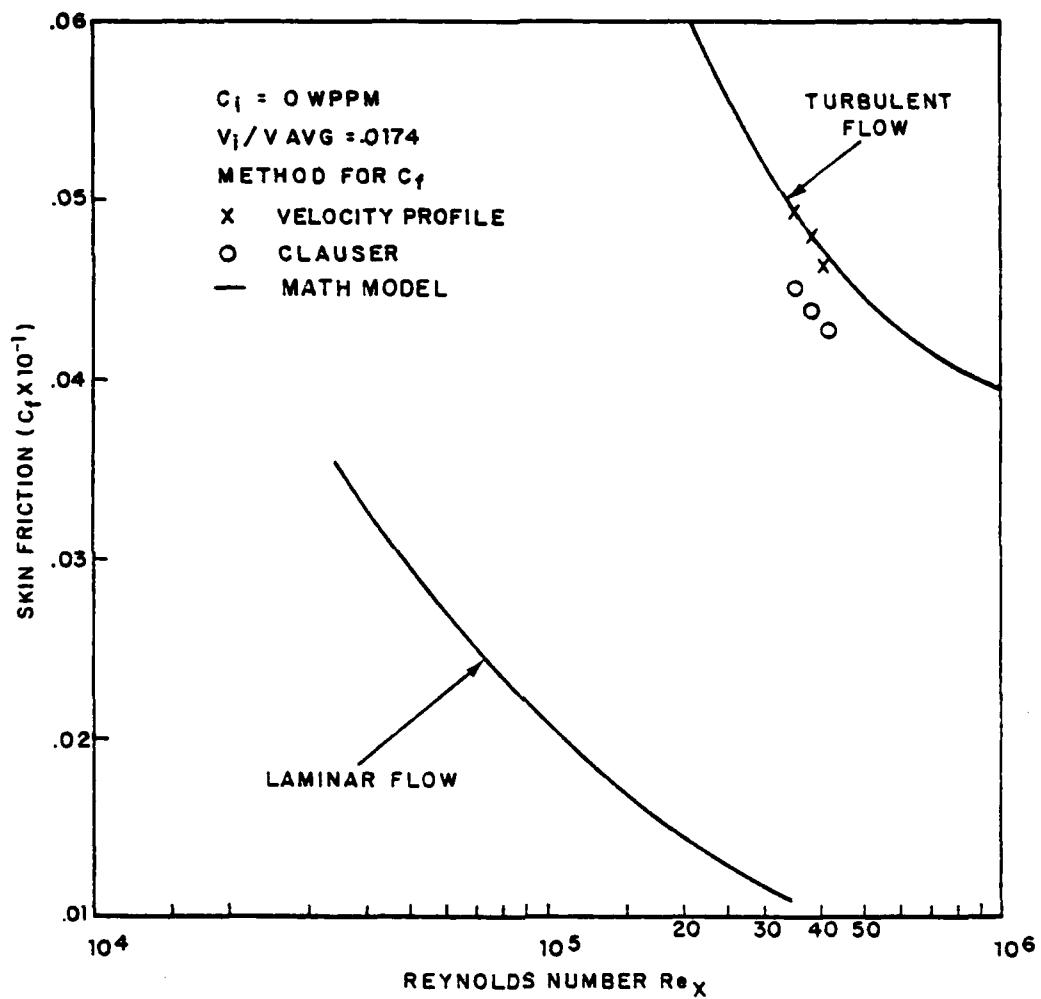
Figure 91. Variation of local skin friction along the plate
compared with experimental results for water flow
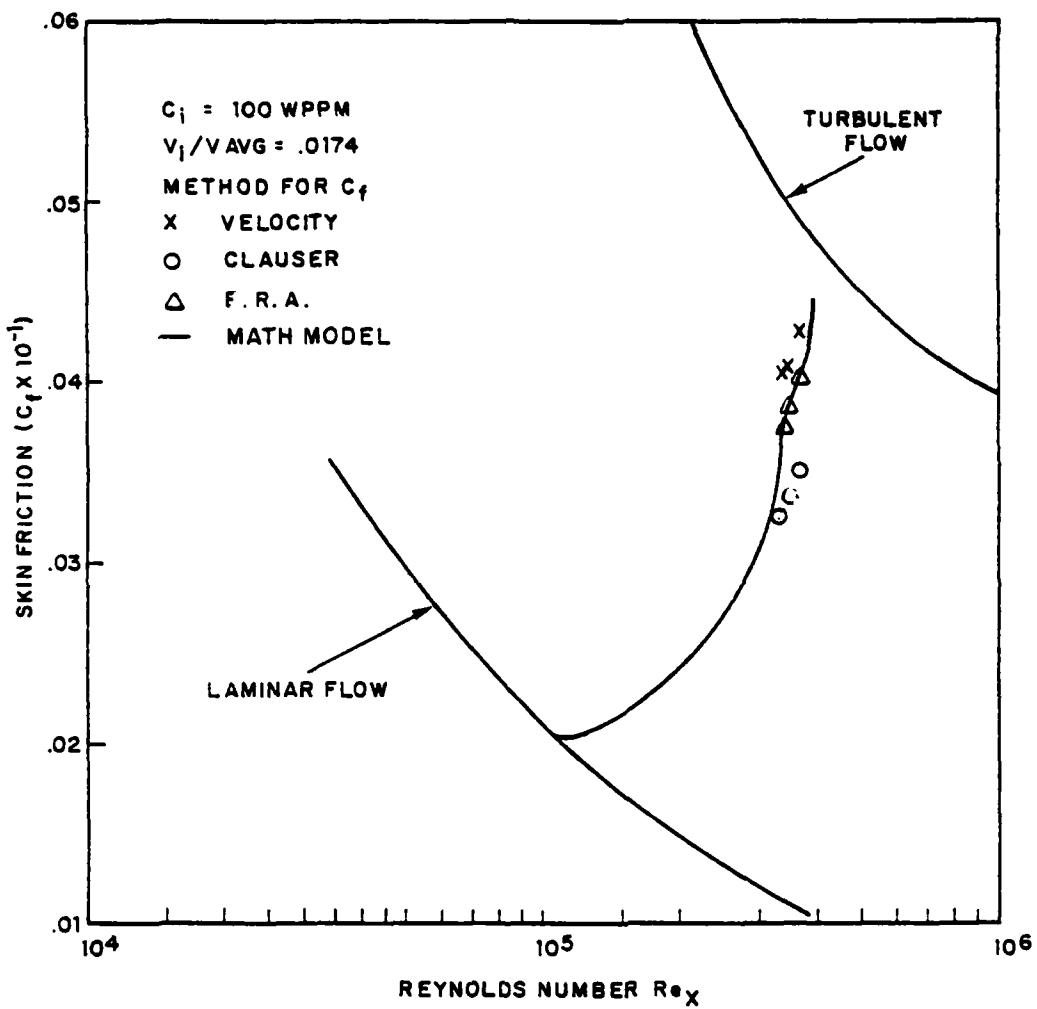
Figure 92. Variation of local skin friction along the plate
compared with experimental results for injected
polymer concentration C_i = 100 WPPM
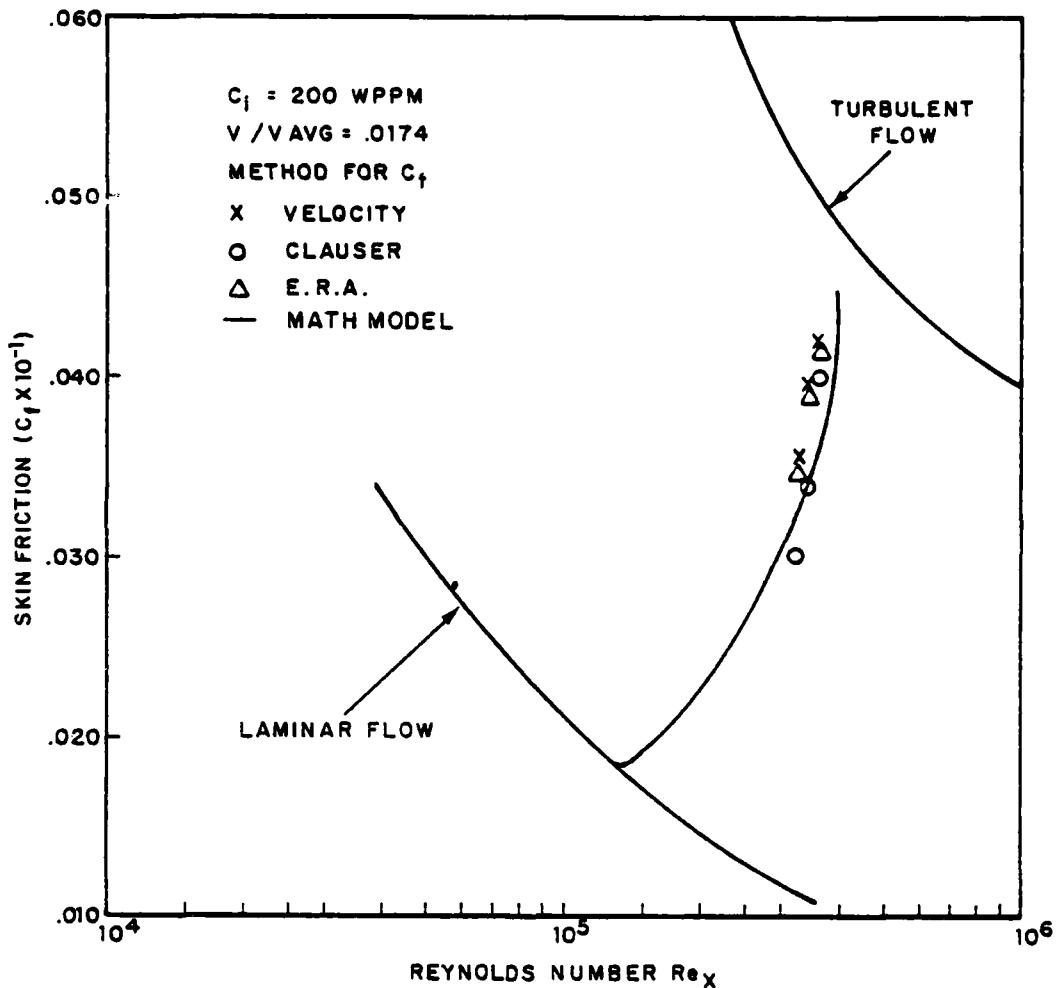
Figure 93. Variation of local skin friction along the plate
compared with experimental results for injected
polymer concentration C_i = 200 WPPM
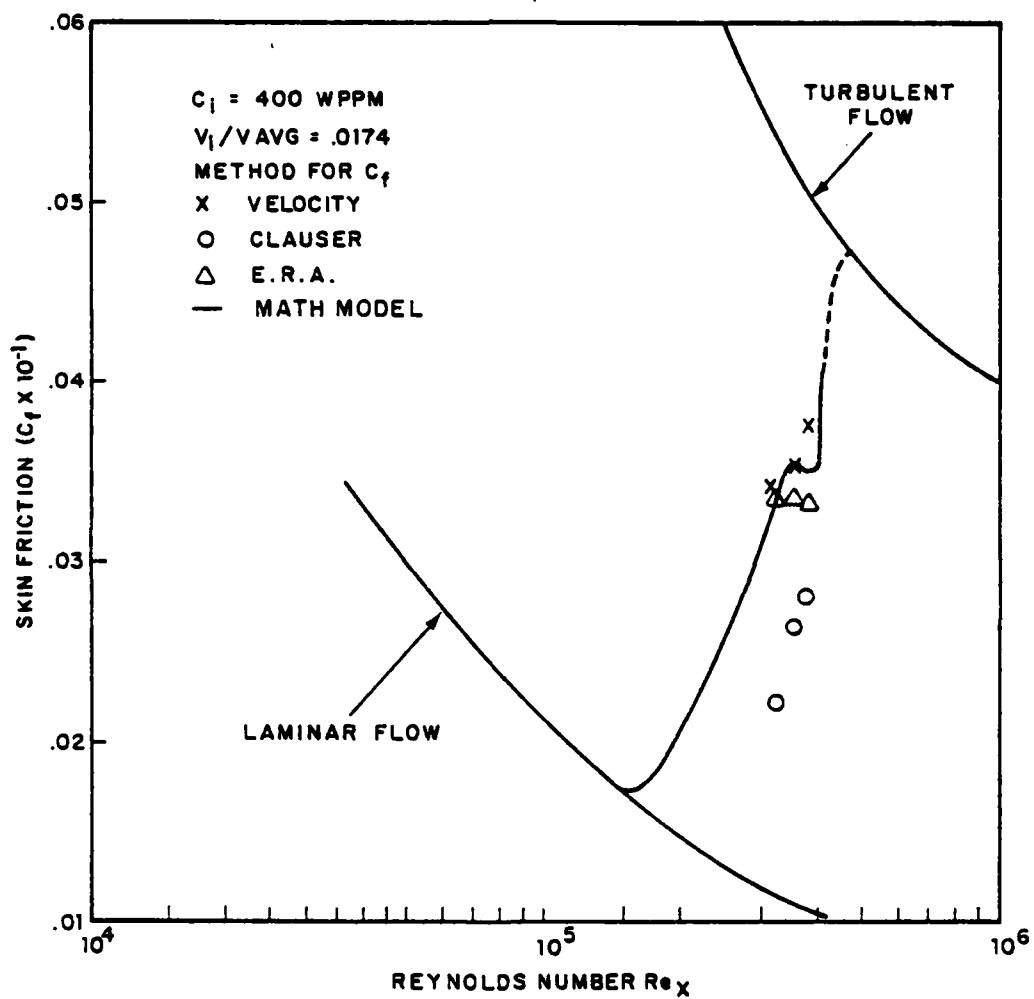
Figure 94. Variation of local skin friction along the plate
compared with experimental results for injected
polymer concentration C_i = 400 WPPM
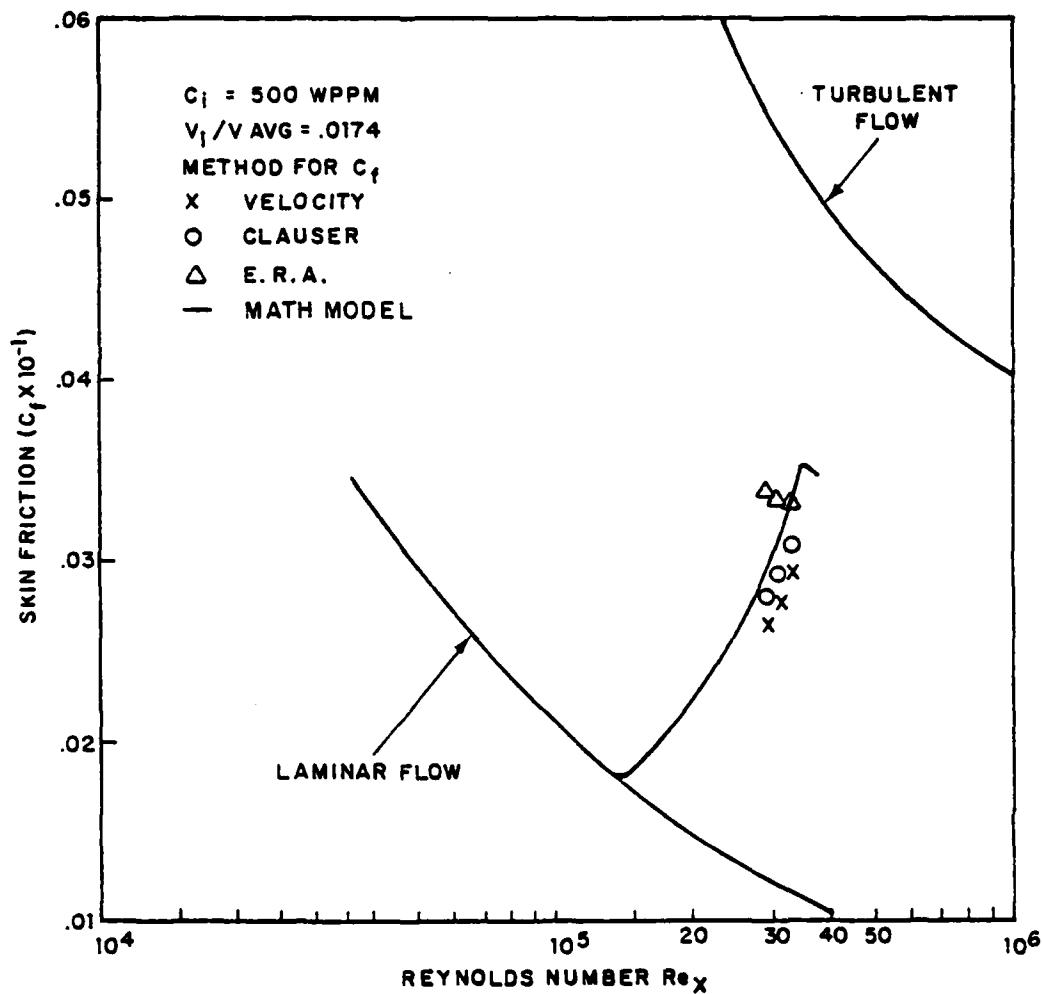
Figure 95. Variation local skin friction along the plate
compared with experimental results for injected
polymer concentration C_i = 500 WPPM
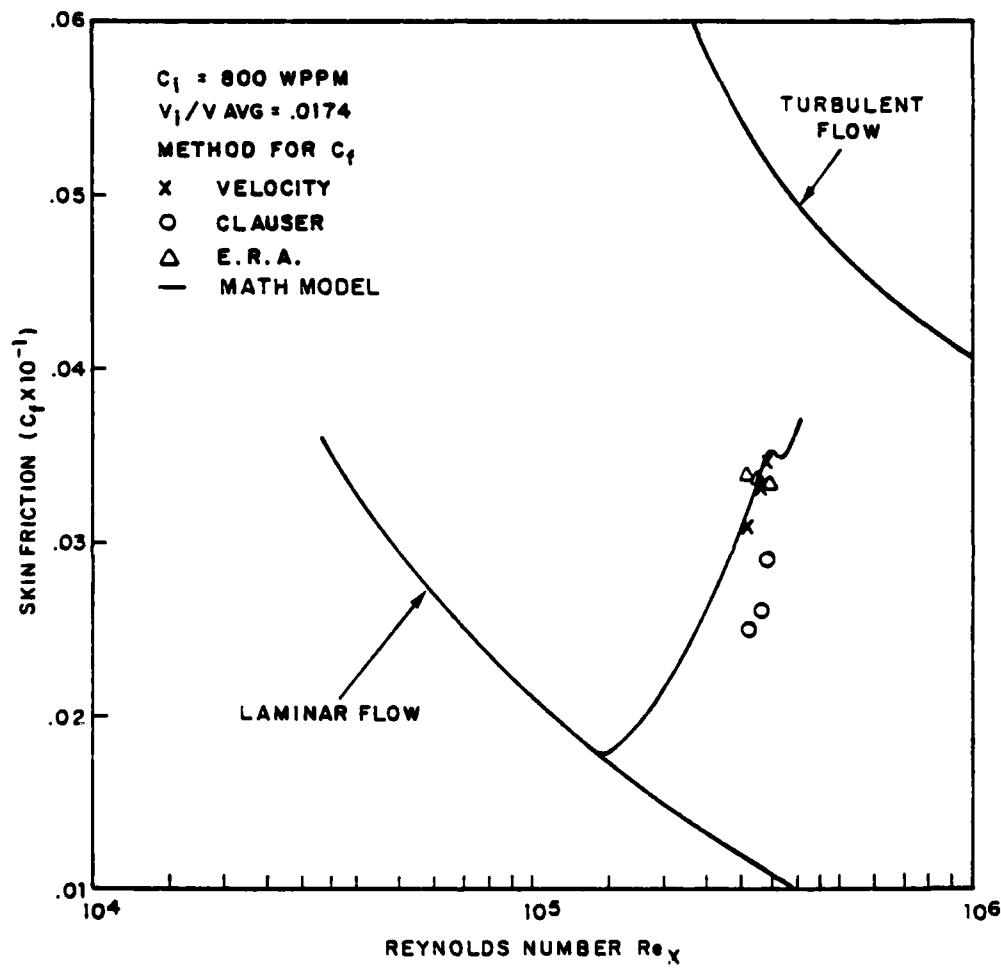
Figure 96. Variation of local skin friction along the plate
compared with experimental results for injected
polymer concentration C_i = 800 WPPM

is assumed with completion of transition to turbulent flow at
$Re_x = 350,000$. The physical limitation of the test setup and
the limited length of the plate precluded the taking of additional
data at larger length Reynolds numbers. More research in this
region with fully-developed turbulent flow is recommended to
complete the development of the flow model.

The effective Reynolds number analogy values of the local
skin friction agree very well with the values predicted by the
analytical model. Figure 97, plotted for the conditions of the
200 WPPM injection case, typically represents the other injection
conditions. Curves of C_f versus Re_x are plotted with C as a
parameter. To obtain a value for the skin friction, the plot
should be entered with the value of Re_x while the value of C_f
is taken off at the appropriate value of the polymer wall concentration.
Utilization of this technique to aid in the solution of equation (63)
has demonstrated its suitability for application to polymer flows.

Polymer Diffusion Patterns

Typical polymer wall concentration distributions normalized
with injection concentration and plotted against normalized axial
length are shown in figures 98 through 99. The polymer flows
injected into the boundary layer showed suppressed diffusion
characteristics in the transition region $0.3 < X/L < 0.7$.
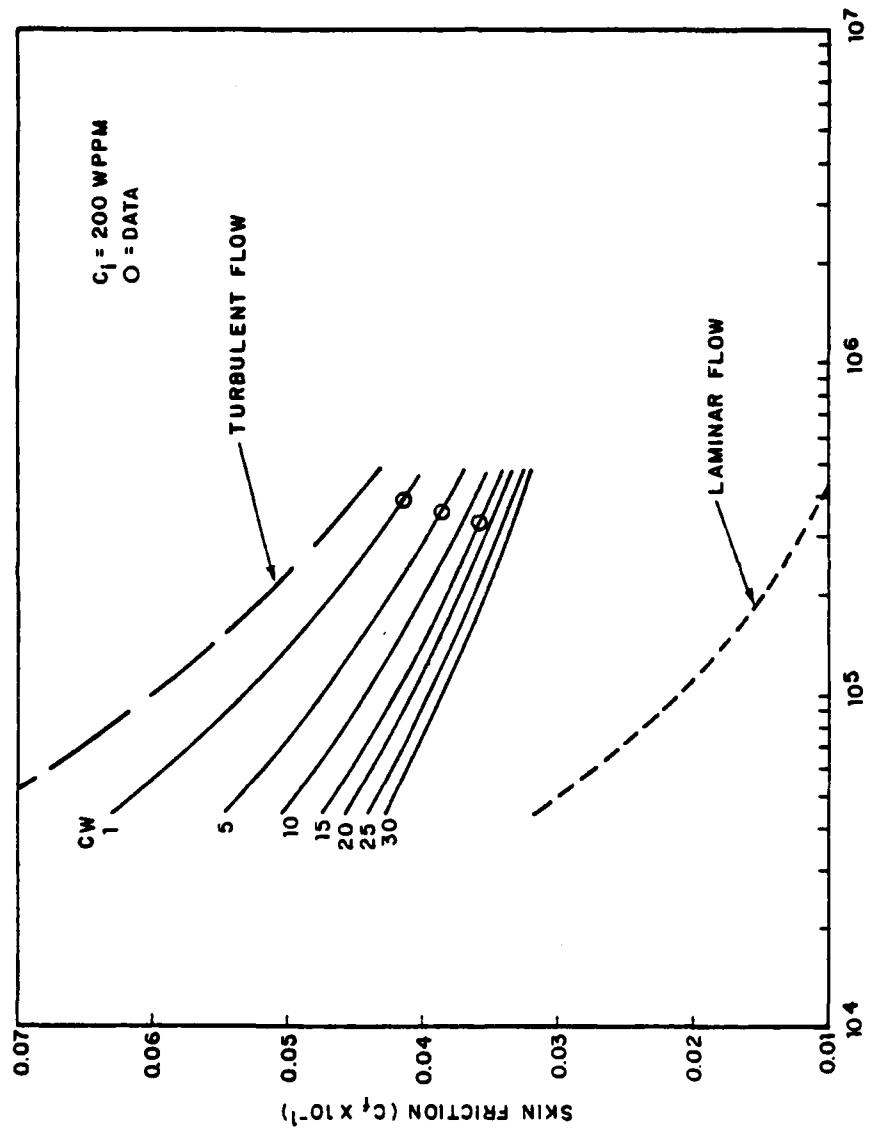
Figure 97. Effective Reynolds number analogy values of local skin friction verses Reynolds number for injected polymer concentration C_1 = 200 WPPM
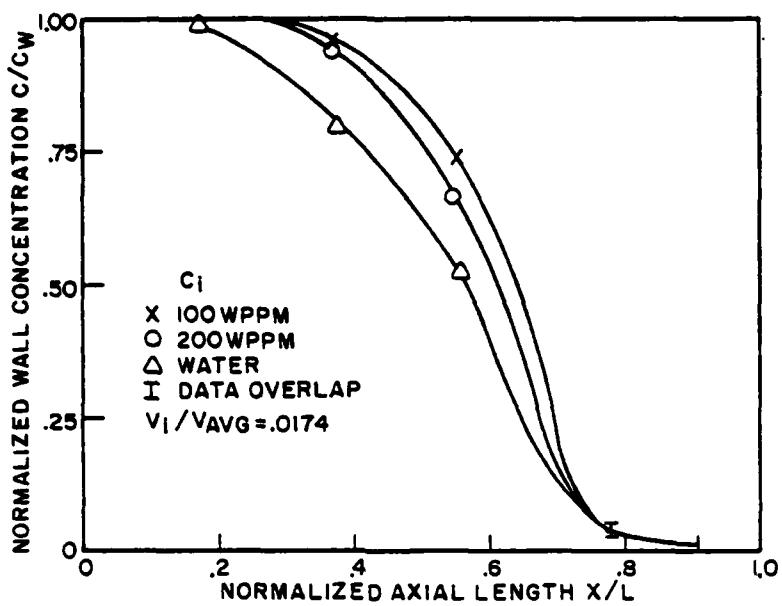
Figure 98. Normalized polymer wall concentration distribution
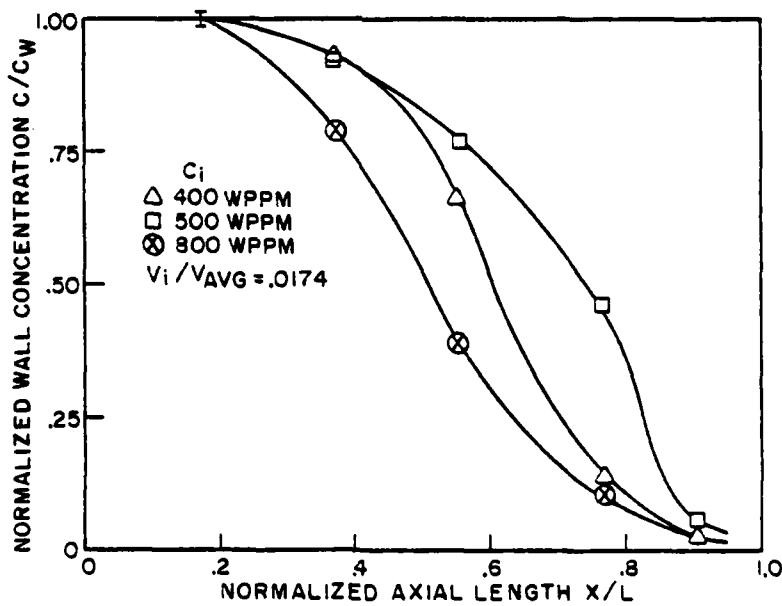
Figure 99. Normalized polymer wall concentration distribution

Higher concentrations of injected polymer resulted in higher
plate trailing edge concentrations. The 500 WPPM injection case
showed the highest wall concentrations for the largest distance along
the plate. It is believed that this distribution was influenced by a
Reynolds number effect due to the lower water temperature of this
run. The lower temperature produced a higher value of the kinematic
viscosity. The resulting lower Reynolds number had an additional
delaying effect on transition.

The polymer appears to affect transition by delaying its
onset, dampening the turbulent transport of material away from
the wall region, and stretching out the transition region.
Figures 100 and 101 show the effect of polymers on transition.
The maximum velocity difference between laminar and turbulent
velocity profiles occured at a height of $y = .020$ inch above
the plate. Axial velocity distribution profiles at this height
along the plate show the characteristic increase in velocity at
transition to turbulent flow for the water injection case. Injec-
tion of polymer showed no such characteristic increase in velocity
indicating delayed transition.

Diffusion patterns normal to the plate were taken at five
stations: 1 = 3.75 inches, 2 = 8.5 inches, 3 = 12.5 inches, 4 = 16.5
inches and 5 = 20.5 inches. No concentration measurements were

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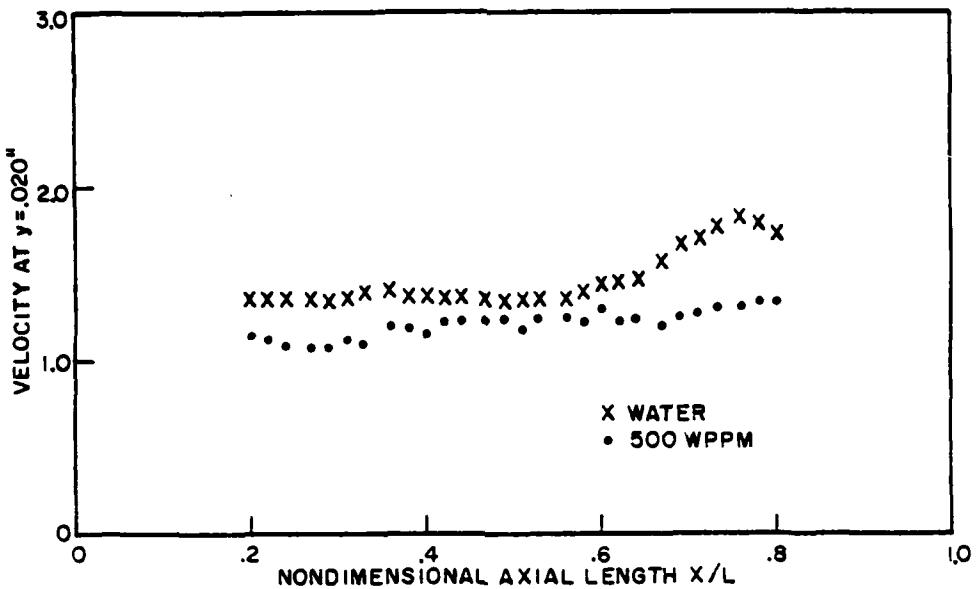
Figure 100. Axial velocity profile at y = .020 inches for injected
polymer concentration C_i = 500 WPPM compared with water
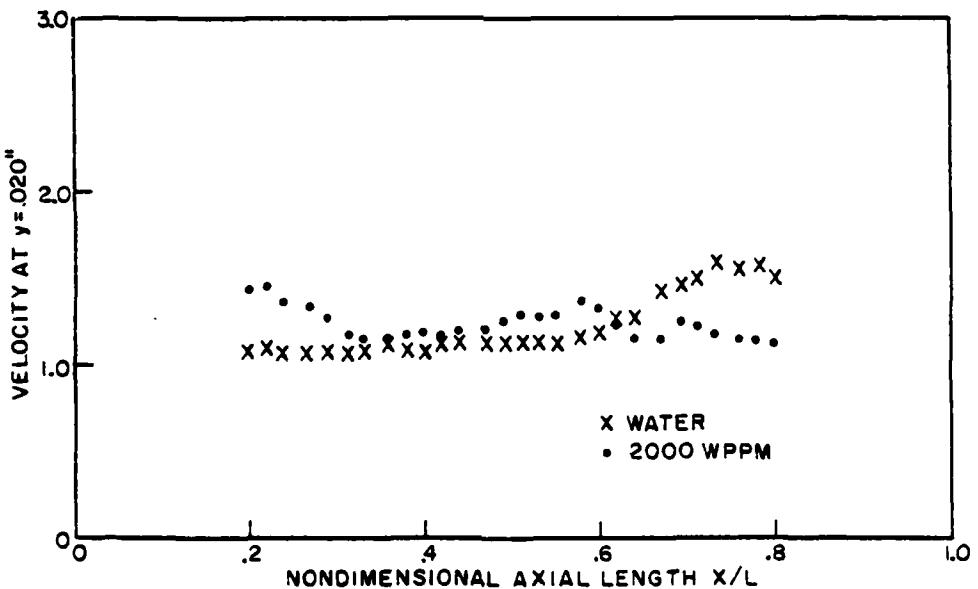
Figure 101. Axial velocity profile at y = .020 inches for injected
polymer concentration C_i = 2000 WPPM compared with water

generally obtainable above y = .010 inch to .015 inch above the
plate for stations 1 and 2, indicating very little to no diffusion
in these laminar regions. Figures 102 through 107, therefore,
compare the three concentration measurement stations along the
plate at x = 12.5, 16.5 and 20.5 inches. The injected polymer
concentrations of C_i = 100 and 200 WPPM indicate a continuous
gradual decay in concentration at stations 3 and 4, with rela-
tively high concentration levels in the near wall region. The
water injected case at these stations shows rapid concentration
decay in the near wall region. At station 4, the diffusion pattern
above y = .015 inch takes on a linear decay characteristic
found in the turbulent flow outer region at station 5. The
concentration distributions normal to the wall at station 5 for
polymer injection of C_i = 100 and 200 WPPM are similar to the water
case at the same station. This is due to the very low wall
concentration at this station.

The reduced diffusion characteristics of polymer flows may
be seen by plotting the growth of the diffusion boundary layer
indicated by a characteristic height, λ_c. This characteristic
height, λ_c, is defined as that height at which the concentration
is equal to one-half of the wall concentration. Increasing
concentration of injected polymer reveals a reduced diffusion
characteristic away from the wall and a thicker layer of higher
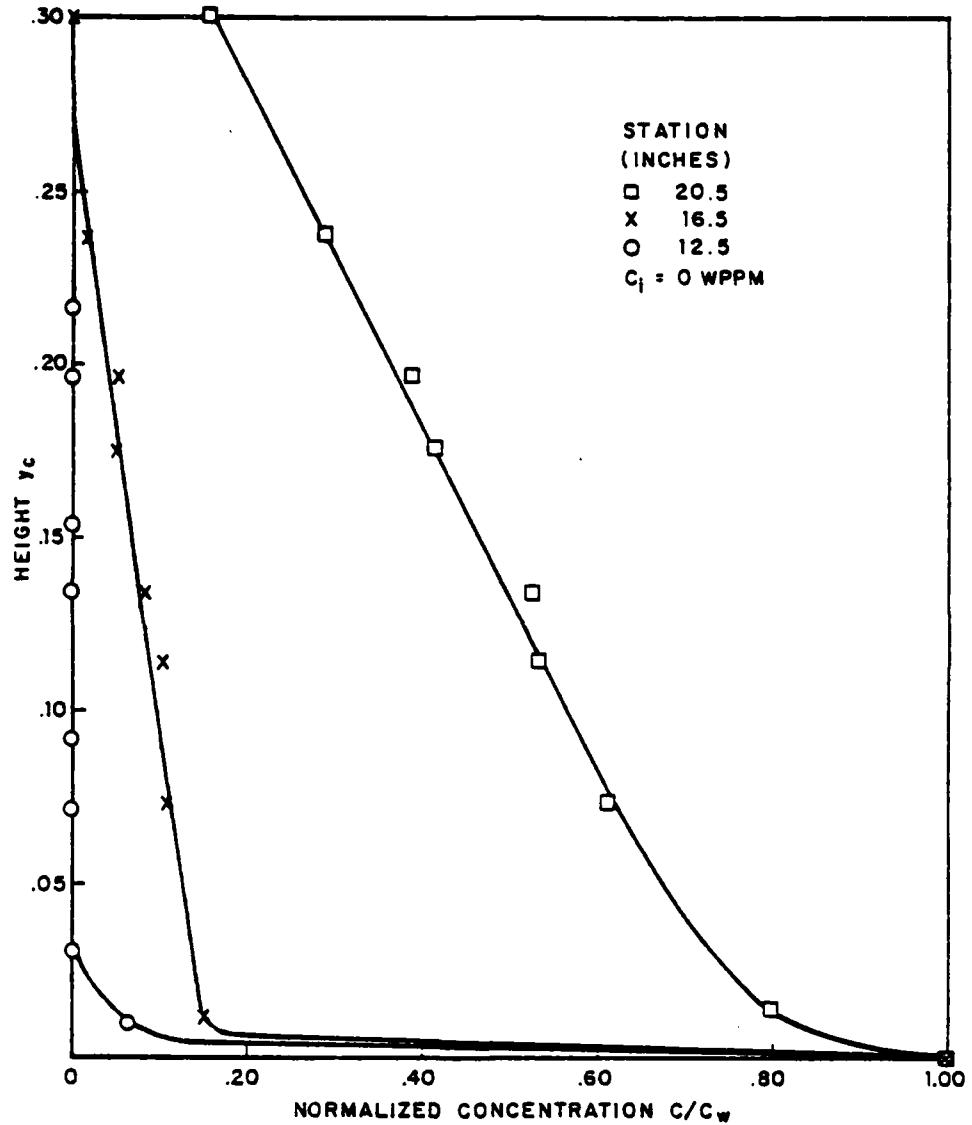
Figure 102. Normalized water diffusion patterns
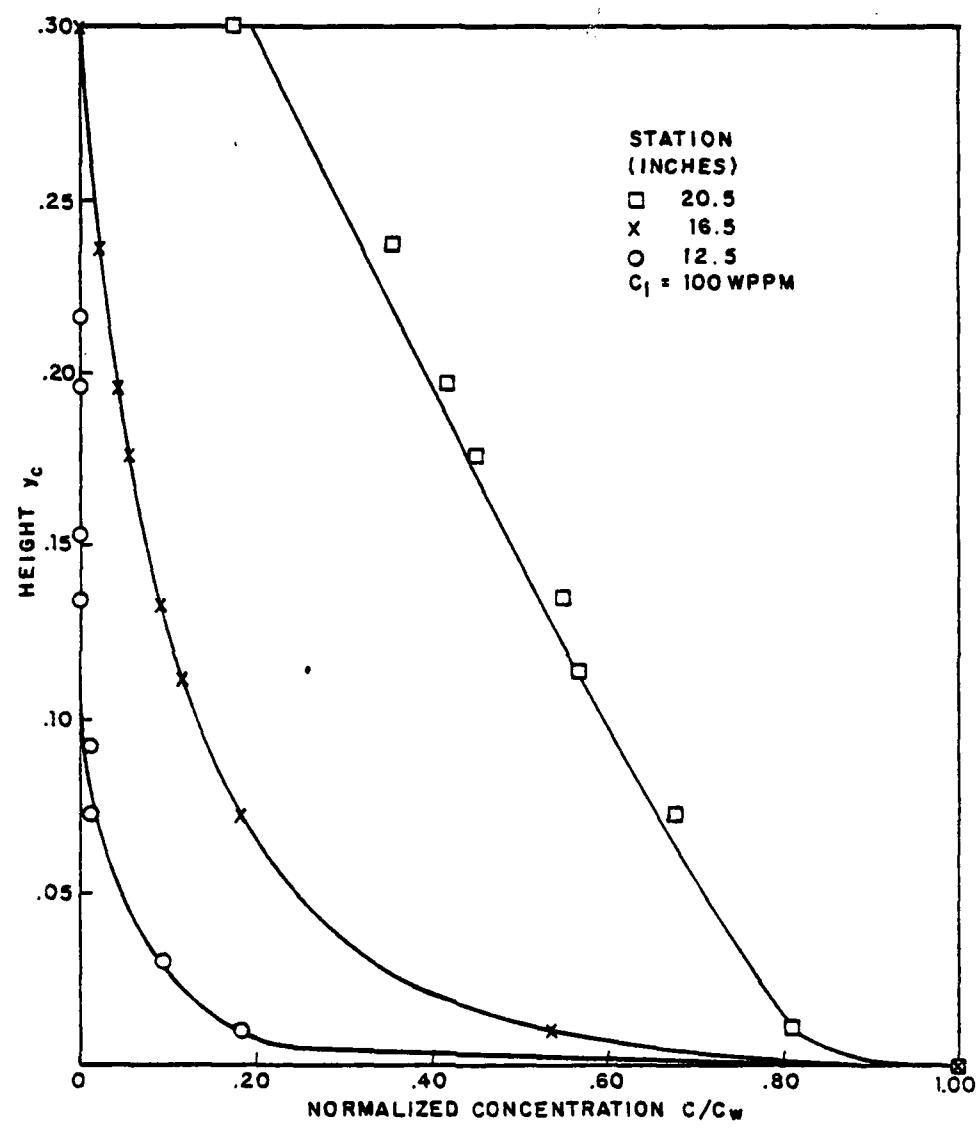
Figure 103.   Normalized polymer diffusion pattern for injected
polymer concentration C_i = 100 WPPM
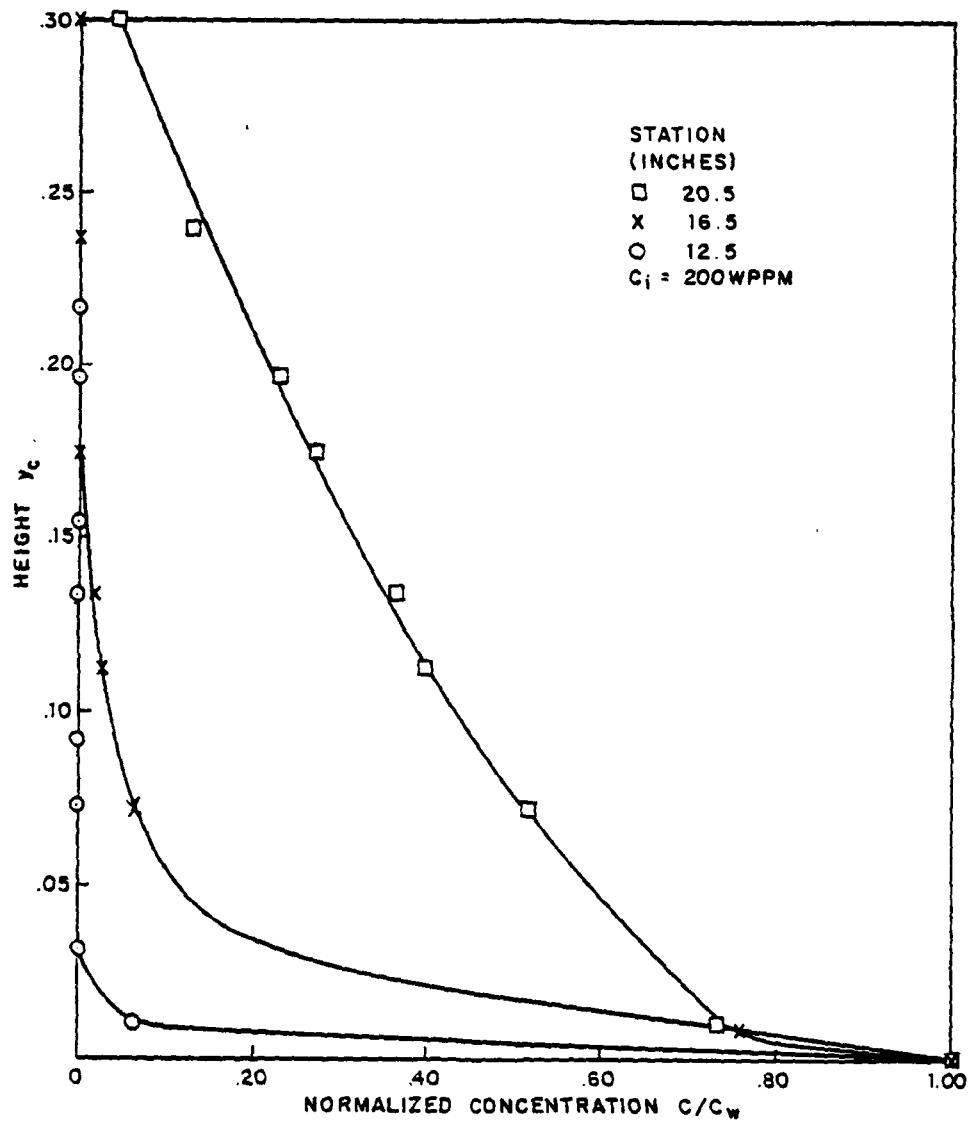
Figure 104. Normalized polymer diffusion pattern for injected
polymer concentration C_i = 200 WPPM
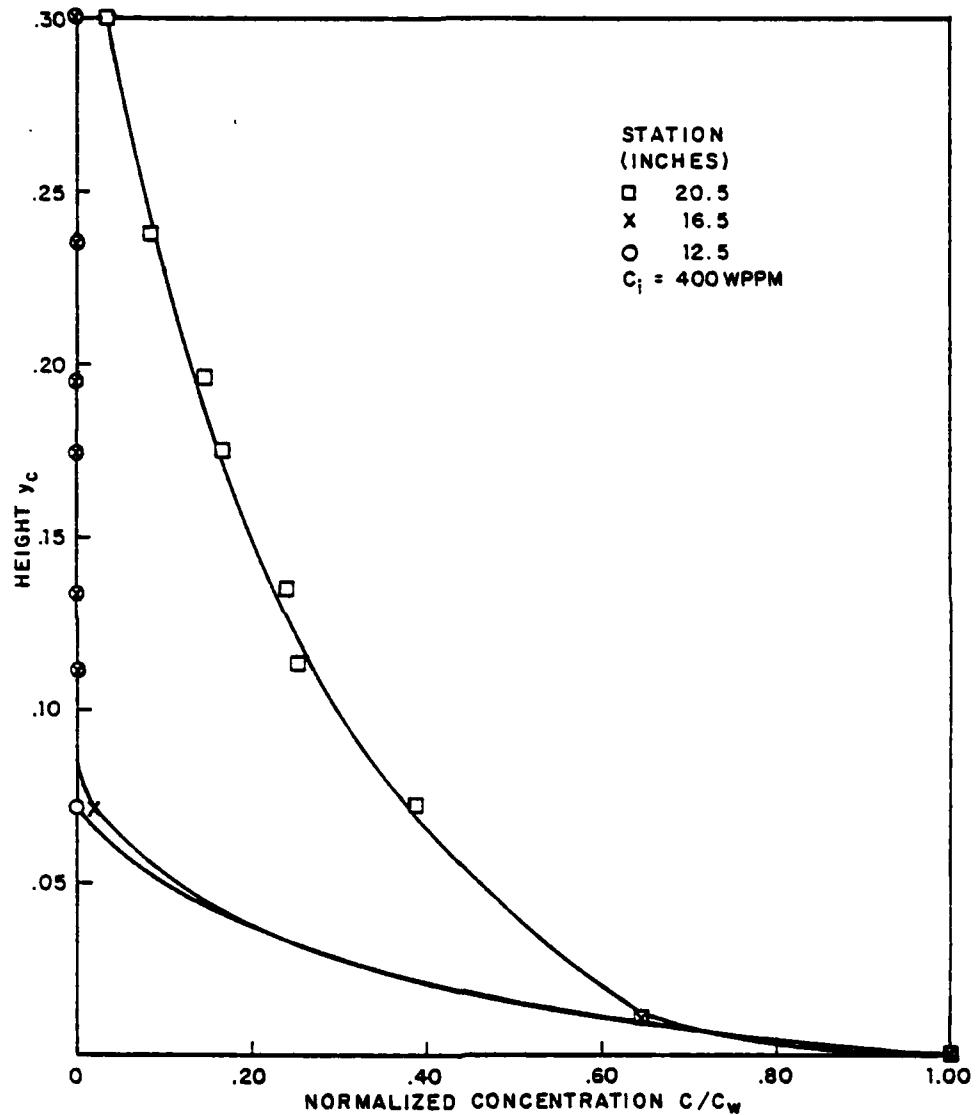
Figure 105.   Normalized polymer diffusion pattern for injected
              polymer concentration C_i = 400 WPPM
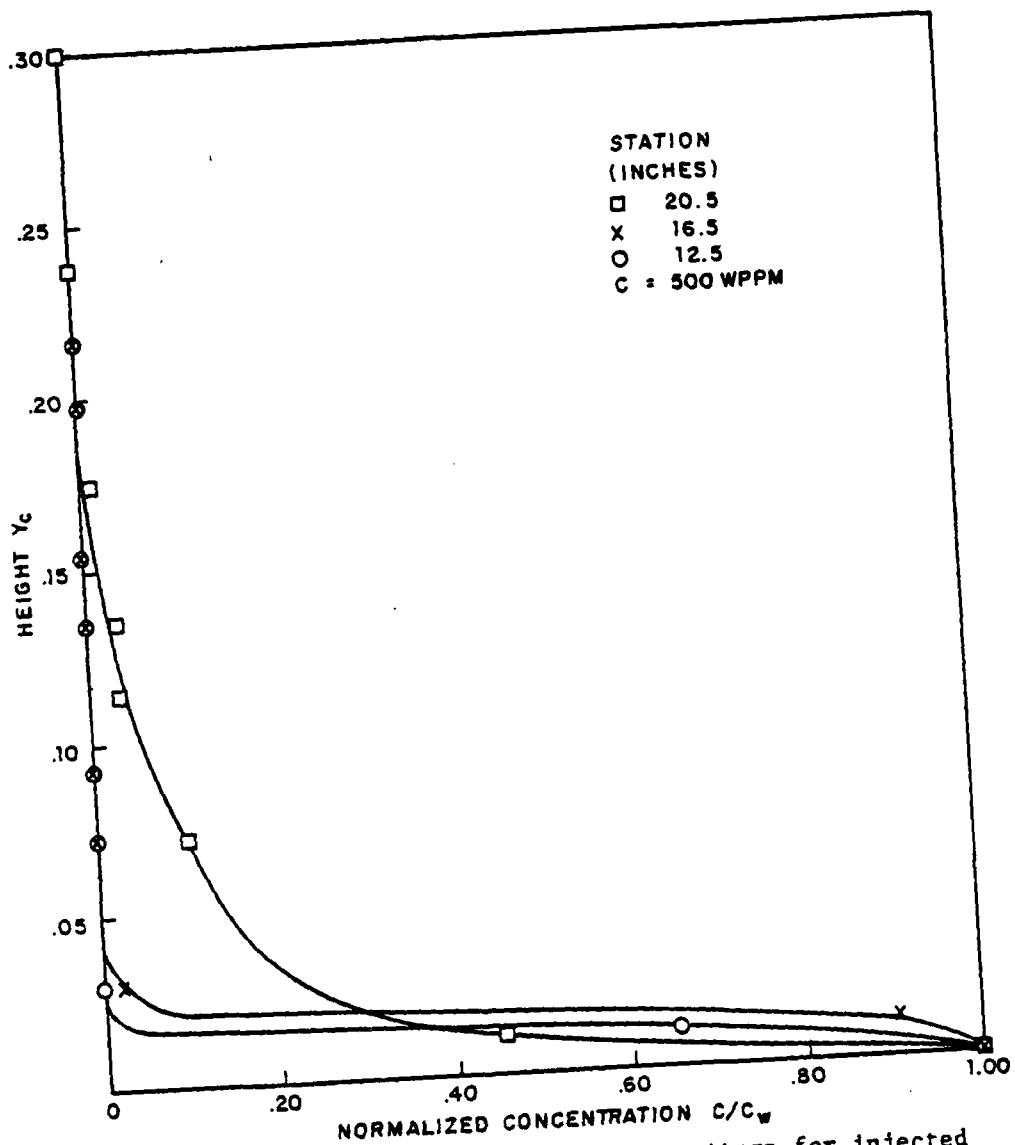
Figure 106. Normalized polymer diffusion pattern for injected
polymer concentration C_i = 500 WPPM
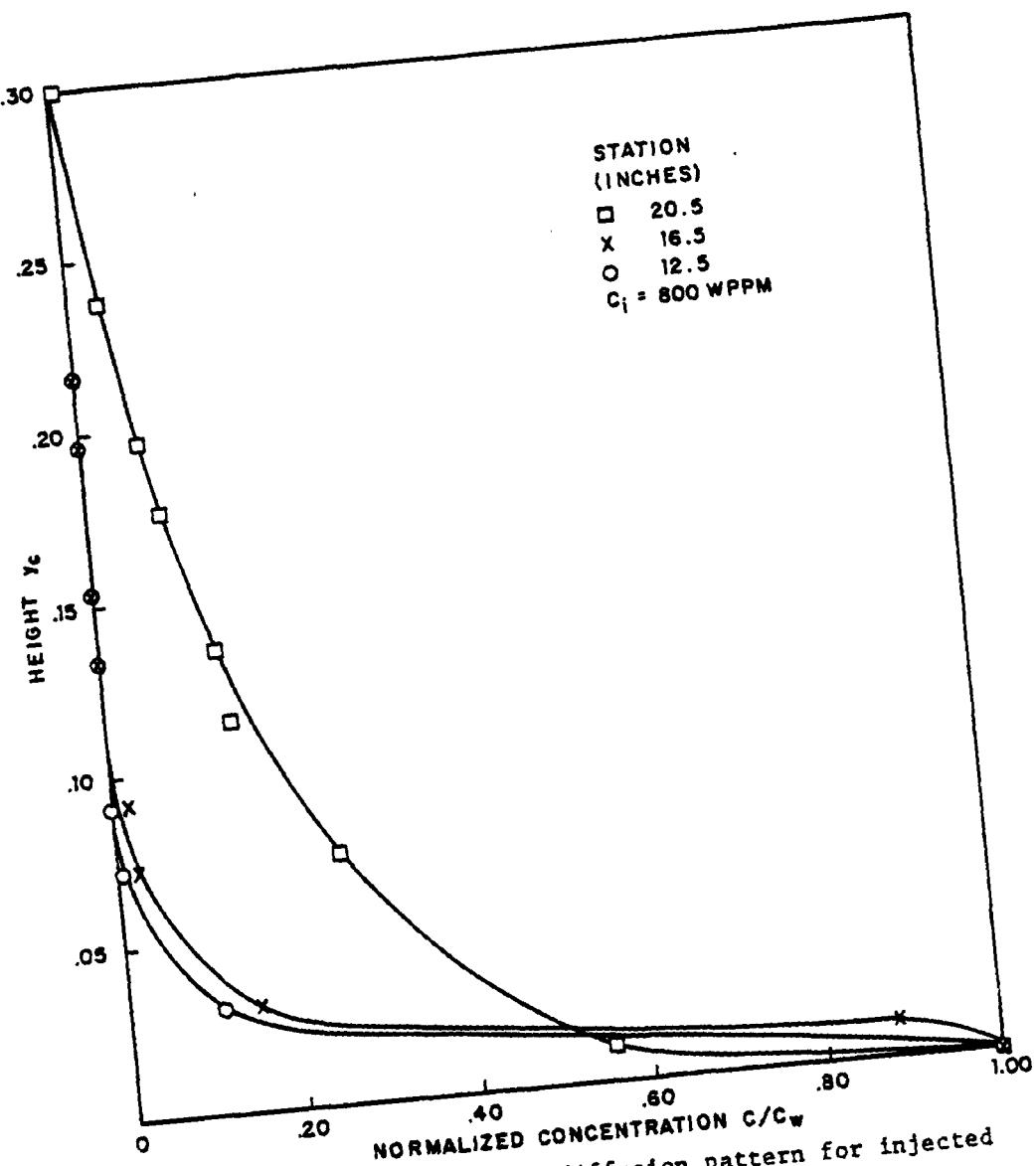
Figure 107. Normalized polymer diffusion pattern for injected polymer concentration C_i = 800 WPPM

concentration in the near wall region as shown in figure 108.

The quantity of polymer or mass flow rate injected into the

boundary layer may be considered to be given by the flow rate

of injection times the concentration. Figure 109 is a plot of

diffusion boundary layer height versus axial length injected at

various flow rates and concentrations in combination to produce

a constant quantity of polymer injected into the boundary layer

of $Q_1C_1 = 3,000$. The lowest rate of injection at the highest

concentration produces the slowest growth of the diffusion boundary

layer. The effect of increasing the quantity of polymer injected

into the boundary layer is shown in figure 110 where the value

of $Q_1C_1 = 16,000$. Again, the lowest rate of injection at the

highest concentration produces the slowest growth of the diffusion

boundary layer. The increased amount of polymer injected into

the boundary layer greatly suppressed the diffusion of polymer

away from the wall. The effect of increasing injection concen-

tration flux into the boundary layer may be seen in figure 111,

which may be considered to present a measure of the injection

"displacement" thickness. At constant velocity of injection,

increased injection concentration flux greatly suppressed the growth

of the diffusion boundary layer. The lowest injection velocity

produced the lowest diffusion boundary layer. In the final zone,

diffusion was found to vary with an exponent of .6 as shown in

figure 112. Sirmalis had predicted that the exponent for similarity

profiles would be lower than his value of .75 as he was unable to

obtain data within .01 inch of the wall. Similarity profiles for the growth of the diffusion boundary layer in the transition region are shown in figure 113. Higher injected concentrations are also shown to have reduced diffusion characteristics in this region.
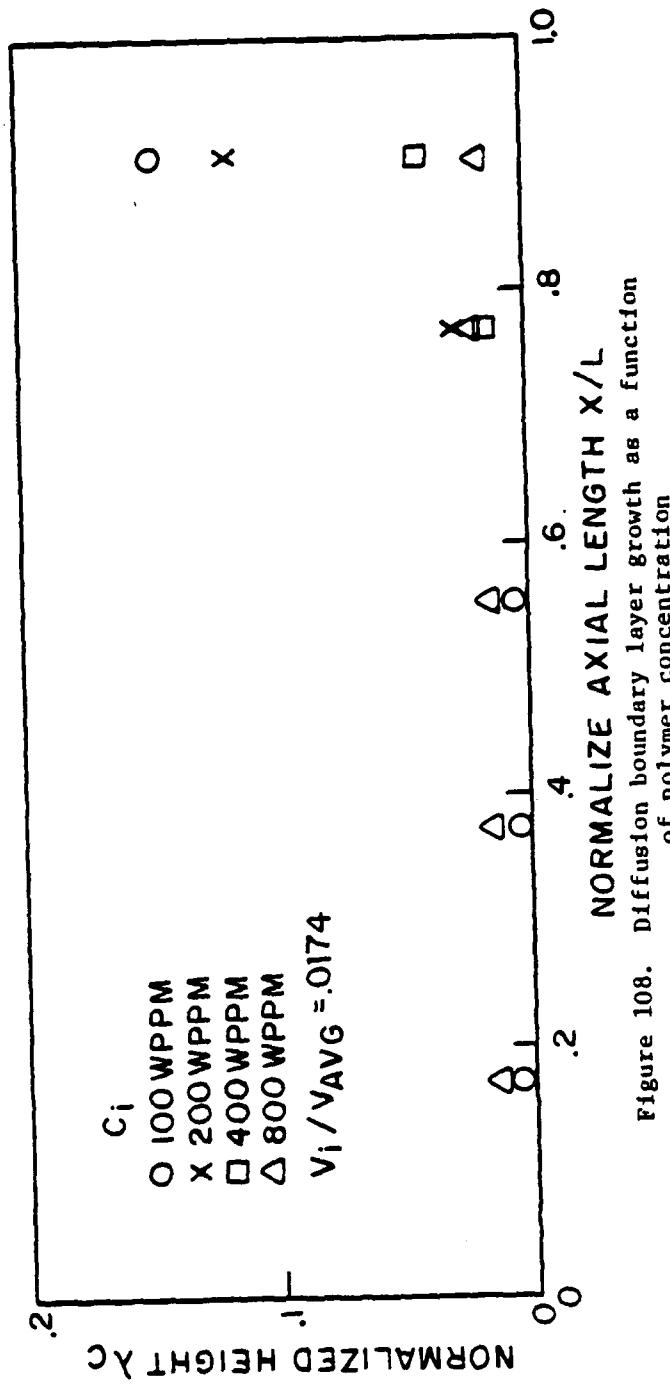
Figure 108. Diffusion boundary layer growth as a function of polymer concentration
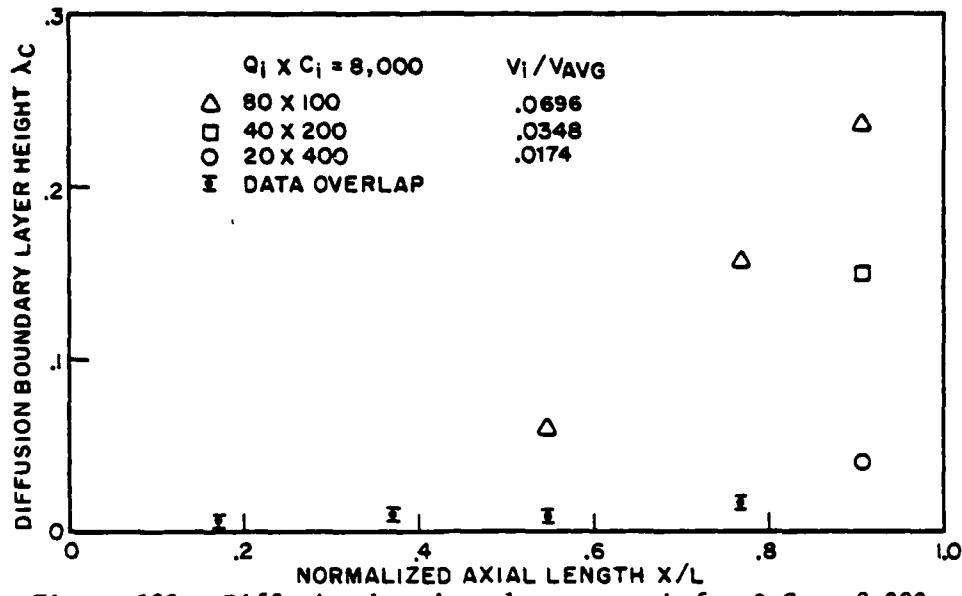
Figure 109. Diffusion boundary layer growth for Q_1C_1 = 8,000
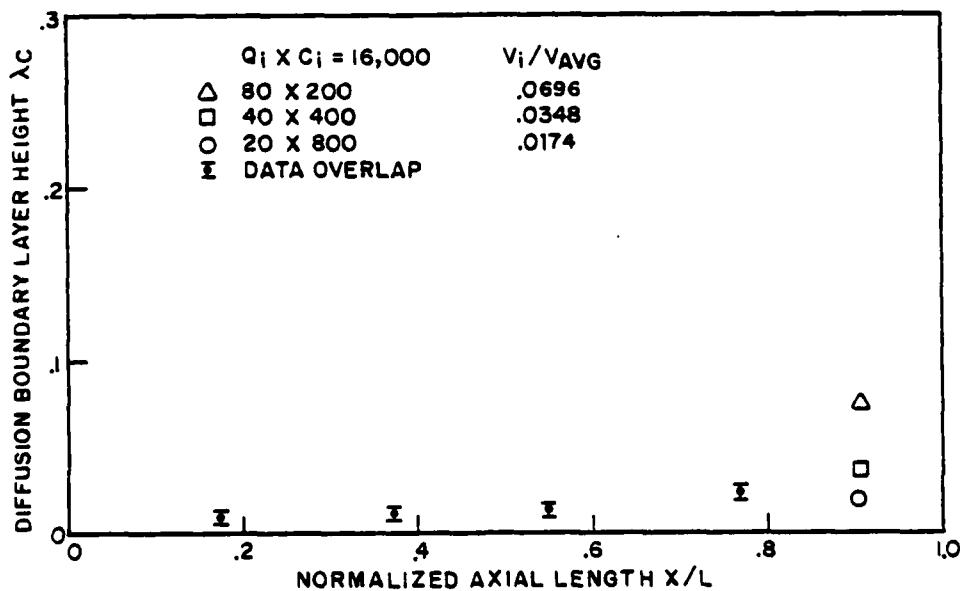
Figure 110. Diffusion boundary layer growth for Q_1C_1 = 16,000
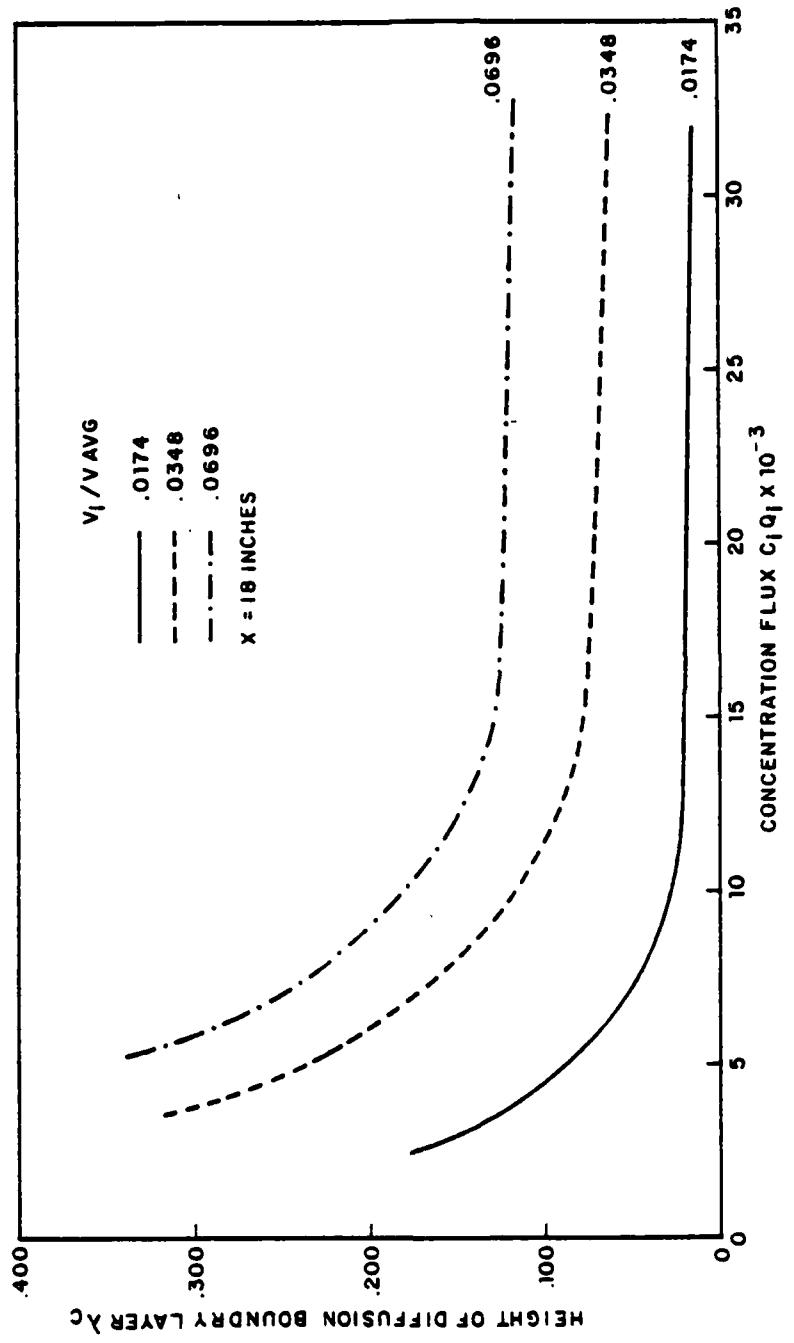
Figure 111. Diffusion boundary layer growth as a function of $Q_1 C_1$ and injection velocity ratio
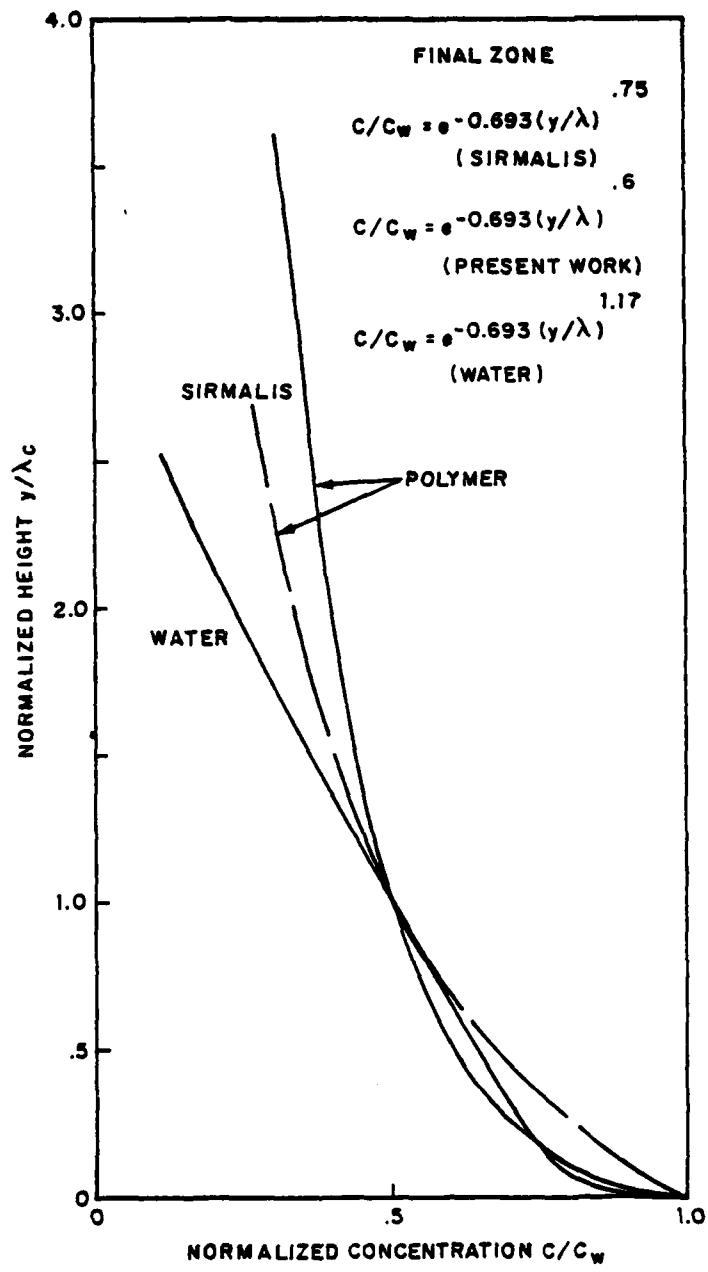
Figure 112. Diffusion boundary layer growth in the final zone
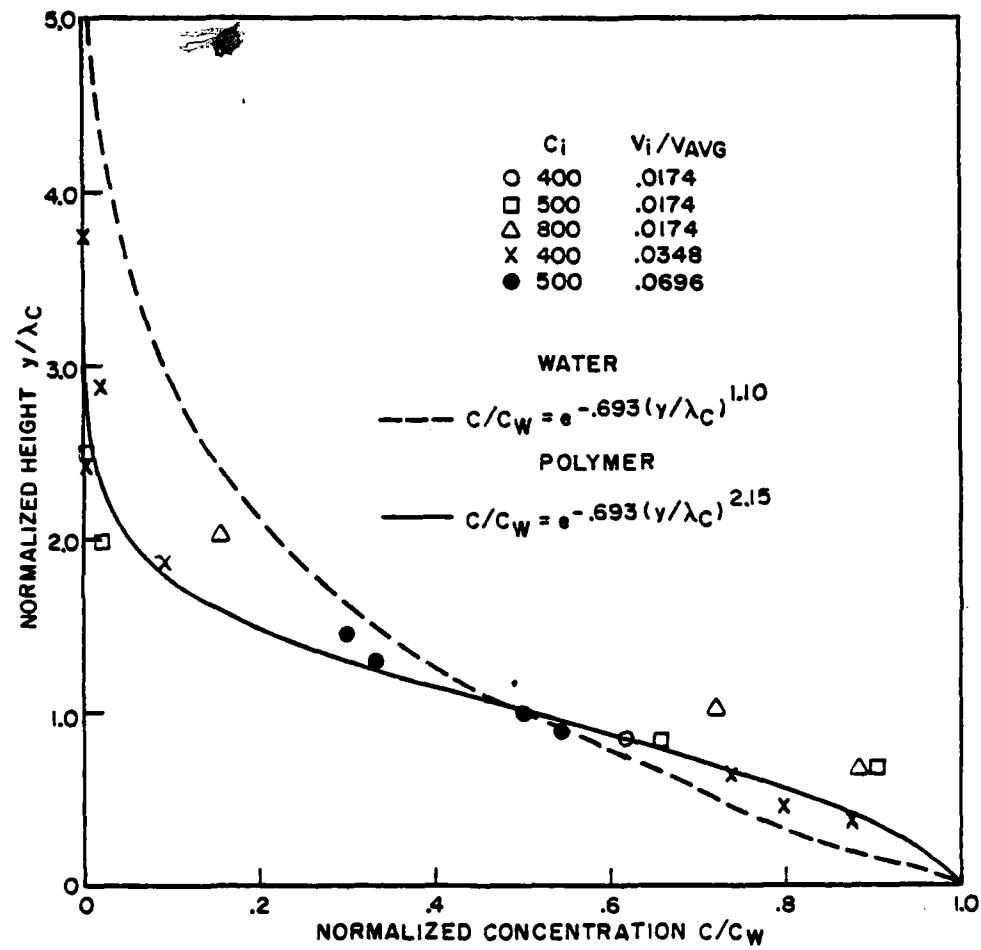
Figure 113. Diffusion boundary layer growth in the transition region

VII. CONCLUSIONS AND RECOMMENDATIONS

General

The object of this research was threefold: (1) to develop an experimental facility and techniques for the study of the effects of drag-reducing polymers on submerged flat plate flow, (2) to qualitatively define the effects of polymers on boundary layers in external flows, and (3) to verify predictive techniques by experiment. A flat plate water tunnel facility was constructed and laser Doppler anemometry techniques were developed for boundary layer velocity measurements. The flow facility is capable of variable injection velocity and concentration of polymer with boundary layer sampling at several heights throughout the boundary layer and at several axial stations. The injection process was unique because injection occurred very close to the leading edge into laminar flow, then developing through transitional to turbulent flow. Application of laser Doppler techniques provided non-disruptive velocity measurements in a low turbulent level facility for a wide range of polymer concentration and velocity of injection.

Analytical Predictive Methods

The case of developing flow with polymer injection has received little attention for external flows. Predictive methods are, therefore, not well developed. Past works on the diffusion of polymer boundary layers have centered on the far downstream region in turbulent flow where the polymer attained the same diffusion characteristics as the medium.

An analytical model was developed for comparison with experimental data. The model is based on integral boundary layer analysis combined with velocity profile relation for flat plate flow at zero pressure gradient and variable polymer wall concentration. Accurate prediction of local skin friction and boundary layer height as a function of variable local wall polymer concentration has been demonstrated over laminar, transitional, and turbulent flow regions by comparison with experimental data.

Polymer Boundary Layer Characteristics

Polymer additives appeared to have no effect in the laminar flow region. However, increasing polymer concentrations delayed and stretched out the transition to turbulent flow. The effect of polymer additives is predominant in the region of maximum turbulence production.

Streamwise turbulence intensity distributions are lower in peak
value and displaced further from the wall over a broadened buffer
region near the wall than found in comparable solvent flows.
Near-wall velocity measurements have shown that the laminar
sublayer thickens with the introduction of polymer. A thicker
velocity profile sublayer reduces the velocity gradient at and
near the wall. The wall shear stress, proportional to the velocity
gradient at the wall, also decreases, thus reducing the frictional
drag of the surface. Although thickening the sublayer, polymers
cause the overall boundary layer to become thinner. Skin friction
prediction by Clauser's technique has been shown applicable, with
modification, to polymer flows.

Diffusion Characteristics

Various polymer concentrations were injected into a laminar
flat plate boundary layer at various injection rates. Transition
to turbulent flow was delayed and extended. Kowalski's (1974)
explanation that polymer molecules seem to act as barriers
restricting communication across the boundary layer, forcing the
liquid to flow in a semi-laminar manner is most appropriate here.
The growth of the diffusion boundary layer, indicated by a
characteristic height, decreased with increasing concentration
and quantity of polymer injected into the boundary layer.

The increased concentration of polymer showed reduced diffusion in the transition region resulting in an extended initial mixing zone. This effect is also supported by the work of Sirmalis (1976).

Conclusion Summary

An experimental facility has been developed that allows laboratory measurements of polymer drag reduction concentration profile distribution for various injector geometries, velocities of injection, and injection concentrations. Hydrodynamic properties of the boundary layers were measured using laser Doppler anemometry techniques. Experiments indicated thickening sublayers and overall thinning of the boundary layer with polymer concentration. Turbulence intensity distributions were lower in peak value and distributed over a broad region. Diffusion of polymer injected into the laminar flow at the leading edge of the flat plate delayed transition and spread the transition region out. Very little diffusion occurred in the laminar region. Diffusion in the turbulent region indicated by a characteristic height of the diffusion boundary layer decreased with increasing concentration and quantity of polymer. Predictive methods were developed building on reliable theories and techniques for the prediction of local skin friction as a function of polymer wall concentration. Comparison with experiment has been performed and good agreement achieved.

Recommendations

1. Additional testing should be performed at boundary layer heights between values of y^+ equal to 7 to 50. This region would supply additional data to define the increasing height of the sublayer with polymer concentration, as well as the polymer-induced ΔB shift. Low concentrations of injected polymer below 50 WPPM should also be tested.

2. Application of the modified Clauser skin friction calculation technique to polymer flows should be continued and will require additional data in the above-mentioned region.

3. Application of traversing laser Doppler techniques should be extended to include rotation of the measuring volume by $90°$ to obtain distribution of the v' turbulence intensity distribution allowing computation of Reynolds stress.

4. Reduction of flow noise indicates that frequency spectrum analysis should be performed to determine which frequencies are attenuated by various polymer concentrations.

5. The experimental facility should be lengthened to allow a more gradual and natural transition to turbulent flow.

6. Multiple horizontal injection stations along the plate should be tested with lower concentration of injection to verify the disentanglement theory. Optimization of the injection system should also include pulsed low concentration injection to determine persistence and possible time scale correlation.

7. Modification of the tunnel passage will allow the study of polymer effects in boundary layers with adverse pressure gradients. The capability for acoustic modulation of the laser beams should be added to the test setup to measure flow reversals using non-disruptive laser Doppler anemometer techniques.

Appendix A

DEVELOPMENT OF SKIN FRICTION RELATIONS

The law of the wall incorporating a polymer term, according to Meyer (1966), is given by

$$u^+ = \left(\frac{1}{K}\right)\ln y^+ + B + \alpha \left(C_W^\gamma\right)\ln\left(\frac{V^*}{V_0^*}\right) \qquad (A1)$$

where $\alpha = 2.3$, $\gamma = .5$.

The conservation of mass and the momentum equations for flow over a flat plate are given by

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad (A2)$$

and

$$\rho u\left(\frac{\partial u}{\partial x}\right) + \rho v\left(\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial \tau}{\partial y}. \qquad (A3)$$

For a flat plate, it may be assumed that the pressure gradient is negligible, such that

$$\frac{\partial P}{\partial x} = 0. \qquad (A4)$$

Therefore, equation (A3) becomes

$$\rho u\left(\frac{\partial u}{\partial x}\right) + \rho v\left(\frac{\partial u}{\partial y}\right) = \frac{\partial \tau}{\partial y}. \tag{A5}$$

It may further be assumed that $\mu = \mathbb{C}$, $\rho = \mathbb{C}$, and $\nu = \mathbb{C} = \frac{\mu}{\rho}$ are all constants. $\tag{A6}$

Equation (A2) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{A7}$$

An expression for v becomes

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$$

$$dv = -\frac{\partial u}{\partial x} dy$$

$$v(y) = v = -\int_0^y \frac{\partial u}{\partial x} dy. \tag{A8}$$

Changing to law of the wall variables we have

$$u^+ = \frac{u}{v^*} \tag{A9}$$

$$u = V^* u^+ \tag{A10}$$

$$\frac{\partial u}{\partial x} = \frac{\partial (V^* u)}{\partial x} \tag{A11}$$

$$y^+ = \frac{y V^*}{\nu} \tag{A12}$$

$$dy^+ = \frac{V^*}{\nu} \, dy \tag{A13}$$

$$dy = \frac{\nu}{V^*} \, dy^+ \tag{A14}$$

$$V^* = V^*(x) = \sqrt{\frac{\tau_W}{\rho}}. \tag{A15}$$

Combining equations (A8, A11, A14), equation (A8) becomes

$$v = -\int_0^{y^+} \frac{d(V^* u^+)}{dx} \left(\frac{\nu}{V^*}\right) dy^+. \tag{A16}$$

Changing equation (A5) into law of the wall variables using equations (A9) thru (A16) we have

$$\rho(V^* u^+)\frac{\partial}{\partial x}(V^* u^+) + \rho\left[-\int_0^{y^+} \frac{\partial(V^* u^+)}{\partial x} \left(\frac{\nu}{V^*}\right) \, dy^+\right] \frac{\partial(V^* u^+)}{\partial y}$$

$$= \frac{\partial \tau}{\partial y}. \tag{A17}$$

The derivatives with respect to x must be handled by the chain rule, because the parameters y^+ and C_W are functions of x in the law of the wall. We are changing independent variables from x and y to x, y^+ and C_W, where $C_W = C_W(x)$ and $y^+ = y^+(x,y)$.

Thus we substitute

$$\frac{\partial}{\partial x} = \frac{\partial y^+}{\partial x} \frac{\partial}{\partial y^+} + \frac{\partial C_W}{\partial x} \frac{\partial}{\partial C_W} . \tag{A18}$$

Now

$$u(x, y) = V^*(x) u^+(y^+, C_W). \tag{A19}$$

Differentiating we have

$$\frac{\partial u}{\partial x} = \frac{\partial (V^* u^+)}{\partial x}$$

$$= u^+ \frac{\partial V^*}{\partial x} + V^* \frac{\partial u^+}{\partial x}$$

$$= u^+ \frac{\partial V^*}{\partial x} + V^* \left[\frac{\partial y^+}{\partial x} \left(\frac{\partial u^+}{\partial y^+} \right) + \frac{\partial C_W}{\partial x} \left(\frac{\partial u^+}{\partial C_W} \right) \right]$$

$$= u^+ \frac{\partial V^*}{\partial x} + V^* \frac{\partial y^+}{\partial x} \left(\frac{\partial u^+}{\partial y^+} \right) + V^* \frac{\partial C_W}{\partial x} \left(\frac{\partial u^+}{\partial C_W} \right). \tag{A20}$$

Substituting equation (A20) into equation (A18),

$$\rho(V^*u^+)\left[u^+\frac{\partial V^*}{\partial x} + V^*\frac{\partial y^+}{\partial x}\left(\frac{\partial u^+}{\partial y^+}\right) + V^*\frac{\partial C_W}{\partial x}\left(\frac{\partial u^+}{\partial C_W}\right)\right]$$

$$-\rho\left[\int_0^{y^+}\frac{\partial}{\partial x}(V^*u^+)\frac{\nu}{V^*}\,dy^+\right]\frac{\partial}{\partial y}(V^*u^+) = \frac{\partial \tau}{\partial y}. \tag{A21}$$

Obtaining an expression for $\frac{\partial u}{\partial y}$ we have

$$\frac{\partial u}{\partial y} = \frac{\partial(V^*u^+)}{\partial y}$$

$$= u^+\frac{\partial V^*}{\partial y} + V^*\frac{\partial u^+}{\partial y}. \tag{A22}$$

However,

$$V^* = V^*(x) \tag{A23}$$

and hence

$$\frac{\partial V^*}{\partial y} = 0. \tag{A24}$$

Equation (A22) then becomes

$$\frac{\partial(V^*u^+)}{\partial y} = V^*\frac{\partial u^+}{\partial y}. \tag{A25}$$

Changing ∂y to ∂y^+ we have

$$\frac{\partial(V^+u)}{\partial y} = V^* \frac{\partial u^+}{\partial y} = V^* \left[\frac{\partial u^+}{\partial y^+} \left(\frac{\partial y^+}{\partial y}\right) + \frac{\partial u^+}{\partial C_W} \left(\frac{\partial C_W}{\partial y}\right) \right] \quad (A26)$$

but

$$\frac{\partial y^+}{\partial y} = \frac{V^*}{\nu} \text{ and } \frac{\partial C_W}{\partial y} = 0$$

and therefore

$$\frac{\partial(V^* u^+)}{\partial y} = \frac{(V^*)^2}{\nu} \left(\frac{\partial u^+}{\partial y^+}\right).$$

Equation (A21) becomes

$$\rho \ (V^* u^+) \left[u^+ \frac{\partial V^*}{\partial x} + V^* \frac{\partial y^+}{\partial x} \frac{\partial u^+}{\partial y^+} + V^* \frac{\partial C_W}{\partial x} \left(\frac{\partial u^+}{\partial C_W}\right) \right]$$

$$-\rho \left[\int_0^{y^+} \frac{\partial}{\partial x} (V^* u^+) \frac{\nu}{V^*} \ dy^+ \right] \frac{(V^*)^2}{\nu} \left(\frac{\partial u^+}{\partial y^+}\right) = \frac{\partial \tau}{\partial y}. \quad (A27)$$

As $\nu = \mathcal{C}$, equation (A6) and $V^* = V^*(x)$, equation (A23) may then be removed from within the integral yielding

$$\rho(V^* u^+) \left[u^+ \frac{\partial v^*}{\partial x} + V^* \frac{\partial y^+}{\partial x} \left(\frac{\partial u^+}{\partial y^+}\right) + V^* \frac{\partial C_W}{\partial x} \left(\frac{\partial u^+}{\partial C_W}\right) \right]$$

$$-\rho \left[\int_0^{y^+} \frac{\partial(V^* u^+)}{\partial x} \ dy^+ \right] V^* \left(\frac{\partial u^+}{\partial y^+}\right) = \frac{\partial \tau}{\partial y}. \quad (A28)$$

After substitution of equation (A20) with equation (A28) we have

$$\rho(V^*u^+) \left[u^+\left(\frac{\partial V^*}{\partial x}\right) + V^*\left(\frac{\partial y^+}{\partial x}\right) \frac{\partial u^+}{\partial y^+} + V^*\left(\frac{\partial C_W}{\partial x}\right) \frac{\partial u^+}{\partial C_W} \right]$$

$$-\rho \left\{ \int_0^{y^+} \left[u^+\left(\frac{\partial V^*}{\partial x}\right) + V^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+} + V^*\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W} \right] dy^+ \right\} V^*\left(\frac{\partial u^+}{\partial y^+}\right) = \frac{\partial \tau}{\partial y}. \quad (A29)$$

Converting $\frac{\partial \tau}{\partial y}$ to law of wall variables we have

$$\frac{\partial \tau}{\partial y} = \frac{\partial \tau}{\partial y^+} \left(\frac{\partial y^+}{\partial y}\right)$$

$$= \frac{V^*}{\nu}\left(\frac{\partial \tau}{\partial y^+}\right).$$

Substituting into equation (A29) we have

$$\rho(V^*u^+) \left[u^+\left(\frac{\partial V^*}{\partial x}\right) + V^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+} + V^*\left(\frac{dC_W}{dx}\right) \frac{du^+}{dC_W} \right]$$

$$-\rho \left[\int_0^{y^+} \left(u^+\left(\frac{\partial V^*}{\partial x}\right) + V^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+} + V^*\left(\frac{\partial C_W}{\partial x}\right) \frac{\partial u^+}{\partial x}\right) \right] y^+ V^*\left(\frac{\partial u^+}{\partial y^+}\right) = \frac{V^*}{\nu}\left(\frac{\partial \tau}{\partial y^+}\right).$$

Dividing then by ρV^* and remembering that $\nu = \frac{\mu}{\rho}$

$$u^+\left[u^+\left(\frac{\partial v^*}{\partial x}\right) + v^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+} + v^*\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W}\right]$$

$$-\left\{\int_0^{y^+}\left[\left(u^+\left(\frac{\partial v^*}{\partial x}\right) + v^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+} + v^*\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W}\right)\right]\partial y^+\right\}\frac{\partial u^+}{\partial y^+}$$

$$= \frac{1}{\rho\nu}\left(\frac{\partial \tau}{\partial y^+}\right) = \frac{1}{\mu}\left(\frac{\partial \tau}{\partial y^+}\right). \tag{A30}$$

Multiplying by dy^+, expanding terms and intergrating we have

$$\int_0^{y^+}(u^+)^2\left(\frac{\partial v^*}{\partial x}\right)dy^+ + \int_0^{y^+}v^*u^+\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+}\,dy^+ + \int_0^{y^+}u^+v^*\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W}\,dy^+$$

$$-\int_0^{y^+}\left\{\left[\int_0^{y^+}u^+\left(\frac{\partial v^*}{\partial x}\right) + v^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+} + v^*\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W}\,dy^+\right]\frac{\partial u^+}{\partial y^+}\right\}dy^+$$

$$= \int_0^{y^+}\frac{1}{\mu}\left(\frac{\partial \tau}{\partial y^+}\right)dy^+ \quad . \tag{A31}$$

From equation (A6) and (A22) upon rearringing equation (A31) and integrating through the boundary layer such that $\tau(y^+) = 0$, we have

$$\frac{\partial v^*}{\partial x}\int_0^{y^+}(u^+)^2 dy^+ + v^*\int_0^{y^+}u^+\left(\frac{\partial y^+}{\partial y^+}\right)\frac{\partial u^+}{\partial y^+}\,dy^+ + \int_0^{y^+}u^+\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u}{\partial C_W}\,dy^+$$

$$-\int_0^{y^+}\left\{\left[\int_0^{y^+}\left(u^+\left(\frac{\partial v^*}{\partial x}\right)+v^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+}\right)dy^+\right]\right\}\frac{\partial u^+}{\partial y^+}\ dy^+=\frac{1}{\mu}\tau_W. \quad \text{(A32)}$$

Defining groups of terms we have

$$G=\int_0^{y^+}(u^+)^2dy^+ \quad \text{(A33)}$$

$$H=\int_0^{y^+}u^+\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+}dy^+ \quad \text{(A34)}$$

$$J=\int_0^{y^+}u^+\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W}dy^+ \quad \text{(A35)}$$

$$M=\int_0^{y^+}\left[u^+\left(\frac{\partial v^*}{\partial x}\right)+v^*\left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+}+v^*\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W}\right]dy^+ \quad \text{(A36)}$$

$$N=\int_0^{y^+}M\left(\frac{\partial u^+}{\partial y^+}\right)dy^+. \quad \text{(A37)}$$

Therefore equation (A32) becomes

$$\frac{\partial v^*}{\partial x}(G)+v^*H+v^*J-N=-\frac{1}{\mu}\tau_W. \quad \text{(A38)}$$

Recalling the law of the wall equation, (A33) becomes

$$G = \int_0^{y^+} \left(\frac{1}{K} \ln y^+ + 5.5 + 2.3\, C_W^{.57} \ln \left(\frac{V^*}{V_0^*} \right) \right)\, dy^+$$

$$= \int_0^{y^+} \left\{ \frac{1}{K^2}(\ln y^+)^2 + (5.5)^2 + (2.3)^2 C_W^{1.14} \ln \left(\frac{V^*}{V_0^*} \right) + 2(5.5)\frac{1}{K}\ln y^+ \right.$$

$$\left. + 2(2.3)\frac{1}{K}\ln y^+\, C_W^{.57} \ln \frac{V^*}{V_0^*} + 2(5.5)\,(2.3) C_W^{.57} \ln \frac{V^*}{V_0^*} \right\} dy^+. \quad (A39)$$

$$G = \frac{1}{K^2} \int_0^{y^+} (\ln y^+)^2 dy^+ + (5.5)^2 y^+ + (2.3)^2\, C_W^{1.14} \ln^2 \left(\frac{V^*}{V_0^*} \right) y^+$$

$$+ \frac{2(5.5)}{K} \int_0^{y^+} \ln y^+\, (dy^+) + \frac{2(2.3)}{K}\, C_W^{.57} \ln \left(\frac{V^*}{V_0^*} \right) \int_0^{y^+} \ln(y^+) dy^+$$

$$+ 2(5.5)(2.3) C_W^{.57} \ln \left(\frac{V^*}{V_0^*} \right) y^+. \quad (A40)$$

Note:
$$(\ln x)^2\, dx = x(\ln x)^2 - 2[x(\ln x - 1)] = x[\ln^2 x - 2\ln x + 2]$$
$$= x[\ln^2 x - 2(\ln x - 1)] \quad (A41)$$

hence

$$G = \frac{1}{K^2} \left\{ y^+ [\ln^2 y^+ - 2(\ln y^+ - 1)] \right\} + (5.5)^2\, y^+$$

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$$+ (2.3)^2 \; C_W^{1.14} \; \ln^2 \left(\frac{V^*}{V_0^*}\right) \; y^+$$

$$+ \frac{2(5.5)}{K} \; [y^+(\ln y^+ - 1)] + \frac{2(2.3)}{K} \; C_W^{.57} \; \ln \left(\frac{V^*}{V_0^*}\right) y^+ (\ln y^+ - 1)$$

$$+ 2(5.5) \; (2.3) \; C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right) \; y^+ \quad 2 \qquad\qquad (A42)$$

$$G = y^+ \left\{ \frac{1}{K^2} \; [\ln^2 y^+ - 2(\ln y^+ - 1)] + \frac{2}{K}(\ln y^+ - 1)\left[5.5 + 2.3 \; C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right)\right] \right.$$

$$\left. + 2.3 \; C_W^{.57} \; \ln\left(\frac{V^*}{V_0^*}\right)\left[2(5.5) + 2.3 \; C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right)\right] + (5.5)^2 . \right\} \quad (A43)$$

Evaluating term H, we have

$$H = \int_0^{y^+} u^+ \left(\frac{\partial y^+}{\partial x}\right)_{\partial y^+} u^+ \; dy^+ \qquad\qquad (A44)$$

$$\frac{\partial y^+}{\partial x} = \frac{\partial}{\partial x} \left(y \; \frac{V^*}{\nu}\right) \;\; = \frac{y}{\nu} \left(\frac{\partial V^*}{\partial x}\right) \qquad\qquad (A45)$$

$$\frac{\partial u^+}{\partial y^+} = \frac{\partial}{\partial y^+} \left[\frac{1}{K} \; \ln y^+ + 5.5 + 2.3 \; C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right)\right] \;\; = \frac{1}{Ky}^+ \qquad\qquad (A46)$$

$$H = \int_0^{y^+} \frac{1}{K} \ln y^+ + 5.5 + 2.3 \, C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right) \left[\frac{y}{\nu} \, \frac{\partial V^*}{\partial x}\right]\left[\frac{1}{Ky^+}\right] \, dy^+. \quad (A47)$$

Now since

$$y^+ = y \, \frac{V^*}{\nu} \quad ,$$

$$\frac{y}{\nu} = \frac{y^+}{V^*}. \quad (A48)$$

Substitute into equation (A47)

$$H = \int_0^{y^+} \left[\frac{1}{K} \ln y^+ + 5.5 + 2.3 C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right)\right]\left[\frac{y^+}{V^*} \, \frac{dV^*}{dx}\right]\left[\frac{1}{Ky^+}\right] \, dy^+ \quad (A49)$$

$$H = \int_0^{y^+} \left[\frac{1}{K} \ln y^+ + 5.5 + 2.3 C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right)\right]\left[\frac{1}{V^* K} \, \frac{\partial V^*}{\partial x}\right] \, dy^+$$

$$= \frac{1}{V^* K} \frac{\partial V^*}{\partial x} \int_0^{y^+} \left[\frac{1}{K} \ln y^+ + 5.5 + 2.3 C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right)\right] \, dy^+$$

$$= \frac{1}{V^* K}\left(\frac{\partial V^*}{\partial x}\right) y^+ \left[\frac{1}{K}(\ln y^+ - 1) + 5.5 + 2.3 C_W^{.57} \ln \left(\frac{V^*}{V_0^*}\right)\right]. \quad (A50)$$

Evaluating term J:

$$J = \int_0^{y^+} u^+ \left(\frac{\partial u^+}{\partial C_W}\right) \frac{\partial C_W}{\partial x} \, dy^+ \tag{A51}$$

$$\frac{\partial u^+}{\partial C_W} = \frac{\partial}{\partial C_W} \left\{ \frac{1}{K} \ln y^+ + 5.5 + 2.3 C_W^{.57} \ln \left(\frac{V^*}{V_0^{\,*}}\right) \right\}$$

$$= (.57)(2.3) \, C_W^{-.43} \ln \left(\frac{V^*}{V_0^{\,*}}\right). \tag{A52}$$

$$J = \frac{\partial C_W}{\partial x} \int_0^{y^+} \left[\frac{1}{K} \ln y^+ + 5.5 + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^{\,*}}\right) \right] \left[(.57)(2.3) C_W^{-.43} \ln\left(\frac{V^*}{V_0^{\,*}}\right) \right] dy^+$$

$$= (.57)(2.3) C_W^{-.43} \ln\left(\frac{V^*}{y^+}\right) \frac{\partial C_W}{\partial x} \int_0^{y^+} \left(\frac{1}{K} \ln y^+ + 5.5 + 2.3 C_W^{.57} \ln \frac{V^*}{V_0^{\,*}} \right) dy^+$$

$$= .57(2.3) C_W^{-.43} \ln\left(\frac{V^*}{V_0^{\,*}}\right) \left(\frac{\partial C_W}{\partial x}\right) y^+ \left[\frac{1}{K}(\ln y^+ - 1) + 5.5 \right.$$

$$\left. + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^{\,*}}\right) \right]. \tag{A53}$$

Evaluating term M, we have:

$$M = \int_0^{y^+} \left[u^+ \left(\frac{\partial V^*}{\partial x} \right) + V^* \left(\frac{\partial y^+}{\partial x} \right) \frac{\partial u^+}{\partial y^+} + V^* \frac{\partial C_W}{\partial x} \left(\frac{\partial u}{\partial C_W} \right) \right] dy^+. \quad \text{(A54)}$$

Let

$$1 = \int_0^{y^+} u^+ \left(\frac{\partial V^*}{\partial x} \right) dy^+ \qquad \text{(A55)}$$

$$2 = \int_0^{y^+} V^* \left(\frac{\partial y^+}{\partial x} \right) \frac{\partial u^+}{\partial y} dy^+ \qquad \text{(A56)}$$

$$3 = \int_0^{Y^+} V^* \left(\frac{\partial C_W}{\partial x} \right) \frac{\partial u^+}{\partial C_W} dy^+. \qquad \text{(A57)}$$

Term 1 becomes

$$\int_0^{y^+} u^+ \left(\frac{\partial V^*}{\partial x} \right) dy^+ = \frac{\partial V^*}{\partial x} \int_0^{y^+} \left[\frac{1}{K} \ln y^+ + 5.5 + 2.3 C_W^{.57} \ln \left(\frac{V^*}{V_0^*} \right) \right] dy^+$$

$$= \frac{\partial V^*}{\partial x} y^+ \left\{ \frac{1}{K} (\ln y^+ - 1) + 5.5 + 2.3 C_W^{.57} \ln \left(\frac{V^*}{V_0^*} \right) \right\}. \qquad \text{(A58)}$$

Term 2 becomes

$$\int_0^{y^+} V^* \left(\frac{\partial y^+}{\partial x}\right)\frac{\partial u^+}{\partial y^+} \, dy^+ = V^* \int_0^{y^+} \left(\frac{y}{\nu} \ \frac{\partial V^*}{\partial x}\right) \ \left(\frac{1}{Ky^+}\right) \, dy^+$$

$$= V^* \int_0^{y^+} \ \frac{y^+}{V^*}\frac{\partial V^*}{\partial x} \frac{1}{Ky^+} \ dy^+$$

$$= \frac{dV^*}{dx} \frac{1}{K} \int_0^{y^+} \ dy^+$$

$$= \frac{1}{K} \frac{dV^*}{dx} \, y^+. \tag{A59}$$

Term 3 becomes

$$\int_0^{y^+} V^*\left(\frac{\partial C_W}{\partial x}\right)\frac{\partial u^+}{\partial C_W} \, dy^+ = V^*\left(\frac{\partial C_W}{\partial x}\right)\int_0^{y^+} \ \frac{\partial u^+}{\partial C_W} \, dy^+$$

$$= V^*\left(\frac{dC_W}{dx}\right) \int_0^{y^+} (2.3)(.57)C_W^{-.43}\ln\left(\frac{V^*}{V_0^*}\right) \ dy^+$$

$$= V^*\left(\frac{dC_W}{dx}\right)(2.3)(.57)C_W^{-.43}\ln\left(\frac{V^*}{V_0^*}\right) y^+. \tag{A60}$$

Now evaluate term N :

$$N = \int_0^{y^+} M \left(\frac{\partial u^+}{\partial y^+}\right) dy^+$$

$$= \int_0^{y^+} \left\{ \frac{\partial v^*}{\partial x} y^+ \left[\frac{1}{K}(\ln y^+ - 1) + 5.5 + 2.3 C_W^{.57}\ln\left(\frac{v^*}{v_0^*}\right)\right] + \frac{1}{K}\frac{\partial v^*}{\partial x} y^+ \right.$$

$$\left. + v^*\left(\frac{\partial C_W}{\partial x}\right)(2.3)(.57)C_W^{-.43}\ln\left(\frac{v^*}{v_0^*}\right) y^+ \right\} \frac{\partial u^+}{\partial y} \ dy^+ \qquad (A61)$$

such that

$$\frac{du^+}{dy^+} = \frac{1}{Ky^+} \qquad (A62)$$

therefore

$$N = \int_0^{y^+} y^+ \left\{ \frac{\partial v^*}{\partial x}\left(\frac{1}{K}(\ln y^+ - 1) + 5.5 + 2.3 C_W^{.57}\ln\left(\frac{v^*}{v_0^*}\right)\right) + \frac{1}{K}\frac{\partial v^*}{\partial x} \right.$$

$$\left. + v^*\left(\frac{\partial C_W}{\partial x}\right)(2.3)(.57)C_W^{-.43}\ln\left(\frac{v^*}{v_0^*}\right) \right\} \ dy \ . \qquad (A64)$$

Now

$$N = \int_0^{y^+} \left[3 + 4 + 5 \right] dy^+ \tag{A65}$$

where

$$3 = \frac{1}{K} \frac{\partial V^*}{\partial x} \left[\frac{1}{K}(\ln y^+ - 1) + 5.5 + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) \right] \tag{A66}$$

$$4 = \frac{1}{K^2} \frac{\partial V^*}{\partial x} \tag{A67}$$

$$5 = \frac{1}{K} V^* \left(\frac{\partial C_W}{\partial x}\right)(2.3)(.57) C_W^{-.43} \ln\left(\frac{V^*}{V_0^*}\right). \tag{A68}$$

And we then have

$$\int_0^{y^+} 3 \, \partial y^+ = \frac{1}{K} \frac{\partial V^*}{\partial x} \int_0^{y^+} \frac{1}{K}(\ln y^+ - 1) + 5.5 + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) \, dy^+$$

$$= \frac{1}{K} \frac{\partial V^*}{\partial x} \left[\frac{1}{K}(y^+ \ln y^+ - 2y^+ + 5.5 \, y^+ + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) y^+ \right]$$

$$= \frac{1}{K} \frac{\partial V^*}{\partial x} y^+ \left[\frac{1}{K}(\ln y^+ - 2) + 5.5 + 2.3 \, C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) \right] \tag{A69}$$

$$\int_0^{y^+} 4 \, \partial y^+ = \frac{1}{K^2} \frac{\partial V^*}{\partial x} \int_0^{y^+} dy^+$$

$$= \frac{1}{K^2} \frac{\partial V^*}{\partial x} y^+ \tag{A70}$$

$$\int_0^{y^+} 5 \, \partial y^+ = \int_0^{y^+} \left[\frac{1}{K} V^*\left(\frac{dC_W}{dx}\right)(2.3)(.57) C_W^{-.43} \ln\left(\frac{V^*}{V_0^*}\right) \right] dy^+$$

$$= \frac{1}{K} V^*\left(\frac{\partial C_W}{\partial x}\right)(2.3)(.57) C_W^{-.43} \ln\left(\frac{V^*}{V_0^*}\right) y^+ \tag{A71}$$

$$N = \left\{ \frac{1}{K} \frac{\partial V^*}{\partial x} y^+ \left[\frac{1}{K}(\ln y^+ - 2) + 5.5 + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) \right] \right.$$

$$+ \frac{1}{K^2} \frac{\partial V^*}{\partial x} y^+$$

$$\left. + \frac{1}{K} V^* \frac{\partial C_W}{\partial x} (2.3)(.57) C_W^{-.43} \ln \frac{V^*}{V_0^*} y^+ \right\} \tag{A72}$$

$$\frac{\partial V^*}{\partial x}(G) = \frac{\partial V^*}{\partial x} y^+ \left\{ \frac{1}{K^2} \ln^2 y^+ - 2(\ln y^+ - 1) \right.$$

$$+ \frac{2}{K}(\ln y^+ - 1)\left[5.5 + 2.3 C_W^{.57}\ln\left(\frac{V^*}{V_0^*}\right)\right]$$

$$+ 2.3 C_W^{.57}\ln\left(\frac{V^*}{V_0^*}\right)\left[2(5.5) + 2.3 C_W^{.57}\ln\left(\frac{V^*}{V_0^*}\right)\right] + (5.5)^2 \left.\right\} \quad (A73)$$

$$V^*(H) = V^*\left[\frac{1}{V^* K}\left(\frac{\partial V^*}{\partial x}\right)y^+ + \frac{1}{K}(\ln y^+ - 1) + 5.5\right.$$

$$\left. + 2.3\, C_W^{.57}\ln\left(\frac{V^*}{V_0^*}\right)\right]$$

$$= \frac{1}{K}\frac{\partial V^*}{\partial x}y^+\left[\frac{1}{K}(\ln y^+ - 1) + 5.5 + 2.3 C_W^{.57}\ln\left(\frac{V^*}{V_0^*}\right)\right] \quad (A74)$$

$$V^*(J) = V^*\,(.57)\,(2.3) C_W^{-.43}\ln\left(\frac{V^*}{V_0^*}\right)\frac{\partial C_W}{\partial x}y^+ \frac{1}{K}(\ln y^+ - 1) + 5.5$$

$$+ 2.3 C_W^{.57}\ln\left(\frac{V^*}{V_0^*}\right). \quad (A75)$$

$$-N = \left\{ \frac{1}{K} \frac{\partial V^*}{\partial x} y^+ \quad \frac{1}{K}(\ln y^+ - 2) + 5.5 + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) \right.$$

$$+ \frac{1}{K^2}\left(\frac{\partial V^*}{\partial x}\right) y^+$$

$$+ \frac{1}{K} V^* \frac{\partial C_W}{\partial x} (2.3)\ (.57)\ C_W^{-.43} \ln\left(\frac{V^*}{V_0^*}\right) y^+ \left. \right\} \qquad \text{(A76)}$$

Combining into equation (A37) we have

$$- \frac{1}{\mu} \tau_W = 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) \left\{ y^+ \frac{\partial V^*}{\partial x} \left[\frac{2}{K}(\ln y^+ - 1) + 2(5.5) \right. \right.$$

$$\left. + 2.3 C_W^{.57} \ln\left(\frac{V^*}{V_0^*}\right) + \frac{1}{K} \right]$$

$$\left. + V^* y^+ (.57)(2.3) C_W^{-.43} \ln \frac{V^*}{V_0^*} \left(\frac{\partial C_W}{\partial x}\right) - \frac{1}{K}\left(\frac{\partial V^*}{\partial x}\right) y^+ \right\}$$

$$+ \frac{(\ln y^+ - 1)}{K} y^+ \left\{ \frac{\partial V^*}{\partial x} \quad - \frac{2}{K} + 2(5.5) + \frac{1}{K} V^* (.57)(2.3) C_W^{-.43} \ln\left(\frac{V^*}{V_0^*}\right) \frac{\partial C_W}{\partial x} \right\}$$

$$+ y^+ \frac{\partial v^*}{\partial x} \left\{ \frac{1}{K^2} (\ln y^+)^2 + (5.5)^2 - (2.3)(.57)^{-.43} \ln \frac{v^*}{v_0^*} \right\}$$

$$+ (2.3)(.57) C_W^{-.43} \ln \left(\frac{v^*}{v_0^*} \right) \frac{\partial C_W}{\partial x} v^* y^+ \left[5.5 - \frac{1}{k} \right]. \tag{A77}$$

Now

$$- \frac{1}{\mu} \tau_W = - \frac{1}{\rho \nu} \left(\frac{\rho \nu}{\lambda^2} \right) = - \frac{U^2}{\nu \lambda^2} \tag{A78}$$

since

$$\tau_W = C_f (\tfrac{1}{2} \rho U^2) \tag{A79}$$

and letting $\lambda = \frac{U}{v^*}$. $\tag{A80}$

Recalling equation (A15)

$$v^* = \sqrt{\frac{\tau_W}{\rho}}$$

$$= \sqrt{\frac{\frac{1}{2}\rho U^2 C_f}{\rho}}$$

resulting in

$$v^* = U \sqrt{\frac{C_f}{2}} . \qquad (A81)$$

from equation (A80)

$$\lambda = \frac{U}{U\sqrt{C_{f/2}}} = \sqrt{\frac{2}{C_f}} \qquad (A82)$$

$$C_f = \frac{2}{\lambda^2} . \qquad (A83)$$

Therefore equation (A79) becomes

$$\tau_w = \frac{\rho U}{\lambda^2} . \qquad (A84)$$

Also

$$\frac{\partial v^*}{\partial x} = - \frac{U}{\lambda^2} \left(\frac{\partial \lambda}{\partial x}\right). \qquad (A85)$$

Substituting equations (A78) and (A85) into equation (A77) and combining terms we have

$$
-\frac{U}{\nu} = y^+ \frac{\partial \lambda}{\partial x} \left\{ 2.3C_w^{.57} \ln\left(\frac{\nu^*}{\nu_0^*}\right) \left[\frac{2}{k}(\ln y^+ - 1) + 2(5.5) \right.\right.
$$

$$
\left.\left. + 2.3 \, C_w^{.57} \ln\left(\frac{\nu^*}{\nu_0^*}\right) \right] + \frac{2}{k}(\ln y^+ - 1)(5.5 - \frac{1}{k}) \right.
$$

$$
\left. + \frac{1}{k^2}(\ln y^+)^2 + (5.5)^2 \right\}
$$

$$
- y^+ \lambda \frac{\partial C_w}{\partial x} (.57)(2.3)C_w^{-.43}\ln\left(\frac{\nu^*}{\nu_0^*}\right) \left[1 + \frac{1}{k^2}(\ln y^+ - 1) \right.
$$

$$
\left. + 5.5 - \frac{1}{k} \right]. \tag{A86}
$$

The value of y^+ in equation (A86) is the value of y^+ at the edge of the boundary layer.

Therefore at the edge of the boundary layer we have

$$
u = U \tag{A87}
$$

and

$$y^+ = \delta^+$$

hence

$$u^+ = \frac{u}{V^*} = \frac{U}{V^*} \quad . \tag{A88}$$

Recalling equation (A80) ,

$$\lambda = \frac{U}{V^*}$$

$$\lambda = \frac{U}{V^*} = u^+, \text{ at } y^+ = \delta^+ \quad . \tag{A89}$$

Whence equation (A1) becomes at $y^+ = \delta^+$

$$\lambda = \frac{1}{k} \ln \delta^+ + B + \alpha \, C_w^\gamma \, \ln\left(\frac{V^*}{V_0^*}\right) \tag{A90}$$

where $\alpha = 2.3$ and $\gamma = .5$

and

$$\ln \delta^+ = K\lambda - KB - \left[\ln\left(\frac{V^*}{V_0^*}\right)\right]^{KZ} \tag{A91}$$

where

$$z = \alpha\, C_W^{\gamma} \tag{A92}$$

and

$$\delta^+ = e^{K(\lambda - B)} \left[\frac{v^*}{v_0^*}\right]^{-Kz} . \tag{A93}$$

Equation (A86) then becomes

$$-\frac{U}{v} = \delta^+ \left\{ z\ln\frac{v^*}{v_0^*}\left[\frac{2}{K}(\ln\delta^+ - 1) + 2B + z\ln\frac{v^*}{v_0^*}\right]\right.$$

$$\left. + \frac{2}{K}(\ln\delta^+ - 1)(B - \frac{1}{K}) + \frac{1}{K^2}(\ln\delta^+)^2 + B^2 \right\}\frac{\partial\lambda}{\partial x}$$

$$- \delta^+\,\lambda\left\{\frac{\gamma z}{C_W}\ \ln\frac{v^*}{v_0^*}\left[1 + \frac{1}{K^2}\ (\ln\delta^+ - 1) + \frac{1}{K}\right]\right\}\frac{\partial C_W}{\partial x} . \tag{A94}$$

Multiplying by dx and integrating, we have

$$R_e(X) - R_e(X_o) = \int_{\lambda(X_o)}^{\lambda(X)} \delta^+\left\{ z\ln\frac{v^*}{v_0^*}\ \left[\frac{2}{K}(\ln\delta^+ - 1) + 2B + z\ln\frac{v^*}{v_0^*}\right]\right.$$

$$\left. + \frac{2}{K}(\ln\delta^+ - 1)(B - \frac{1}{K}) + \frac{1}{K^2}(\ln\delta^+)^2 + B^2 \right\}\ d\lambda$$
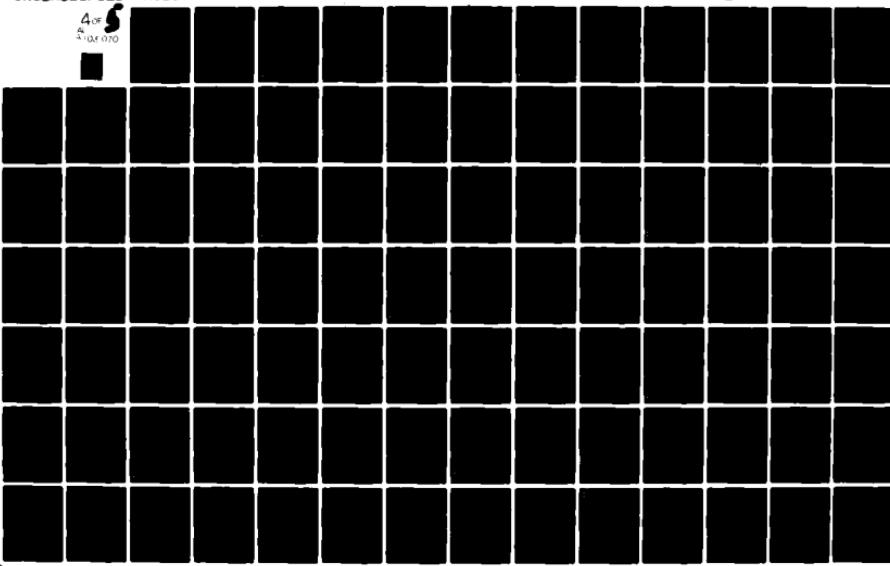
$$-\int_{C_w(X_o)}^{C_w(X)} \delta^+\lambda \left\{ \frac{\gamma Z}{C_w} \ln\left(\frac{V^*}{V_0^*}\right)\left[1 + \frac{1}{K^2}(\ln\delta^+ - 1) + B - \frac{1}{K} \right] \right\} dC_w. \quad \text{(A95)}$$

For injection of polymer at or close to the leading edge we may assume $X_o = 0$, hence $Re(X_o) = 0$ and $C_w(X_o) = C_1$. Equation (A95) is numerically integrated in the flow model described in chapter V.

APPENDIX B

COMPUTER PROGRAM DATAPAC AND SAMPLE LISTING

```
                    RESEQUENCED LISTING

      PROGRAM DATABAC                                          000001
      OVERLAY(90)                                              000002
      COMMON/AREA/M,H,S,KWALL,KBLAS                            000003
      COMMON/IOVER/IENO,IPASS                                  000004
      COMMON YLAM(31,2),YC(31,2)                               000005
      COMMON CI,QI,UI,CW,SLOT                                  000006
      COMMON DELTA,DELTA1,DELTA2,HYDIA,H12,H32,IDENT,II,MC,MCM,NPLOT, RE  000007
     1DELTA,REDELTA1,REDELTA2,REHYDIA,REX,UFS,XIN              000007
      COMMON IFMT(10),MCASE(12),NXCHAR(12),NYCHAR(12),NTCHAR(12), NCHAR(  000008
     1120),T(31)                                              000008
      COMMON XAX(9,12),YAX(9,12),TITLE(9,12),CHARAC(9,20)      000009
      COMMON AETA(40,2),DUUTAU(31,2),REYL(31,2),U(31,2),UPLUS(31,2), URA  000010
     1TIO(40,2),URATIO2(31,2),Y(31,2),YDELTA(31,2),YDELT1(31,2), YDELT2   000010
     22(31,2),YPLUSL(31,2)                                    000010
      COMMON CONC(31,2)                                        000011
      DIMENSION CNU(31),D1(31),D2(31),D3(31),FREQ(31),REY(31),TX(31), UM  000012
     1(31),URATIO2(31),U2(31),U1(31),X(31),XF(31),YM(31),YM(31),YY(31),Y2  000012
     22(31),NNN(12,2),YMM(31,2),YPLUS(31,2)                   000012
      DIMENSION CONC(31),Z(31),FLOW(31)                        000013
      DIMENSION WCCM(31),WY(31)                                000014
      REAL LAMBDA                                              000015
      REWIND 1                                                 000016
      IF(IPASS.EQ.1) GO TO 80                                  000017
      IPASS=1                                                  000018
      READ 10,KBLAS,KWALL                                      000019
      READ 10,NPLOT,NCM,(MCASE(I),I=1,11)                      000020
   10 FORMAT(12I5)                                             000021
      READ 20,(IFMT(JK),JK=1,10)                               000022
   20 FORMAT(10A4)                                             000023
      READ IFMT,(URATIO(I,2),I=1,KBLAS)                        000024
      READ IFMT,(AETA(I,2),I=1,KBLAS)                          000025
      READ 30,(YPLUSL(I,2),I=1,KWALL)                          000026
      READ 30,(UPLUS(I,2),I=1,KWALL)                           000027
   30 FORMAT(6F10.5)                                           000028
      DO 40 K=1,NPLOT                                          000029
      READ 50,(XAX(JK,K),JK=1,9),NXCHAR(K)                     000030
      READ 50,(YAX(JK,K),JK=1,9),NYCHAR(K)                     000031
   40 READ 50,(TITLE(JK,K),JK=1,9),NTCHAR(K)                   000032
   50 FORMAT(9A3,I3)                                           000033
      DO 60 K=1,NCM                                            000034
   60 READ 50,(CHARAC(JK,K),JK=1,9),NCHAR(K)                   000035
      READ 70,H,M,S                                            000036
   70 FORMAT (3F10.5)                                          000037
      SLOT=S                                                   000038
   80 READ 90,IDENT,CI,QI                                      000039
   90 FORMAT (I6,4X,2F10.5)                                    000040
      IF (IDENT-01790)790,100                                  000041
  100 CONTINUE                                                 000042
      MM=1                                                     000043
      IF(QI.LE.1.E-10) GO TO 170                               000044
      J=1                                                      000045
  110 READ 120,CONC(J),YC(J,MM),DUM,XC,IC,ISTOP                000046
  120 FORMAT(4E10.5,I10,I5)                                    000047
      IF(ISTOP.GT.0) GO TO 130                                 000048
      J=J+1                                                    000049
      GO TO 110                                                000050
  130 JCONC=J                                                  000051
      CW=CONC(1)                                               000052
      CWCI=CW/CI                                               000053
```

```
      DO 140 K=1,JCONC                                            000054
140   CCX(K,MM)=CONC(K)/CX                                        000055
      DO 150 K=1,JCONC                                            000056
      M=K                                                         000057
      IF(CCX(K,MM).GE.0.5) GO TO 160                              000058
150   CONTINUE                                                    000059
160   M2=M-2                                                      000060
      NPLOT=11                                                    000061
      GO TO 180                                                   000062
170   NPLOT=9                                                     000063
180   I=1                                                         000064
190   READ 200,Y(I,MM),V(I),FREQ(I),X(I),T(I),TX(I),Z(I),FLOW(I) 000065
200   FORMAT(8F10.3)                                             000066
      IF(T(I).LE.1.E-10) GO TO 210                               000067
      I=I+1                                                       000068
      GO TO 190                                                   000069
210   CONTINUE                                                    000070
      QFLOW=FLOW(1)                                              000071
      QVOL=QFLOW*8.33/(60.*62.4)                                 000072
      N=I-1                                                       000073
      DO 730 LL=1,NPLOT                                          000074
      MC=MCASE(LL)                                               000075
      DO 780 MM=1,MC                                             000076
      IF((MM.EQ.2 .AND. KBLAS.NE.0) .AND. LL.EQ.6) GO TO 750     000077
      IF((MM.EQ.2 .AND. KWALL.NE.0) .AND. LL.EQ.7) GO TO 760     000078
      IF(LL.NE.1) GO TO 770                                      000079
      VFS=V(1)                                                   000080
      UFS=FREQ(1)*VFS                                            000081
      UFSM=UFS*12.*2.54*.01                                      000082
      SF=S/12.                                                   000083
      ASLOT=S*W                                                  000084
      ASLOTF=ASLOT/144.                                          000085
      UI=QI*.0610237/(ASLOT*12.*60.)                             000086
      DO 220 J=1,N                                               000087
      CNU(J)=3.60901355E-05-(7.17973611E-07)*(T(J))+(7.15690612E-0  000088
     1 3)*(T(J)**2)-(3.50477166E-11)*(T(J)**3)+(6.62377924E-14)*(T(  00004A
     2 J)**4)                                                    000088
      U(J,MM)=FREQ(J)*V(J)                                       000089
      URATIO(J,MM)=U(J,MM)/UFS                                   000090
220   CONTINUE                                                    000091
      DO 230 K=1,N                                               000092
      IF(URATIO(K,MM)-.99) 240,240,230                           000093
230   CONTINUE                                                    000094
240   DELTA=Y(K-1,MM)                                            000095
      J=N+1                                                      000096
      DO 250 M=1,N                                               000097
      L=J-M                                                      000098
      Y2(M)=Y(L,MM)                                              000099
      U2(M)=U(L,MM)                                              000100
      URATIO2(M)=URATIO(L,MM)                                    000101
250   CONTINUE                                                    000102
      DO 260 I=1,N                                               000103
      Y(I,MM)=Y2(I)                                              000104
      U(I,MM)=U2(I)                                              000105
      URATIO(I,MM)=URATIO2(I)                                    000106
      UM(I)=U(I,MM)*12.*2.54*.01                                 000107
      YM(I)=Y(I,MM)*2.54                                         000108
      REY(I)=UFS*Y(I,MM)/(12.*CNU(I))                            000109
      REYL(I,MM)=ALOG10(REY(I))                                  000110
260   CONTINUE                                                    000111
      U1(1)=1.                                                   000112
      U2(1)=1.E-10                                               000113
```

```
          D3(1)=1.E-10                                          000114
          DO 270 L=2,M                                          000115
            D1(L)=(1.-URATIO(L,MM))                             000116
            D2(L)=URATIO(L,MM)*(1.-URATIO(L,MM))               000117
            D3(L)=URATIO(L,MM)*(1.-(URATIO(L,MM)**2.))         000118
  270     CONTINUE                                              000119
          DO 280 L=2,N                                          000120
            YY(L)=Y(L,MM)                                       000121
  280     CONTINUE                                              000122
          YY(1)=1.E-10                                          000123
          DO 290 L=1,4                                          000124
            MCCM(L)=CCM(M2,MM)                                  000125
            AY(L)=YC(M2,MM)                                     000126
  290       M2=M2+1                                             000127
          CALL INTEGRAL(N,YY,D1,DELTA1)                         000128
          CALL INTEGRAL(N,YY,D2,DELTA2)                         000129
          CALL INTEGRAL(N,YY,D3,DELTA3)                         000130
          DO 300 I=1,N                                          000131
            YDELTA(I,MM)=Y(I,MM)/DELTA                          000132
            YDELTA1(I,MM)=Y(I,MM)/DELTA1                        000133
            YDELTA2(I,MM)=Y(I,MM)/DELTA2                        000134
            URATIO2(I,MM)=(YDELTA(I,MM))**(1./7.)              000135
  300     CONTINUE                                              000136
          AREA=(H*H1/144.                                       000137
          UAVG=QVOL/AREA                                        000138
          UAVGM=UAVG*12.*2.54*.01                              000139
          PERIM=((2.*H)+(2.*H))/12.                            000140
          HYDIA=4.*AREA/PERIM                                   000141
          HYDIAM=HYDIA*.3048                                    000142
          H12=DELTA1/DELTA2                                     000143
          H32=DELTA3/DELTA2                                     000144
          REX=UFS*X(1)/(CNU(1)*12.L                            000145
          REHALF=UFS*(H/24.)/CNU(1)                            000146
          REDELTA=UFS*DELTA/CNU(1)                             000147
          REDELTA1=UFS*DELTA1/CNU(1)                           000148
          REDELTA2=UFS*DELTA2/CNU(1)                           000149
          REHYDIA=UAVG*HYDIA/CNU(1)                            000150
          CFFW=0.455/(ALOG(0.06*REX)**2.)                     000151
          DELTAM=DELTA*2.54                                    000152
          DELTA1M=DELTA1*2.54                                  000153
          DELTA2M=DELTA2*2.54                                  000154
          DELTA3M=DELTA3*2.54                                  000155
          XIN=X(1)                                             000156
          XM=XIN*2.54                                          000157
          DO 310 I=1,N                                         000158
            YM(I)=Y(I,MM)/12.                                  000159
            XF(I)=X(1)/12.                                     000160
            AETA(I,MM)=YM(I)*SQRT(UFS/(CNU(1)*XF(I)))         000161
  310     CONTINUE                                             000162
          DO 320 I=1,N                                         000163
            UTAU=UFS*SQRT(CFFW/2.)                             000164
            QUUTAU(I,MM)=(UFS-U(I,MM))/UTAU                   000165
            UPLUS(I,MM)=U(I,MM)/UTAU                          000166
            YPLUS(I,MM)=Y(I,MM)*UTAU/(12.*CNU(1))            000167
            YPLUSL(I,MM)=ALOG10(YPLUS(I,MM))                 000168
            YHM(I,MM)=Y(I,MM)/(H2.5)                         000169
  320     CONTINUE                                             000170
          IF(QI.LE.1.E-10) GO TO 340                          000171
          CALL POLY(LAMBDA,MCCM,AY,2)                         000172
          DO 330 I=1,JCONC                                    000173
  330     YLAM(I,MM)=YC(I,MM)/LAMBDA                         000174
  340     PRINT 350,IDENT                                    000175
```

```
350 FORMAT(1H1,20H        TEST NUMBER  ,I5)              000175
    PRINT 360,XIN,XM                                     000177
    PRINT 500, UFS,UFSM                                  000178
    PRINT 510,UAVG,UAVGM                                 000179
    PRINT 370,DELTA,DELTAM                               000160
    PRINT 380,DELTA1,DELTA1M                             000181
    PRINT 390,DELTA2,DELTA2M                             000182
    PRINT 400,DELTA3,DELTA3M                             000183
    PRINT 520,HYOIA ,HYOIAM                              000184
    PRINT 410,H12                                        000185
    PRINT 420,H32                                        000186
    PRINT 430,REX                                        000187
    PRINT 540,REHALF                                     000188
    PRINT 530, REHYOIA                                   000189
    PRINT 440,REDELTA                                    000190
    PRINT 450,REDELTA1                                   000191
    PRINT 460,REDELTA2                                   000192
    PRINT 480, TX(1)                                     000193
    PRINT 470, T(1)                                      000194
    PRINT 490, CNU(1)                                    000195
    PRINT 550,CFFW                                       000196
C...UTAU IS BASED ON FRANK WHITE S FORMULA               000197
    PRINT 560,UTAU                                       000198
    PRINT 570,CI                                         000199
    PRINT 580,QI                                         000200
    PRINT 610,QFLOW                                      000201
    PRINT 620,QVOL                                       000202
    PRINT 590,UI                                         000203
    PRINT 600,S                                          000204
    PRINT 630,CW                                         000205
    PRINT 640,CWCI                                       000205
    PRINT 650,LAMBDA                                     000207
360 FORMAT (1X,11H       X= ,F9.3,4H IN.,F9.3,4H CM.)    000208
370 FORMAT (1X,11H   DELTA = ,F9.3,4H IN.,F9.3,4H CM.)   000209
380 FORMAT (1X,11H  DELTA1 = ,F9.3,4H IN.,F9.3,4H CM.)   000210
390 FORMAT (1X,11H  DELTA2 = ,F9.3,4H IN.,F9.3,4H CM.)   000211
400 FORMAT (1X,11H  DELTA3 = ,F9.3,4H IN.,F9.3,4H CM.)   000212
410 FORMAT (1X,11H     H12 = ,F9.3)                      000213
420 FORMAT (1X,11H     H32 =-,F9.3)                      000214
430 FORMAT (1X,11H     REX = ,F9.0)                      000215
440 FORMAT(1X,11HREDELTA  = ,F9.0)                       000216
450 FORMAT (1X,11HREDELTA1 = ,F9.0)                      000217
460 FORMAT (1X,11HREDELTA2 = ,F9.0)                      000218
470 FORMAT (1X,11HH2O TEMP = ,F9.3)                      000219
480 FORMAT (1X,11H  T WALL = ,F9.3)                      000220
490 FORMAT (1X,11H      NU = ,F9.6)                      000221
500 FORMAT (1X,11H      UFS = ,F9.2,8H FT/SEC ,F9.3,8H M/SEC  )  000222
510 FORMAT(1X,*   UAVG= *,F9.3,* FT/SEC*,F9.3,* M/SEC*)  000223
520 FORMAT (1X,11H   HYOIA = ,F9.3,4H FT.,F9.3,4H M.)    000224
530 FORMAT (1X,11H REHYOIA = ,F9.1)                      000225
540 FORMAT(1X,11H  REHALF = ,F9.0)                       000225
550 FORMAT(1X,11H      CF = ,F9.6)                       000227
560 FORMAT(1X,11H  UTAUFW = ,F9.6)                       000229
570 FORMAT(1X,* CONC IN = *,F9.2,* PPM*)                 000229
580 FORMAT(1X,*POLYFLOW = *,F9.2,* CC/MIN*)              000230
590 FORMAT(1X,*POLY UIN = *,F9.6,* FT/SEC*)              000231
600 FORMAT(1X,*    SLOT = *,F9.4,* IN*)                  000232
610 FORMAT(1X,*    QFLOW = *,F9.3,* GPM*)                000233
620 FORMAT(1X,*    QVOL= *,F9.3,* FT3/SEC*)              000234
630 FORMAT(1X,*   CWALL = *,F9.2,* PPM*)                 000235
640 FORMAT(1X,*   CW/CI = *,F9.4)                        000236
650 FORMAT(1X,*  LAMBDA = *,F9.4)                        000237
```

```
      PRINT 390,IDENT                                              000235
      PRINT 660                                                    000237
  660 FORMAT(//*      Y(IN.)     Y(CM.)     U(FT/S)     U(M/S)     U/UFS     Y/  0002-3
     1DELTA1   Y/DELTA2    REY      CG REY   Y/DELT1     JET1     X(IN.)    000240
     2  Z(IN.)*/)                                                  0002-0
      DO 670 I=1,N                                                 0002-1
  670     PRINT 680,Y(I,MM),YM(I),U(I,MM),UM(I),URATIO(I,MM),YOELTA1(I,M  0002-2
     1M),YOELTA2(I,MM),REY(I),REYL(I,MM),YOELT1(I,MM),JET(I,MM),X(  0002-2
     2  I),Z(I)                                                    0002-2
  680 FORMAT(1X,13F10.3)                                           0002-3
      PRINT 690                                                    0002-4
  690 FORMAT(//.*    Y(IN.)    Y/DELTA    U/UFS     (1/7)     UPLUS    0002-5
     1YPLUS    DEL UTAU   Y/HALFH  LOG(YPLUS)*/)                    0002-5
      DO 710 I=1,N                                                 0002-6
          PRINT 700,Y(I,MM),YOELTA(I,MM),URATIO(I,MM),URATIO7(I,MM), U  0002-7
     1    PLUS(I,MM),YPLUS(I,MM),OUUTAU(I,MM),YHH(I,MM),YPLUS(I,MM)  0002-7
  700 FORMAT(1X,3F10.3)                                            0002-8
  710 CONTINUE                                                     0002-9
      PRINT 720                                                    000250
  720 FORMAT(//*     YC(IN.)    CONC(PRM)   C/CM     Y/L(MBD)      XC*.)    000251
      DO 730 J=1,JCONC                                             000252
  730     PRINT 740,YC(J,MM),CONC(J,CCM(J,MM),YL1X(J,MM),XC         000253
  740 FORMAT(1X,3F10.3)                                            000254
      GO TO 770                                                    000255
  750 NNN(LL,MM)=KBLAS                                             000256
      GO TO 780                                                    000257
  760 NNN(LL,MM)=KWALL                                             000258
      GO TO 780                                                    000259
  770 NNN(LL,MM)=N                                                 000260
      IF(LL.EQ.10.OR.LL.EQ.11) NNN(LL,MM)=JCONC                    000261
  780 CONTINUE                                                     000262
      CALL PUNPLOT(NNN)                                            000263
      CM=0.                                                        000264
      CWCI=0.                                                      000265
      QI=0.                                                        000266
      II=0.                                                        000267
      REWIND 1                                                     000268
      GO TO 800                                                    000269
  790 CONTINUE                                                     000270
      IENO=93                                                      000271
  800 CONTINUE                                                     000272
      END                                                          000273
```

```
                    RESEQUENCED LISTING

$JOB,N3130,I2,DATAPAC,1,2000,,CORE=55,SCR=10,CLASS=C,R,W,I2T,GS31,DAT,BA    000000
     IC1,01,TG,OG,ALL) A,W,I2T,GS83,DATAPAC1,01,TG,OG,1280,730,771230,,S    000000
     2,A211 3,W,TAG1,I2TAGS83,DATAPAC1,01,TG,01 0,W,PLC8,MASTER,LIBRARY,    000000
     3C2,****,I) 3,W,POIR,MASTER,LIBRARY-DIRECTORY,C2,****,I) L,X)          000000
     PROGRAM MAIN                                                          000001
     OVERLAY(C)                                                            000002
     COMMON/AD98/H,4,S,KHALL,KBLAS                                         000003
     COMMON/IOVER/IEND,IPASS                                               000004
     IEND=0                                                                000005
     IPASS=0                                                               000006
 10  CONTINUE                                                              000007
     CALL UFOVER(98,4HTAG1)                                                000008
     IF(IEND,EQ,33) GO TO 20                                               000009
     CALL UFOVER(99,4HTAG1)                                                000010
     GO TO 10                                                              000011
 20  CONTINUE                                                              000012
     END                                                                   000013
```

```
                    RESEQUENCED LISTING

      SUBROUTINE PUNPLOT(IZ)                                            010001
      COMMON YLAM(31,2),YCC(31,2)                                       000002
      COMMON CI,QI,UI,CW,S                                             000003
      COMMON DELTA,DELTA1,DELTA2,HYDIA,H12,H32,IDENT,LL,MC,NCH,NPLOT, RE 000004
     1DELTA,REDELTA1,REDELTA2,REHYDIA,REX,UES,XIN                      000004
      COMMON IFNT(10),MCASE(12),NXCHAR(12),NYCHAR(12),NTCHAR(12), NCHAR( 000005
     120),T(31)                                                        000005
      COMMON XAX(9,12),YAX(9,12),TITLE(9,12),CHARAC(9,20)              000006
      COMMON AETA(40,2),QUUTAU(31,2),REYL(31,2),UI(31,2),UPLUS(31,2), UPA 000007
     1TIO(40,2),URATIO7(31,2),Y(31,2),YDELTA(31,2),YDELTA1(31,2), YDELTA 000007
     22(31,2),YPLUSL(31,2)                                             000007
      COMMON CCW(31,2)                                                 000008
      DIMENSION FACT(12),KGRID(12),KLABEL(12),KSYM(12,2),LGRIDX(12), LGR 000009
     1IDY(12),NO(20),NLABEL(12),NLOGX(12),NLOGY(12),NLOG1(12), NLOG2(12) 000009
     2,NSYM(12,2),NXT(12),NYT(12),PX(12),PY(12),XAXL(12), X81(12),X82(12 000009
     3),XC1(20),XENO(12),XGRID(12),XNUM(20),XOF(12), XUNIT(12),YAXL(12), 000009
     4YSL(12),Y92(12),YC1(20),YENO(12),YGRID(12), YOF(12),YUNIT(12)     000009
      DIMENSION P1(40,11,2),Q1(40,11,2),P(20),YNUM(20),IPZ(12),IZ(12,2) 000010
      DATA (NLOGX(I),I=1,11)/6*0,3,4*0/                                000011
      DATA (NLOG1(I),I=1,11)/11*0/                                     000012
      DATA (NLOGY(I),I=1,11)/11*0/                                     000013
      DATA (NLOG2(I),I=1,11)/11*0/                                     000014
      DATA (LGRIDX(I),I=1,11)/6*0,1,4*0/                               000015
      DATA (LGRIDY(I),I=1,11)/11*0/                                    000016
      DATA (KGRID(I),I=1,11)/11*1/                                     000017
      DATA (NSYM(I,1),I=1,11)/11*67000000000000009/                   000018
      DATA (NSYM(I,2),I=1,11)/2*50000000000000009,,600000000000000008, 2*6 000018
     10000000000000009/                                               000018
      DATA KSINT,HGT,H/1.,1.,05/                                      000020
      DATA (XGRID(I),I=1,11)/6*.5,0.,,4*.5/                           000021
      DATA (YGRID(I),I=1,11)/11*.5/                                   000022
      DATA (FACT(I),I=1,11)/11*1./                                    000023
      DATA (KLABEL(I),I=1,11)/11*1/                                   000024
      DATA (NLABEL(I),I=1,11)/11*20/                                  000025
      DATA (NXT(I),I=1,11)/4*11,13,11,0,4*11/                         000026
      DATA (NYT(I),I=1,11)/11*11/                                     000027
      DATA (PX(I),I=1,11)/4*3,,2,5,3,,2,,4*3,/                        000028
      DATA (PY(I),I=1,11)/11*2.5/                                     000029
      DATA (XAXL(I),I=1,11)/4*5,,6,,3,,7,3,,4*5,/                     000030
      DATA (YAXL(I),I=1,11)/11*5,/                                    000031
      DATA (XUNIT(I),I=1,11)/5,,1,,5,,10,,3,,1,,3,,4*1,/              000032
      DATA YUNIT(1)/0.250/,(YUNIT(I),I=2,11)/4*1,,5,,40,,20,,1,,30.5./ 000033
      DATA (XOF(I),I=1,11)/4*0,,2,,6*0,/                              000034
      DATA (YOF(I),I=1,11)/11*0./                                     000035
      DATA (XENO(I),I=1,11)/2*-3,,12,,-3,,-2,5,12,,-2,,-3,,12,,-3,,12,/ 000036
      DATA (YENO(I),I=1,11)/2*7.,-21.5,2*7.,-21.5,2*7.,-21.5,7.,-12./  000037
      DATA (X81(I),I=1,11)/11*11./                                    000038
      DATA (X82(I),I=1,11)/11*0./                                     000039
      DATA (Y81(I),I=1,11)/11*0./                                     000040
      DATA (Y82(I),I=1,11)/11*6.3/                                    000041
      DATA (KSYM(I,1),I=1,11)/11*2/                                   000042
      DATA (KSYM(I,2),I=1,11)/3*0,2,2*0/                              000043
      DATA (XC1(I),I=1,5)/3*1,275/,(XC1(I),I=6,11)/5*3,775/, (XC1(I),I=1 000044
     11,15)/3*5.873/,(XC1(I),I=16,20)/5*7.375/                        000044
      DATA (YC1(I),I=1,16,3)/4*1,25/                                  000045
      DATA (YC1(I),I=2,17,3)/4*1.00/                                  000046
      DATA (YC1(I),I=3,18,3)/4*0.75/                                  000047
      DATA (YC1(I),I=4,19,3)/4*0.50/                                  000048
      DATA (YC1(I),I=5,20,3)/4*0.25/                                  000049
```

```
      DATA (NO(I),I=1,20)/0.4*3.5*0.5*3.1.3.1.2*5/                    J00050
      DATA (XNUM(I),I=1.5)/5*2.425/,(XNUM(I),I=6.10)/5*4.273/,(XNUM(I), J00051
     1I=11.15)/5*6.679/,(XNUM(I),I=16,20)/5*6.679/                     J00051
      P(1)=IDENT                                                       J00052
      P(2)=XIN                                                         J00053
      P(3)=UFS                                                         J00054
      P(4)=T(1)                                                        J00055
      P(5)=HYOIA                                                       J00056
      P(6)=REX                                                         J00057
      P(7)=REHYOIA                                                     J00058
      P(8)=REDELTA                                                     J00059
      P(9)=REDELTA1                                                    J00060
      P(10)=REDELTA2                                                   J00061
      P(11)=DELTA                                                      J00062
      P(12)=DELTA1                                                     J00063
      P(13)=DELTA2                                                     J00064
      P(14)=H12                                                        J00065
      P(15)=H32                                                        J00066
      P(16)=CI                                                         J00067
      P(17)=QI                                                         J00068
      P(18)=CW                                                         J00069
      P(19)=UI                                                         J00070
      P(20)=S                                                          J00071
      DO 11 I=1,NPLOT                                                  J00072
      NC=NCASE(I)                                                      J00073
      DO 11 M=1,NC                                                     J00074
      ISZ=IZ(I,M)                                                      J00075
      DO 10 J=1,ISZ                                                    J00076
      P1(J,1,M)=U(J,M)                                                 J00077
      Q1(J,1,M)=Y(J,M)                                                 J00078
      P1(J,2,M)=YDELTA(J,M)                                            J00079
      Q1(J,2,M)=URATIO(J,M)                                            J00080
      P1(J,3,M)=YDELTA1(J,M)                                           J00081
      Q1(J,3,M)=URATIO(J,M)                                            J00082
      P1(J,4,M)=YDELTA2(J,M)                                           J00083
      Q1(J,4,M)=URATIO(J,M)                                            J00084
      P1(J,5,M)=REYL(J,M)                                              J00085
      Q1(J,5,M)=URATIO(J,M)                                            J00086
      P1(J,6,M)=URATIO(J,M)                                            J00067
      Q1(J,6,M)=AETA(J,M)                                              J00068
      P1(J,7,M)=YPLUSL(J,M)                                            J00069
      Q1(J,7,M)=UPLUS(J,M)                                             J00030
      P1(J,8,M)=YDELTA(J,M)                                            J00091
      Q1(J,8,M)=OUUTAU(J,M)                                            J00092
      P1(J,9,1)=URATIO(J,1)                                            J00093
      P1(J,9,2)=URATIO7(J,1)                                           J00094
      Q1(J,9,M)=YDELTA(J,1)                                            J00095
      P1(J,10,M)=CCW(J,M)                                              J00096
      Q1(J,10,M)=VCC(J,M)                                              J00097
      P1(J,11,M)=CCW(J,M)                                              J00098
      Q1(J,11,M)=YLAM(J,M)                                             J00099
   10 CONTINUE                                                         J00100
      DO 20 I=1,NCH                                                    J00101
   20 YNUM(I)=YC1(I)                                                   J00102
      WRITE(1,70) NPLOT                                                J00103
      DO 60 I=1,NPLOT                                                  J00104
      NC=NCASE(I)                                                      J00105
      DO 30 M=1,NC                                                     J00106
   30 IPZ(M)=IZ(I,M)                                                   J00107
      XNTCHAR=NTCHAR(I)                                                J00108
      XC=.5*(XB1(I)-HGT*XNTCHAR)                                       J00109
      YC=1.625                                                         J00110
```

```
      WRITE(1,80) FACT(I),XGRID(I),YGRID(I)                      000111
      WRITE(1,70) XLABEL(I),NLABEL(I),XGRID(I),NLOGX(I),NLOG1(I), NLOG  000112
     1 Y(I),NLOG2(I),LGRIDX(I),LGRIDY(I)                         000112
      IF(KLABEL(I).EQ.0) GO TO 50                                000113
      DO 40 J=1,NCH                                              000114
      WRITE(1,90) XC1(J),YC1(J),XNUM(J),YNUM(J)                  000115
      WRITE(1,100) (CHARAC(JK,J),JK=1,9),NCHAR(J)                000116
   40 WRITE(1,110) P(J),HGT,NO(J)                                000117
   50 CONTINUE                                                   000118
      WRITE(1,120) NCASE(I),NXT(I),NYT(I),HGT                    000119
      WRITE(1,130) PX(I),PY(I),XAXL(I),YAXL(I),XUNIT(I),YUNIT(I), XOF(  000120
     1 I),YOE(I)                                                 000120
      WRITE(1,130) XENO(I),YENO(I),XB1(I),XB2(I),YB1(I),YB2(I),XC,YC    000121
      WRITE(1,100) (XAX(L,I),L=1,9),NXCHAR(I)                    000122
      WRITE(1,100) (YAX(L,I),L=1,9),NYCHAR(I)                    000123
      WRITE(1,100) (TITLE(L,I),L=1,9),NTCHAR(I)                  000124
      WRITE(1,150) (KSYM(I,M),M=1,MC)                            000125
      WRITE(1,150) (IZ(I,L),L=1,MC)                              000126
      WRITE(1,150) (IPZ(L),L=1,MC)                               000127
      WRITE(1,160) (IFMT(L),L=1,10)                              000128
      DO 60 M=1,MC                                               000129
      ISZ=IZ(I,M)                                                000130
      WRITE(1,IFMT) (P1(J,I,M),J=1,ISZ)                          000131
      WRITE(1,IFMT) (Q1(J,I,M),J=1,ISZ)                          000132
      IF(KSYM(I,M).EQ.0) GO TO 60                                000133
      WRITE(1,140) NSYM(I,M),KSIN(I,M                            000134
   60 CONTINUE                                                   000135
   70 FORMAT(9I5)                                                000136
   80 FORMAT(3F10.5)                                             000137
   90 FORMAT(4F10.5)                                             000138
  100 FORMAT(9A4,I3)                                             000139
  110 FORMAT(F12.5,F10.5,I5)                                     000140
  120 FORMAT(3I3,F5.3)                                           000141
  130 FORMAT(8F10.3)                                             000142
  140 FORMAT(A5,I3,F5.3)                                         000143
  150 FORMAT(16I5)                                               000144
  160 FORMAT(10A8)                                               000145
      RETURN                                                     000146
      END                                                        000147
```

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```
                   INPUT LISTING

        SUBROUTINE INTEGRAL (NDATA,XDATA,YDATA,SUM)              1
        DOUBLE PRECISION A,B                                     2
        DIMENSION A(4,5),B(3,1),AREA(29,29),AV(29)              3
        DIMENSION XDATA(31),YDATA(31)                           4
        K=0                                                     5
        K1=0                                                    6
        NDATA1=NDATA+1                                          7
        NDATA2=NDATA-2                                          8
        DO 10 I=1,3                                             9
        K1=K1+1                                                10
        A(I,1)=XDATA(K1)**2                                    11
        A(I,2)=XDATA(K1)                                       12
        A(I,3)=1.                                              13
10      A(I,4)=YDATA(K1)                                       14
        K1=K1-2                                                15
        CALL GAUSJ(A,3,4)                                      16
        K=K+1                                                  17
        DO 20 I=1,3                                            18
20      B(I,1)=A(I,4)                                          19
        KP1=K                                                  20
        DO 30 I=1,2                                            21
        KP2=KP1+1                                              22
        AREA(KP1,K1=B(1,1)*(XDATA(KP2)**3-XDATA(KP1)**3)/3.+B(2,1)*(XDATA(   23
       $KP2)**2-XDATA(KP1)**2)/2.+B(3,1)*(XDATA(KP2)-XDATA(KP1))   23
30      KP1=KP1+1                                              24
        IF(K.LT.NDATA2) GO TO 1                                25
        DO 50 I=2,K                                            26
50      AV(I)=(AREA(I,I-1)+AREA(I,I))*.5                       27
        AV(1)=AREA(1,1)                                        28
        AV(NDATA1)=AREA(NDATA1,NDATA2)                         29
        SUM=0.                                                 30
        DO 60 I=1,NDATA1                                       31
60      SUM=SUM+AV(I)                                          32
        RETURN                                                 33
        END                                                   34
    END CARD DETECTED
```

```
                RESEQUENCED LISTING

      SUBROUTINE INTEGRAL (NDATA,XDATA,YDATA,SUM)          000001
      DOUBLE PRECISION A,B                                 000002
      DIMENSION A(4,3),B(3,1),AREA(29,29),AV(29)           000003
      DIMENSION XDATA(31),YDATA(31)                        000004
      K=0                                                  000005
      K1=0                                                 000006
      NDATA1=NDATA-1                                       000007
      NDATA2=NDATA-2                                       000008
   10 DO 20 I=1,3                                          000009
      K1=K1+1                                              000010
      A(I,1)=XDATA(K1)**2                                  000011
      A(I,2)=XDATA(K1)                                     000012
      A(I,3)=1.                                            000013
   20 A(I,4)=YDATA(K1)                                     000014
      K1=K1-2                                              000015
      CALL GAUSJ(A,3,4)                                    000016
      K=K+1                                                000017
      DO 30 I=1,3                                          000018
   30 B(I,1)=A(I,4)                                        000019
      KP1=K                                                000020
      DO 40 I=1,2                                          000021
      KP2=KP1+1                                            000022
      AREA(KP1,K)=B(1,1)*(XDATA(KP2)**3-XDATA(KP1)**3)/3.+B(2,1)*(XDAT  000023
     1 A(KP2)**2-XDATA(KP1)**2)/2.+B(3,1)*(XDATA(KP2)-XDATA(KP1))       000023
   40 KP1=KP1+1                                            000024
      IF(K.LT.NDATA2) GO TO 10                             000025
      DO 50 I=2,K                                          000026
   50 AV(I)=(AREA(I,I-1)+AREA(I,I))*.5                     000027
      AV(1)=AREA(1,1)                                      000028
      AV(NDATA1)=AREA(NDATA1,NDATA2)                       000029
      SUM=0.                                               000030
      DO 60 I=1,NDATA1                                     000031
   60 SUM=SUM+AV(I)                                        000032
      RETURN                                               000033
      END                                                 000034
```

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```
      SUBROUTINE POLY(B,X,Y,NK)                                    101031
      DOUBLE PRECISION A                                           000002
      DIMENSION A(4,5),X(31),Y(31)                                 101003
      K=0                                                          300004
      DO 10 I=1,4                                                  103005
      K=K+1                                                        300006
      A(I,1)=X(K)**3                                               300007
      A(I,2)=X(K)**2                                               300008
      A(I,3)=X(K)                                                  101009
      A(I,4)=1.                                                    300010
   10 A(I,5)=Y(K)                                                  103011
      CALL GAUSJ(A,4,5)                                            300012
      GO TO (20,30),NK                                             000013
   20 X2=X(2)/2.                                                   303014
      P=A(1,5)*X2**3+A(2,5)*X2**2+A(3,5)*X2+A(4,5)                 101015
      B=12.*(P-Y(1))/(X2-Y(1))                                     300016
      GO TO 40                                                     103017
   30 B=A(1,5)*.5**3+A(2,5)*.5**2+A(3,5)*.5+A(4,5)                 303018
   40 RETURN                                                       103019
      END                                                          300020
```

```
                        RESEQUENCED LISTING

      SUBROUTINE GAUS(H,N,NP1)                                   100001
      DOUBLE PRECISION H                                         000002
      DIMENSION H(...3)                                          000003
      DO 10 K=1,N                                                000004
      KK=K+1                                                     000005
        DO 10 J=KK,NP1                                           000006
        H(K,J)=H(K,J)/H(K,K)                                     000007
        DO 10 I=1,N                                              000008
        IF(K.EQ.I) GO TO 10                                      000009
        H(I,J)=H(I,J)-H(I,K)*H(K,J)                              000010
  10  CONTINUE                                                   000011
      RETURN                                                     000012
      END                                                        000013
```

```
                        RESEQUENCED LISTING

          PROGRAM PLOTIM                                              000001
          OVERLAY(99)                                                 000002
          COMMON/ICVER/IENO,IPASS                                     000003
          COMMON/G/PX,PY                                              000004
          COMMON/SYM/U(50,2),V(50,2)                                  000005
          COMMON/SIM/XLIM1(12),XLIM2(12),YLIM1(12),YLIM2(12),KML      000006
          COMMON/LOG/NLOGX,YAXL,YAX,NXCHAR,NLOG2,LGRIDX,LGRIDY        000007
          DIMENSION ISZ(16),IPZ(16),KSYM(16)                         000008
          DIMENSION XAX(9),YAX(9),TITLE(9),IFMT(10)                   000009
      C...NPLOT IS THE NUMBER OF SEPARATE PLOTS                       000010
      C...FACT IS A FACTOR BY WHICH THE ENTIRE PLOT IS TO BE MULTIPLIED 000011
      C...XGRID,YGRID ARE THE GRID WIDTHS                             000012
      C...KLABEL INDICATES WHETHER SPECIFIC VALUES OF VARIABLES ARE TO LABELE 000013
      C...ON THE PLOT                                                 000014
      C...NLABEL IS THE NUMBER OF LABELED VARIABLES                   000015
      C....IF KGRID = 1, A GRID WILL BE PLOTTED                       000016
      C...NLOGX IS THE HIGHEST POWER OF THE LOG X-AXIS                000017
      C...NLOG1 IS THE LOWEST POWER OF THE LOG X-AXIS                 000018
      C...NLOGY IS THE HIGHEST POWER OF THE LOG Y-AXIS                000019
      C...NLOG2 IS THE LOWEST POWER OF THE LOG Y-AXIS                 000020
      C...LGRIDX,LGRIDY INDICATE WHETHER LOG GRIDS ARE TO BE DRAWN    000021
      C...MCASE=NUMBER OF FIGURES ON THE PLOT                         000022
      C...NXT,NYT ARE NUMBER OF TICK MARKS                            000023
      C...HGT=HEIGHT OF THE CHARACTERS TO BE PRINTED IN TITLE         000024
      C...PX,PY ARE BEGINNING COORDINATES OF THE AXES                 000025
      C....XAXL,YAXL ARE THE AXIS LENGTHS IN INCHES                   000026
      C....XUNIT,YUNIT ARE THE SCALED LENGTHS OF THE AXES             000027
      C...XOF,YOF ARE THE OFFSETS                                     000028
      C...XENO,YENO ARE THE COORDINATES OF ENPLT                      000029
      C...X81,X82,Y91,Y82 ARE THE BORDER LENGTHS IN INCHES            000030
      C....XG,YG ARE THE BEGINNING COORDINATES OF THE TITLE           000031
      C....XAX IS THE TITLE OF THE X-AXIS                             000032
      C...NXCHAR IS THE NUMBER OF CHARACTERS IN XAX                   000033
      C...YAX IS THE TITLE OF THE Y-AXIS                              000034
      C...NYCHAR IS THE NUMBER OF CHARACTERS IN YAX                   000035
      C....TITLE IS THE TITLE OF THE PLOT                             000036
      C...NTCHAR IS THE NUMBER OF CHARACTERS IN THE TITLE             000037
      C...KSYM=1 INDICATES THAT SYMBOLS AND LINES ARE TO BE PLOTTED   000038
      C...KSYM=2 INDICATES THAT ONLY SYMBOLS ARE TO BE PLOTTED        000039
      C....ISZ IS THE NUMBER OF POINTS TO BE READ                     0000-0
      C....IPZ IS THE NUMBER OF POINTS TO BE PLOTTED                  0000-1
      C...IFMT IS THE FORMAT OF THE X,Y DATA TO BE READ               000042
          CALL PLOTS(0,0,10)                                         1000-1
          READ(1,20) NPLOT                                           0000-4
          DO 110 KLM=1,NPLOT                                         1000-5
          KML=KLM                                                    000046
          READ(1,10) FACT,XGRID,YGRID                                1000-7
       10 FORMAT(3F10.5)                                             0000-8
          CALL FACTOR(FACT)                                          1000-9
          READ(1,20) KLABEL,NLABEL,KGRID,NLOGX,NLOG1,NLOGY,NLOG2,LGRIDX, L 000050
        1 GRIDY                                                      000050
       20 FORMAT(9I3)                                                000051
          IF(KLABEL.EQ.1) CALL LABEL(NLABEL)                         000052
          READ(1,30) MCASE,NXT,NYT,HGT                               000053
       30 FORMAT(3I3,F5.3)                                           000054
          READ(1,40) PX,PY,XAXL,YAXL,XUNIT,YUNIT,XOF,YOF             000055
          READ(1,40) XENO,YENO,X81,X82,Y91,Y92,XG,YG                 000056
       40 FORMAT(8F10.5)                                             000057
          READ(1,50) (XAX(L),L=1,9),NXCHAR                           000058
```

```
        READ(1,50) (YAX(L),L=1,9),NYCHAR              000059
        READ(1,50) (TITLE(L),L=1,3),NTCHAR            000060
   50 FORMAT(9A8,I4)                                  000061
        J=NCASE                                       000062
        READ(1,60) (KSYM(L),L=1,M)                    000063
        READ(1,60) (ISZ(L),L=1,M)                     000064
        READ(1,60) (IPZ(L),L=1,M)                     000065
   60 FORMAT(16I3)                                    000066
        READ(1,70) (IFMT(L),L=1,10)                   000067
   70 FORMAT(10A8)                                    000068
        XLIM1(KLM)=XOF                                000069
        XLIM2(KLM)=XUNIT+XOF+.0001                    000070
        YLIM1(KLM)=YOF                                000071
        YLIM2(KLM)=YUNIT+YOF+.0001                    000072
        CALL PLOT(XB2,YB1,3)                          000073
        CALL PLOT(XB1,YB1,2)                          000074
        CALL PLOT(XB1,YB2,2)                          000075
        CALL PLOT(XB2,YB2,2)                          000076
        CALL PLOT(XB2,YB1,2)                          000077
        IF(KGRID.EQ.1) CALL GRID(XAXL,YAXL,XGRID,YGRID)  000078
        CALL CHAR(XC,YC,0.,HGT,TITLE,NTCHAR)          000079
        IF(NLOGX.NE.0 .OR. NLOGY.NE.0) CALL LOGAXIS(NLOGX,PX,PY,XAXL, XA  000080
    1  X,NXCHAR,NLOG1)                                000080
        XSF=XAXL/XUNIT                                000081
        YSF=YAXL/YUNIT                                000082
        CALL PLOT(PX,PY,-3)                           000083
        CALL OFFST(XOF,YOF,1)                         000084
        CALL SCALE(XSF,YSF,1)                         000085
        IF(NLOGX.NE.0) GO TO 80                       000086
        CALL AXIS(0.,0.,XAX,NXCHAR,XAXL,NXT,1.0.)     000087
   80 IF(NLOGY.NE.0) GO TO 90                         000088
        CALL AXIS(0.,0.,YAX,NYCHAR,YAXL,NYT,0.,1.16153/2.)  000089
   90    DO 100 J=1,M                                 000090
        N=ISZ(J)                                      000091
        IZ=IPZ(J)                                     000092
        READ(1,IFMT) (U(I,J),I=1,N)                   000093
        READ(1,IFMT) (V(I,J),I=1,N)                   000094
        KSYMB=KSYM(J)                                 000095
        IF(KSYMB.NE.0) CALL SYMBOM(IZ,J)              000096
        IF(KSYMB.EQ.2) GO TO 100                      000097
        CALL PLOT(U(1,J),V(1,J),3)                    000098
        DO 100 I=1,IZ                                 000099
          IF(U(I,J).LT.XLIM1(KLM) .OR. U(I,J).GT.XLIM2(KLM)) GO TO  000100
    1     100                                         000100
          IF(V(I,J).LT.YLIM1(KLM) .OR. V(I,J).GT.YLIM2(KLM)) GO TO  000101
    1     100                                         000101
          CALL PLOT(U(I,J),V(I,J),2)                  000102
  100    CONTINUE                                     000103
        CALL RESET                                    000104
  110 CALL PLOT(XENO,YENO,-3)                         000105
        CALL RESET                                    000106
        CALL ENPLT(XENO,YENO)                         000107
        END                                           000108
```

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                    RESEQUENCED LISTING

      SUBROUTINE GRID(XAXL,YAXL,XGRID,YGRID)            100001
      COMMON/G/PX,PY                                    000002
      IF(XGRID.LT.1.E-5) GO TO 10                       000003
      LX=XAXL/(2.*XGRID)+.9999                          000004
   10 IF(YGRID.LT.1.E-5) GO TO 20                       000005
      LY=YAXL/(2.*YGRID)+.9999                          000006
   20 XA=PX+XAXL                                        000007
      YA=PY+YAXL                                        000008
      IF(XGRID.LT.1.E-5) GO TO 30                       000009
      CALL PLOT(XA,PY,3)                                000010
      CALL PLOT(XA,YA,2)                                000011
   30 IF(YGRID.LT.1.E-5) GO TO 40                       000012
      IF(XGRID.LT.1.E-5) CALL PLOT(XA,YA,3)             000013
      CALL PLOT(PX,YA,2)                                000014
      CALL PLOT(PX,PY,3)                                000015
   40 P=PX                                              100016
      IF(XGRID.LT.1.E-5) GO TO 70                       000017
      DO 50 I=1,LX                                      000018
      P=P+XGRID                                         100019
      CALL PLOT(P,PY,3)                                 000020
      CALL PLOT(P,YA,2)                                 100021
      IF(I.EQ.LX) GO TO 60                              000022
      P=P+XGRID                                         100023
      CALL PLOT(P,YA,3)                                 000024
      CALL PLOT(P,PY,2)                                 100025
   50 CONTINUE                                          000026
   60 CALL PLOT(PX,PY,3)                                000027
   70 Q=PY                                              000028
      IF(YGRID.LT.1.E-5) GO TO 90                       000029
      DO 80 I=1,LY                                      000030
      Q=Q+YGRID                                         100031
      CALL PLOT(PX,Q,3)                                 000032
      CALL PLOT(XA,Q,2)                                 100033
      IF(I.EQ.LY) RETURN                                000034
      Q=Q+YGRID                                         100035
      CALL PLOT(XA,Q,3)                                 000036
      CALL PLOT(PX,Q,2)                                 100037
   80 CONTINUE                                          000038
   90 CONTINUE                                          120039
      RETURN                                            000040
      END                                               100041
```

```
                    RESEQUENCED LISTING

    SUBROUTINE LABEL(N)                                            000001
C...XC,YC ARE THE BEGINNING COORDINATES OF THE VARIABLES           000002
C...XNUM,YNUM ARE THE BEGINNING COORDINATES OF THE NUMBERS         000003
C...CHARAC IS THE ALPHABETIC CHARACTERS                            000004
C...NCHAR IS THE NUMBER OF CHARACTERS IN CHARAC                    000005
C...VALUE IS THE VALUE OF THE VARIABLE                             000006
C...HGT IS THE HEIGHT OF THE CHARACTERS                            000007
C...NO IS THE NUMBER OF DIGITS TO THE RIGHT OF THE DECIMAL POINT   000008
      DIMENSION CHARAC(9)                                          000009
      DO 40 I=1,N                                                  000010
      READ(1,10) XC,YC,XNUM,YNUM                                   000011
   10 FORMAT(4F10.5)                                               000012
      READ(1,20) (CHARAC(J),J=1,9),NCHAR                           000013
   20 FORMAT(9A8,I8)                                               000014
      READ(1,30) VALUE,HGT,NO                                      000015
   30 FORMAT(F12.5,F10.5,I3)                                       000016
      CALL CHAR(XC,YC,0.,HGT,CHARAC,NCHAR)                         000017
   40 CALL NUMPLT(XNUM,YNUM,0.,HGT,VALUE,NO)                       000018
      RETURN                                                       000019
      END                                                          000020
```

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```
                    RESEQUENCED LISTING

        SUBROUTINE SYMBOM(IZ,J)                                   100001
C...NSYM IS THE SYMBOL                                            000002
C...KSINT IS THE INTERVAL AT WHICH SYMBOLS ARE TO BE PLOTTED      100003
C...H IS THE HEIGHT OF THE SYMBOLS                                300004
        COMMON/SYM/U(50,2),V(50,2)                                300005
        COMMON/SIM/XLIM1(12),XLIM2(12),YLIM1(12),YLIM2(12),KML    300006
        READ(1,10) NSYM,KSINT,H                                   100007
   10 FORMAT(A5,I5,F5.3)                                          300008
        DO 20 I=1,IZ,KSINT                                        100003
        IF(U(I,J).LT.XLIM1(KML) .OR. U(I,J).GT.XLIM2(KML)) GO TO 20  000010
        IF(V(I,J).LT.YLIM1(KML) .OR. V(I,J).GT.YLIM2(KML)) GO TO 20  130011
        CALL VECTOR(U(I,J),V(I,J),1,1,-1,NSYM)                    000012
   20 CONTINUE                                                    100013
        RETURN                                                    300014
        END                                                       100015
```

```
                    RESEQUENCED LISTING

        SUBROUTINE LOGAXIS(NLOGX,PX,PY,XAXL,XAX,NXCHAR,NLOG1)      000001
        COMMON/LOG/NLOGY,YAXL,YAX,NYCHAR,NLOG2,LGRIDX,LGRIDY        000002
        DIMENSION YAX(9),YLOG(10)                                   000003
        DIMENSION XLOG(10),IX(1),XAX(9)                            000004
C...PRINT X-AXIS POWERS OF 10                                       000005
        PXAX=PX                                                     000006
        PYAX=PY                                                     000007
        IF(NLOGX.EQ.0) GO TO 70                                     000008
        XAXP=PX+XAXL                                                000009
        CALL PLOT(PX,PY,3)                                          000010
        CALL PLOT(XAXP,PY,2)                                        000011
        PXX=PX                                                      000012
        PX3=PX                                                      000013
        PX=PX-.1                                                    000014
        PY=PY-.25                                                   000015
        X1=XAXL+1.E-5                                               000016
        NLOG=NLOG1+2**2                                             000017
        XNLOGX=NLOGX-NLOG1                                          000018
        X2=X1/XNLOGX                                                000019
        PX2=PX+.25                                                  000020
        PY2=PY+.07                                                  000021
        IF(NLOG1.LT.10) NL=1                                        000022
        IF(NLOG1.GE.10) NL=2                                        000023
        CALL CHAR(PX,PY,0.,.10,2H10,2)                             000024
        CALL CHAR(PX2,PY2,0.,.07,NLOG,NL)                          000025
        NLOGX1=NLOG1+1                                              000026
        DO 10 I=NLOGX1,NLOGX                                        000027
        IF(I.LT.10) NL=1                                            000028
        IF(I.GE.10) NL=2                                            000029
        IX(1)=I*2**2                                                000030
        PX=PX+X2                                                    000031
        PX2=PX+.25                                                  000032
        CALL CHAR(PX,PY,0.,.10,2H10,2)                             000033
   10   CALL CHAR(PX2,PY2,0.,.07,IX,NL)                            000034
C...DRAW X-AXIS TICK MARKS UNLESS GRID IS TO BE DRAWN               000035
        PY=PY+.25                                                   000036
        IF(LGRIDX.EQ.1) GO TO 60                                    000037
        PY1=PY-.075                                                 000038
        PY3=PY-.1                                                   000039
        DO 50 I=NLOGX1,NLOGX                                        000040
        DO 40 J=1,10                                                000041
        XJ=J+1                                                      000042
        CALL PLOT(PXX,PY,3)                                         000043
        IF(J.NE.1 .AND. J.NE.10) GO TO 20                           000044
        CALL PLOT(PXX,PY3,2)                                        000045
        GO TO 30                                                    000046
   20   CALL PLOT(PXX,PY1,2)                                        000047
   30   XLOG(J)=ALOG10(XJ)                                          000048
        IF(J.EQ.10) GO TO 50                                        000049
   40   PXX=PX3+XLOG(J)*X2                                          000050
   50   PX3=PXX                                                     000051
C...LABEL X-AXIS                                                    000052
   60   PX=PXAX                                                     000053
        PY=PYAX                                                     000054
        NXCHAR=IABS(NXCHAR)                                         000055
        XNX=NXCHAR                                                  000056
        PX5=.5*(XAXL-(.14*XNX))+PX                                  000057
        PY5=PY-.45                                                  000058
        CALL CHAR(PX5,PY5,0.,.14,XAX,NXCHAR)                       000059
```

```
    70 IF(NLOGY.EQ.0) GO TO 140                                    000060
       YAXP=PY+YAXL                                                000061
       CALL PLOT(PX,PY,3)                                          000062
       CALL PLOT(PX,YAXP,2)                                        000063
C....PRINT Y-AXIS POWERS OF 10                                     000064
       PYY=PY                                                      000065
       PY3=PY                                                      000066
       PX=PXR+.0                                                   000067
       Y1=YAXL+1.E-3                                               000068
       NLOG=NLOG2=20+42                                            000069
       YNLOGY=NLOGY-NLOG2                                          000070
       Y2=Y1+YNLOGY                                                000071
       PX2=PX+.22                                                  000072
       PY2=PY+.07                                                  000073
       IF(NLOG2.LT.10) NL=1                                        000074
       IF(NLOG2.GE.10) NL=2                                        000075
       CALL CHAR(PX,PY,0.,.10,2H10,2)                             000076
       CALL CHAR(PX2,PY2,0.,.07,NLOG,NL)                          000077
       NLOGY1=NLOG2+1                                              000078
       DO 80 I=NLOGY1,NLOGY                                        000079
       IF(I.LT.10) NL=1                                            000080
       IF(I.GE.10) NL=2                                            000081
       IX(1)=I*20+42                                               000082
       PY=PY+Y2                                                    000083
       PY2=PY+.07                                                  000084
       CALL CHAR(PX,PY,0.,.10,2H10,2)                             000085
    80 CALL CHAR(PX2,PY2,0.,.07,IX,NL)                            000086
C....DRAW Y-AXIS TICK MARKS UNLESS GRID IS TO BE DRAWN             000087
       PX=PX+.0                                                    000088
       IF(LGRIDY.EQ.1) GO TO 130                                  000089
       PX1=PX-.375                                                 000090
       PX3=PX-.1                                                   000091
       DO 120 I=NLOGY1,NLOGY                                      000092
       DO 110 J=1,10                                               000093
       YJ=J+1                                                      000094
       CALL PLOT(PX,PYY,3)                                        000095
       IF(J.NE.1 .AND. J.NE.10) GO TO 90                          000096
       CALL PLOT(PX3,PYY,2)                                       000097
       GO TO 100                                                  000098
    90 CALL PLOT(PX1,PYY,2)                                       000099
   100 YLOG(J)=ALOG10(YJ)                                         000100
       IF(J.EQ.10) GO TO 120                                      000101
   110 PYY=PY3+YLOG(J)*Y2                                         000102
   120 PY3=PYY                                                    000103
C....LABEL Y-AXIS                                                  000104
   130 PX=PYAX                                                    000105
       PY=PYAX                                                    000106
       NYCHAR=IABS(NYCHAR)                                        000107
       YNY=NYCHAR                                                 000108
       PX3=PX-.5                                                  000109
       PY5=.5*(YAXL-(.14*YNY))+PY                                000110
       CALL CHAR(PX3,PY5,1.14153/2.,.14,YAX,NYCHAR)             000111
   140 IF(LGRIDX.NE.1) GO TO 170                                  000112
C....DRAW X-GRID                                                   000113
       PYAX=PY+YAXL                                               000114
       PX5=PX                                                     000115
       PY3=PY                                                     000116
       PX7=PX                                                     000117
       PY7=PY                                                     000118
       CALL PLOT(PX6,PY6,3)                                       000119
       NXLOG=NLOGX-NLOG1                                          000120
       DO 150 I=1,NXLOG                                           000121
```

```
      XJ=1.                                           000122
      DO 150 J=1,3                                     000123
      XJ=XJ+1.                                         000124
      PX6=PX7+X2*ALOG10(XJ)                            000125
      CALL PLOT(PX6,PY6,3)                             000126
      CALL PLOT(PX6,PYAX,2)                            000127
      XJ=XJ+1.                                         000128
      IF(J.EQ.3) GO TO 150                             000129
      PX6=PX7+X2*ALOG10(XJ)                            000130
      CALL PLOT(PX6,PYAX,3)                            000131
      CALL PLOT(PX6,PY6,2)                             000132
150   CONTINUE                                         000133
160   PX7=PX7+X2                                       000134
170   IF(LGRIDY.NE.1) RETURN                           000135
C...DRAW Y-GRID                                        000136
      PXAX=PX+XAXL                                     000137
      PX6=PX                                           000138
      PY6=PY                                           000139
      CALL PLOT(PX6,PY6,3)                             000140
      NYLOG=NLOGY-NLOGZ                                000141
      DO 190 I=1,NYLOG                                 000142
      YJ=1.                                            000143
      DO 180 J=1,3                                     000144
      YJ=YJ+1.                                         000145
      PY6=PY7+Y2*ALOG10(YJ)                            000146
      CALL PLOT(PX6,PY6,3)                             000147
      CALL PLOT(PXAX,PY6,2)                            000148
      YJ=YJ+1.                                         000149
      IF(J.EQ.3) GO TO 180                             000150
      PY6=PY7+Y2*ALOG10(YJ)                            000151
      CALL PLOT(PXAX,PY6,3)                            000152
      CALL PLOT(PX6,PY6,2)                             000153
180   CONTINUE                                         000154
190   PY7=PY7+Y2                                       000155
      RETURN                                           000156
      END                                              000157
```

```
TEST NUMBER  51206
       X=    1C...0 IN.  25.400 CM.
     UFS =    2.62 FT/SEC    0.797 M/SEC
    UAVG=    2.206 FT/SEC   0.672 M/SEC
   DELTA =    0.140 IN.     0.355 CM.
  DELTA1 =    0.032 IN.     0.083 CM.
  DELTA2 =    0.016 IN.     0.040 CM.
  DELTA3 =    0.026 IN.     0.066 CM.
   HYDIA =    0.071 FT.     0.022 M.
     H12 =    2.005
     H32 =    1.670
     REX =  185584.
  REHALF =    4742.
 REHYDIA =   12927.9
 REDELTA =   30295.
REDELTA1 =    7040.
REDELTA2 =    3377.
  T WALL =  100.000
 H2, TEMP =   60.000
      MU =  .00001207
      CF =  0.005271
  UTAUFH =    0.1343
 CONC IN =  800.000 PPM
POLYFLOW =   25.00 CC/MIN
   QFLOW =    8.000 GPM
    QVOL =    0.010 FT3/SEC
POLY UIN =   0.0363 FT/SEC
    SLOT =    0.0200 IN
   CWALL =    0.000 PPM
   CW/CI =   0.0000 PPM
  LAMBDA =   0.0118
```

TEST NUMBER 512-6

Y(IN.)	Y(CM.)	U(E(/S)	U(M/S)	U/UES	Y/DELIA1	Y/DELTA2	REX	LOG REY	Y/DELTA	AETA	X(IN.)	Z(IN.)
0.010	0.024	0.912	0.270	0.349	0.296	0.816	173.361	2.239	0.069	0.428	16.000	1.108
0.023	0.051	1.253	0.381	0.478	0.622	1.296	364.781	2.562	0.144	0.858	16.030	1.136
0.040	0.112	1.746	0.532	0.667	1.231	2.567	722.330	2.859	0.286	1.700	16.000	1.136
0.060	0.152	2.143	0.653	0.819	1.847	3.856	1283.567	3.035	0.429	2.550	16.000	1.136
0.031	0.224	2.395	0.734	0.916	2.472	5.153	1658.093	3.161	0.574	3.412	16.003	1.136
0.105	0.254	2.531	0.771	0.968	3.178	6.417	1835.846	3.257	0.715	4.257	16.000	1.136
0.148	0.310	2.581	0.787	0.987	3.639	7.585	2134.530	3.329	0.845	5.323	16.003	1.136
0.148	0.355	2.607	0.795	0.997	4.303	8.971	2524.570	3.402	1.003	5.941	16.907	1.136
0.170	0.431	2.613	0.796	0.999	5.227	10.896	3366.324	3.487	1.215	7.216	16.000	1.136
0.210	0.509	2.614	0.797	0.999	6.163	12.847	3615.300	3.558	1.432	8.508	16.000	1.136
0.230	0.545	2.615	0.252	1.003	7.086	14.772	4157.053	3.619	1.647	9.782	16.026	1.136

Y(IN.)	Y/DELTA	U/UFS	(1/7)	UPLUS	VPLUS	DEL UTRU	V/HALFH	LOG(YPLUS)
0.013	0.069	0.349	0.642	6.791	8.900	12.680	0.037	0.949
0.029	0.144	0.478	0.759	9.132	14.728	19.171	0.977	1.272
0.043	0.206	0.667	0.836	13.000	37.106	6.678	0.152	1.569
0.068	0.429	0.819	0.886	15.956	55.626	3.522	0.228	1.745
0.093	0.574	0.916	0.925	17.431	74.447	1.646	0.336	1.072
0.105	0.715	0.968	0.953	18.845	92.711	0.633	0.361	1.967
0.118	0.845	0.987	0.976	19.218	139.584	0.261	0.450	2.040
0.148	0.846	0.997	0.819	19.511	129.618	0.067	0.532	2.113
0.170	1.215	0.999	1.028	19.456	157.623	0.022	0.647	2.197
0.231	1.432	0.999	1.053	19.463	185.607	3.015	0.762	2.269
0.230	1.647	1.003	1.074	19.473	213.420	2.030	0.677	2.329

YC(IN.)	COM(IPH)	C/CM	Y/LAMBDA	XC
0.000	3.610	2.006	0.010	0.020

APPENDIX C

COMPUTER PROGRAM DATAPAC OUTPUT CURVES

VELOCITY PROFILE

Y DISTANCE ABOVE FLAT PLATE (INCHES)
U VELOCITY (FT/SEC)

TEST NO.	51206.	REX=	180504.	DELTA=	0.140 IN	CIN=	800.0 PPM
X=	10.000IN	REDYDIA=	12928.	DELTA1=	0.032 IN	QIN=	20.000CC/M
UFS=	2.616F/S	REDELTA=	30295.	DELTA2=	0.016 IN	CWALL=	0.0 PPM
WATER TEMP=	60.000 F	REDELTA1=	7040.	H12=	2.085	UIN=	0.03832F/S
HYDIA=	0.071 FT	REDELTA2=	3377.	H32=	1.670	SLOT=	0.02000 IN

NORMALIZED VELOCITY PROFILE

Y/DELTA DISTANCE ABOVE FLAT PLATE NORMALIZED WITH DELTA

TEST NO.	51206.	REX=	180584.	DELTA=	0.140 IN	CIN=	800.0 PPM
X=	10.000IN	REHYDIA=	12920.	DELTA1=	0.032 IN	OIN=	20.060CC/M
UFS=	2.616F/S	REDELTA=	30295.	DELTA2=	0.016 IN	CWALL=	0.0 PPM
WATER TEMP= 60.000 F		REDELTA1=	7040.	H12=	2.006	UIN=	0.03032F/S
HYDIA=	0.071 FT	REDELTA2=	3377.	H32=	1.670	SLOT=	0.02000 IN

Y/DELTA1 DISTANCE ABOVE FLAT PLATE NORMALIZED WITH DELTA1

NORMALIZED VELOCITY PROFILE

TEST NO.	51206.	REX=	1801484.	DELTA=	0.140 IN	CIN=	800.0 PPM
Y=	10.000IN	REHYDIA=	12928.	DELTA1=	0.032 IN	UIN=	20.000CC/M
UFS:	2.616F/S	REDITTH=	30295.	DELTA2=	0.016 IN	CWALL=	0.0 PPM
WATER TEMP=	60.000 F	REDELTA1=	7040.	H12=	2.085	UIN=	0.03832F/S
HYDIA=	0.071 FI	REDELTA2=	3377.	H32=	1.670	SLOT=	0.02000 IN

Y/DELTA2 DISTANCE ABOVE FLAT PLATE NORMALIZED WITH DELTA2

/UFS VELOCITY RATIO

NORMALIZED VELOCITY PROFILE

TEST NO.	51206.	REX=	180584.	DELTA=	0.140 N	CIN=	80.0 PPM
X=	10.000IN	REHYDIA=	12928.	DELTA1=	0.032 IN	OIN=	20.000CC/M
UFS=	2.616F/S	REDELTA=	30295.	DELTA2=	0.016 IN	CWAII=	0.0 PPM
WATER TEMP=	60.000 F	REDELTA1=	7040.	H12=	2.085	UIN=	0.03832F/S
HYDIA=	0.671 FT	REDELTA2=	3377.	H32=	1.670	SI0I=	0.02000 IN

NORMALIZED VELOCITY PROFILE

TEST NO.	51206.	REFX=	180584.	DELTA=	0.140 IN	UIN=	800.0 PPM
X=	10.000IN	REHYDIA=	12928.	DELTA1=	0.032 IN	OIN=	20.000CC/M
UFS=	2.616/S	REDELTA=	30295.	DELTA2=	0.016 IN	CHALL=	0.0 PPM
WATER TEMP= 60.000 F		REDELTA1=	7040.	H12=	2.085	UIN=	0.63832F/S
HYDIA=	0.071 FT	REDELTA2=	3377.	H32=	1.670	SLOT=	0.02000 IN

NORMALIZED VELOCITY PROFILES

TEST NO.	51206.	REX=	180584.	DELTA=	0.140 IN	CIN=	800.0 PPM
t=	10.000IN	REHYDIA=	12928.	DELTA1=	0.032 IN	Q'N=	20.000CC/M
UFS=	2.616F/S	REDELTA=	30295	DELTA2=	0.016 IN	CWALL=	0.0 PPM
WATER TEMP=	60.000 F	REDELTA1=	7040.	H12=	2.085	UIN=	0.03832F/S
HYDIA=	0.071 FT	REDELTA2=	3377.	H32=	1.670	SLOT=	0.02000 IN

WAKE DEFICIT

Y/DELTA

(UFS-U)/UTAU

TEST NO.	51206.	REX=	180584.	DELTA=	0.140 IN	CIN=	860.0 PPM
X=	10.000IN	REHYDIA=	12928.	DELTA1=	0.032 IN	QIN=	20.000CC/M
UFS=	2.616F/S	REDELTA=	36295.	DELTA2=	0.016 IN	CWALL=	0.0 PPM
WATER TEMP=	60.000 F	REDELTA1=	7040.	H12=	2.080	UIN=	0.03832F/S
HYDIA=	0.071 FT	REDELTA2=	3377.	H32=	1.670	SIOI=	0.0200 IN

LAW OF THE WALL

YPLUS — UPLUS

TEST NO. 51206.	REX= 186584.	DELTA= 0.140 IN	CIN= 810.0 PPM
4= 10.000IN	REHYDIA= 12928.	DELTA1= 0.032 IN	QIN= 20.000CC/H
UFS= 2.616F/S	REDELTA= 30295.	DELTA2= 0.016 IN	CWALL= 0.0 PPM
WATER TEMP= 60.000 F	REDELTA1= 7040.	H12= 2.085	UIN= 0.038326F/S
HYDIA= 0.071 FT	REDELTA2= 3377.	H32= 1.670	SLOT= 0.02000 N

APPENDIX D

HYDRODYNAMIC PARAMETER TABULATIONS

TABLE 20

BOUNDARY LAYER PARAMETERS FOR WATER INJECTED AT $V_1 = .0383$ FT/SEC

C1 = 0.0 WPPM   Q1 = 20 cc/min   Qw = 8.00 gal/min   S = .02 in.

X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.575	75761	.080	.022	.009	14693	19395	5343	2217	2.410	1.659
6.00	2.651	124795	.099	.027	.012	14693	24759	6783	2872	2.362	1.632
8.00	2.691	168904	.099	.030	.013	14693	25082	7670	3332	2.302	1.634
10.00	2.731	214268	.160	.027	.019	14693	41088	6931	4893	1.416	1.590
14.00	2.795	307005	.179	.026	.018	14693	47051	6850	4701	1.457	1.808
15.00	2.787	327992	.220	.029	.020	14693	57674	7547	5295	1.425	1.813
16.00	2.822	354252	.200	.033	.021	14693	53085	8749	5450	1.605	1.816
17.00	2.825	376793	.158	.027	.017	14693	42023	7102	4494	1.580	1.764
18.00	2.847	402064	.200	.033	.019	14693	53662	8766	5183	1.691	1.784

TABLE 21

BOUNDARY LAYER PARAMETERS FOR WATER INJECTED AT V_1 = .0766 FT/SEC

C_1 = 0.0 WPPM			Q_1 = 40 cc/min			Q_w = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	Re_{δ_1}	Re_{δ_2}	H_{12}	H_{32}
3.75	2.563	73373	.080	.023	.010	14297	18807	5480	2322	2.360	1.658
6.00	2.641	120970	.140	.034	.014	14297	33872	7657	3434	2.229	1.652
8.00	2.681	163736	.170	.036	.017	14297	41704	8861	4234	2.093	1.671
10.00	2.722	207800	.119	.034	.015	14297	29774	8431	3789	2.225	1.630
14.00	2.802	299470	.159	.037	.018	14297	40711	9471	4654	2.035	1.662
15.00	2.822	323151	.180	.037	.021	14297	46430	9640	5338	1.806	1.724
16.00	2.825	345061	.179	.036	.022	14297	46428	9311	5763	1.616	1.719
17.00	2.804	363902	.230	.037	.023	14297	59081	9600	5825	1.648	1.777
18.00	2.842	390530	.220	.035	.023	14297	57278	9073	6077	1.493	1.774

TABLE 22

BOUNDARY LAYER PARAMETERS FOR WATER INJECTED AT V_1 = .1532 FT/SEC

C_1 = 0.0 WPPM			Q_1 = 80 cc/min			Q_w = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.574	70643	.090	.026	.012	13706	20390	5787	2623	2.206	1.607
6.00	2.638	115840	.110	.030	.013	13706	25485	6913	3038	2.276	1.636
8.00	2.677	156736	.120	.031	.014	13706	28213	7232	3332	2.171	1.652
10.00	2.704	197896	.140	.036	.016	12706	33223	8448	3866	2.185	1.638
14.00	2.786	235456	.180	.037	.020	13706	44042	9104	5010	1.817	1.718
15.00	2.796	306944	.230	.039	.023	14706	56576	9568	5674	1.686	1.751
16.00	2.813	329398	.230	.040	.025	13706	56772	9822	6088	1.613	1.779
17.00	2.817	350483	.263	.042	.027	13706	65116	10347	6640	1.558	1.796
18.00	2.810	370177	.266	.041	.026	13706	65595	10159	6457	1.573	1.794

TABLE 23

BOUNDARY LAYER PARAMETERS FOR WATER INJECTED AT V_1 = .3074 FT/SEC

| C_1 = 0.0 WPPM | | Q_1 = 40 cc/min | | | | Q_w = 8.00 gal/min | | | | S = .005 in. | |
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.570	70533	.120	.028	.012	13706	27085	6331	2611	2.425	1.612
6.00	2.640	115927	.120	.032	.014	13706	27776	7476	3271	2.286	1.634
8.00	2.677	156736	.120	.035	.016	13706	28165	8217	3654	2.249	1.627
10.00	2.703	197823	.120	.032	.015	13706	28534	7628	3574	2.135	1.658
14.00	2.784	285252	.160	.033	.017	13706	39071	8052	4158	1.937	1.703
15.00	2.801	307493	.179	.034	.018	13706	44131	8312	4496	1.849	1.725
16.00	2.825	330803	.200	.035	.020	13706	49620	8624	5019	1.718	1.764
17.00	2.836	352847	.220	.036	.022	13706	54745	8903	5393	1.651	1.777
18.00	2.839	373997	.220	.035	.022	13706	54853	8805	5460	1.613	1.800

TABLE 24

BOUNDARY LAYER PARAMETERS FOR WATER INJECTED AT V_1 = .6148 FT/SEC

C_1 = 0.0 WPPM			Q_1 = 80 cc/min			Q_w = 8.00 gal/min				S = .005 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.578	70753	.100	.030	.013	13706	22573	6819	2860	2.384	1.614
6.00	2.634	115664	.120	.034	.015	13706	27759	7752	3392	2.286	1.018
8.00	2.671	156385	.120	.035	.016	13706	28149	8134	3679	2.211	1.628
10.00	2.705	197969	.120	.036	.016	13706	28508	8565	3888	2.203	1.629
14.00	2.789	285764	.170	.038	.020	13706	41591	9224	4817	1.915	1.690
15.00	2.804	307822	.180	.035	.019	13706	44277	8644	4798	1.802	1.728
16.00	2.830	331388	.200	.037	.022	13706	49708	9152	5389	1.698	1.762
17.00	2.833	352473	.220	.037	.023	13706	54687	9214	5623	1.638	1.778
18.00	2.890	380716	.220	.038	.024	13706	55838	9729	6028	1.614	1.786

TABLE 25

BOUNDARY LAYER PARAMETERS FOR 100 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

C_1 = 100 WPPM			Q_1 = 20 cc/min			Q_w = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	Re_{δ_1}	Re_{δ_2}	H_{12}	H_{32}
3.75	2.581	71342	.080	.027	.011	13804	18218	60676	2549	2.384	1.636
6.00	2.658	117553	.100	.032	.014	13804	23464	7479	3216	2.325	1.624
8.00	2.693	158801	.118	.032	.015	13804	28155	7728	3518	2.197	1.643
10.00	2.737	201745	.140	.035	.017	13804	33845	8403	4030	2.085	1.659
14.00	2.802	289151	.190	.039	.021	13804	47065	9624	5092	1.890	1.700
15.00	2.813	311020	.180	.038	.021	13804	44812	9496	5246	1.810	1.720
16.00	2.805	330312	.205	.035	.021	13804	50862	8808	5289	1.665	1.777
17.00	2.817	352991	.220	.040	.025	13804	54768	10020	6147	1.630	1.767
18.00	2.828	375215	.263	.040	.025	13804	65738	9994	6352	1.573	1.794

TABLE 26.

BOUNDARY LAYER PARAMETERS FOR 100 WPPM POLYMER INJECTED AT V_1 = .0766 FT/SEC

C1 = 100 WPPM			Q1 = 40 cc/min			Qw = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	Re_{61}	Re_{62}	H_{12}	H_{32}
3.75	2.426	74280	.070	.023	.009	15291	16639	5490	2188	2.509	1.596
6.00	----	----	----	----	----	----	----	----	----	----	----
8.00	----	----	----	----	----	----	----	----	----	----	----
10.00	----	----	----	----	----	----	----	----	----	----	----
14.00	2.669	305087	.169	.024	.024	15291	44299	6252	4101	1.524	1.771
15.00	2.668	326756	.170	.025	.014	15291	44439	6516	3575	1.823	1.801
16.00	2.676	349585	.170	.022	.015	15291	63043	8090	5851	1.441	1.814
17.00	2.676	371434	.180	.023	.016	15291	47194	5962	4308	1.384	1.816
18.00	2.681	394018	.240	.031	.022	15291	44572	5716	3967	1.383	1.813

TABLE 27

BOUNDARY LAYER PARAMETERS FOR 200 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

C1 = 200.0 WPPM   Q1 = 20 cc/min   Qw = 8.00 gal/min   S = .02 in.

X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	Re_{δ_1}	Re_{δ_2}	H_{12}	H_{32}
3.75	2.570	69526	.080	.021	.009	13510	17843	4601	1915	2.403	1.676
6.00	2.631	113883	.100	.025	.011	13510	22731	5665	2328	2.334	1.656
8.00	2.664	153748	.100	.028	.012	13510	22970	6349	2820	2.251	1.652
10.00	2.705	195143	.100	.029	.013	13510	23417	6688	3048	2.195	1.661
14.00	2.792	281987	.140	.031	.016	13510	33838	7468	3763	1.984	1.702
15.00	----	----	----	----	----	----	----	----	----	----	----
16.00	2.818	345601	.160	.034	.018	13510	39033	8203	4457	1.840	1.730
17.00	2.823	346215	.200	.035	.021	13510	48877	8442	5029	1.679	1.776
18.00	2.817	365801	.200	.032	.019	13510	48822	7682	4644	1.654	1.805

TABLE 28

BOUNDARY LAYER PARAMETERS FOR 200 WPPM POLYMER INJECTED AT V_1 = .0766 FT/SEC

C1 = 200 WPPM   Q1 = 40 cc/min   Qw = 8.00 gal/min   S = .02 in.

X	UFS	Re_λ	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.581	69824	.080	.027	.010	13510	17875	5926	2313	2.562	1.589
6.00	2.642	114359	.100	.029	.012	13510	22872	6534	2731	2.392	1.623
8.00	2.677	154498	.100	.029	.013	13510	23128	6792	2942	2.308	1.634
10.00	2.716	195937	.120	.031	.014	13510	28215	7260	3268	2.221	1.644
14.00	2.807	283502	.180	.034	.018	13510	43740	8321	4436	1.876	1.715
15.00	2.815	304618	.160	.031	.017	13510	38991	7614	4228	1.801	1.750
16.00	2.810	324348	.180	.033	.019	13510	43787	8102	4713	1.719	1.772
17.00	2.819	345724	.200	.036	.022	13510	48833	8783	5354	1.640	1.789
18.00	2.834	368009	.220	.037	.022	13510	53975	8967	5510	1.628	1.788

TABLE 29

BOUNDARY LAYER PARAMETERS FOR 200 WPPM POLYMER INJECTED AT V_1 = .1532 FT/SEC

C1 = 200 WPPM			Q1 = 80 cc/min			Qw = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.42	73620	.080	.024	.009	15691	19497	5934	2118	2.801	1.593
6.00	----	----	----	----	----	----	----	----	----	----	----
8.00	----	----	----	----	----	----	----	----	----	----	----
10.00	----	----	----	----	----	----	----	----	----	----	----
14.00	2.657	311656	.140	.032	.016	15691	37452	8564	4163	2.057	1.705
15.00	2.671	335677	.170	.029	.018	15691	45652	7869	4936	1.594	1.230
16.00	2.690	360602	.180	.029	.020	15691	48735	7802	5364	1.455	1.764
17.00	2.698	284279	.240	.030	.021	15691	65101	8133	5829	1.395	1.793
18.00	2.716	409598	.238	.032	.023	15691	64935	8653	6306	1.372	1.797

TABLE 30

BOUNDARY LAYER PARAMETERS FOR 400 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

C_1 = 400 WPPM			Q_1 = 20 cc/min			Q_w = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.451	68231	.080	.029	.011	13902	17467	6286	2364	2.659	1.582
6.00	----	----	----	----	----	----	----	----	----	----	----
8.00	----	----	----	----	----	----	----	----	----	----	----
10.00	----	----	----	----	----	----	----	----	----	----	----
14.00	2.684	278945	.170	.035	.019	13902	40551	8445	4631	1.823	1.663
15.00	2.702	300874	.170	.038	.021	13902	40919	9058	5057	1.791	1.658
16.00	2.715	322476	.180	.038	.022	13902	43438	9138	5439	1.680	1.682
17.00	2.720	343262	.200	.038	.022	13902	48485	9298	5434	1.711	1.746
18.00	2.734	365325	.210	.037	.024	13902	51145	9004	5956	1.512	1.758

TABLE 31

BOUNDARY LAYER PARAMETER FOR 400 WPPM POLYMER INJECTED AT V_1 = .766 FT/SEC

C_1 = 400 WPPM Q_1 = 40 cc/min Q_w = 8.00 gal/min S = .02 in.

X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.484	64302	.080	.022	.010	12928	16461	4506	2051	2.197	1.624
6.00	----	---	---	---	---	----	---	---	---	---	---
8.00	----	---	---	---	---	----	---	---	---	---	---
10.00	---	---	---	---	---	----	---	---	---	---	---
14.00	2.714	262289	.200	.036	.019	12928	44919	8013	4362	1.837	1.648
15.00	2.731	282784	.160	.037	.020	12928	36151	8315	4455	1.866	1.672
16.00	2.729	304116	.160	.031	.019	12928	36125	7029	4199	1.674	1.754
17.00	2.732	320606	.200	.034	.021	12928	45353	7651	4799	1.594	1.747
18.00	2.746	341205	.200	.033	.022	12928	45494	7563	4926	1.535	1.767

TABLE 32

BOUNDARY LAYER PARAMETERS FOR 400 WPPM POLYMER INJECTED AT V_1 = .1532 FT/SEC

C_1 = 400 WPPM			Q_1 = 80 cc/min				Q_w = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	Re_{δ_1}	Re_{δ_2}	H_{12}	H_{32}
3.75	2.468	64844	.100	.030	.011	13121	20708	6172	2312	2.670	1.582
6.00	2.538	106693	.100	.031	.014	13121	21339	6657	2910	2.288	1.629
8.00	2.578	144499	.140	.028	.015	13121	30345	6052	3168	1.910	1.738
10.00	2.610	182866	.180	.029	.018	13121	39499	6264	3915	1.600	1.750
14.00	2.673	262191	.200	.031	.021	13121	44947	6991	4718	1.482	1.789
15.00	2.680	281655	.220	.032	.022	13121	49616	7290	4900	1.488	1.776
16.00	2.675	299871	.219	.032	.021	13121	49164	7236	4832	1.498	1.780
17.00	2.666	317541	.218	.033	.022	13121	48864	7309	4918	1.486	1.786
18.00	2.670	336725	.220	.034	.022	13121	49499	7522	4936	1.524	1.780

TABLE 33

BOUNDARY LAYER PARAMETERS FOR 400 WPPM POLYMER INJECTED AT V_1 = .6148 FT/SEC

C1 = 400 WPPM   Q1 = 80 cc/min   Qw = 8.00 gal/min   S = .005 in.

X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	Re_{δ_1}	Re_{δ_2}	H_{12}	H_{32}
3.75	2.603	71439	.199	.027	.009	13706	45584	6157	2078	2.963	1.463
6.00	2.660	116806	.120	.029	.014	13706	27987	6807	3325	2.047	1.691
8.00	2.695	157790	.170	.028	.015	13706	40284	6584	3647	1.805	1.778
10.00	2.730	199799	.200	.030	.018	13706	47856	7304	4352	1.678	1.799
14.00	2.799	286788	.263	.036	.023	13706	64749	8875	5531	1.605	1.806
15.00	2.790	306285	.230	.034	.021	13706	56257	8243	5175	1.593	1.820
16.00	2.779	325416	.230	.031	.019	13706	56134	7463	4695	1.589	1.835
17.00	2.775	345257	.230	.031	.020	13706	56060	7630	4771	1.599	1.832
18.00	2.798	368596	.240	.032	.020	13706	58975	7886	4891	1.612	1.832

TABLE 34

BOUNDARY LAYER PARAMETERS FOR 500 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

C1 = 500 WPPM		Q1 = 20 cc/min			Qw = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.485	61468	.080	.023	.010	12353	15736	4600	1982	2.321	1.671
6.00	2.561	101357	.100	.025	.012	12353	20231	5091	2338	2.177	1.690
8.00	2.608	137623	.119	.029	.013	12353	24607	5911	2782	2.124	1.679
10.00	2.713	178955	.119	.031	.016	12353	25398	6589	3439	1.916	1.718
14.00	2.724	251553	.140	.038	.019	12353	30186	8285	3996	2.073	1.656
15.00	2.744	271500	.160	.039	.019	12353	34708	8448	4212	2.006	1.658
16.00	2.765	291816	.160	.041	.020	12353	35062	8925	4308	2.071	1.650
17.00	2.780	311736	.160	.037	.020	12353	35252	8143	4417	1.843	1.711
18.00	2.790	331261	.180	.033	.019	12353	39707	7261	4089	1.775	1.756

TABLE 35

BOUNDARY LAYER PARAMETERS FOR 500 WPPM POLYMER INJECTED AT V_1 = .0766 FT/SEC

$C1$ = 500 WPPM			$Q1$ = 40 cc/min			Qw = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.474	64043	.100	.026	.011	12928	20535	5273	2194	2.404	1.636
6.00	2.550	105617	.120	.033	.014	12928	25306	6900	2888	2.389	1.613
8.00	2.596	143363	.120	.033	.015	12928	25762	7021	3197	2.196	1.644
10.00	2.633	181758	.040	.032	.015	12928	30535	6913	3379	2.046	1.681
14.00	2.717	262579	.160	.038	.019	12928	35921	8504	4309	1.973	1.680
15.00	2.736	283302	.180	.040	.021	12928	40796	8982	4682	1.918	1.692
16.00	2.759	304729	.200	.040	.022	12928	45709	9161	5077	1.804	1.680
17.00	2.769	324948	.200	.042	.023	12928	45875	9579	5182	1.849	1.704
18.00	2.786	346175	.199	.042	.023	12928	45972	9627	5314	1.812	1.713

TABLE 36

BOUNDARY LAYER PARAMETERS FOR 500 WPPM POLYMER INJECTED AT V_1 = .1532 FT/SEC

C_1 = 500 WPPM			Q_1 = 80 cc/min			Q_w = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	Re_{61}	Re_{62}	H_{12}	H_{32}
3.75	----	----	----	----	----	----	----	----	----	----	----
6.00	2.611	129581	.120	.029	.024	15491	31048	7567	3570	2.119	1.673
8.00	2.633	174231	.169	.169	.028	15491	44272	7415	4301	1.724	1.725
10.00	2.674	221180	.200	.200	.027	15491	53030	7226	4819	1.500	1.795
14.00	2.752	318684	.220	.220	.031	15491	60095	8353	5806	1.439	1.800
15.00	2.751	341324	.220	.220	.031	15491	60128	8508	5886	1.446	1.799
16.00	2.753	364343	.220	.032	.022	15491	60117	8744	5954	1.469	1.794
17.00	2.736	384724	.240	.051	.015	15491	65122	13864	4191	3.308	1.885
18.00	2.760	410928	.219	.034	.021	15491	60105	9276	5793	1.601	1.814

TABLE 37

BOUNDARY LAYER PARAMETERS FOR 500 WPPM POLYMER INJECTED AT V_1 = .6148 FT/SEC

C_1 = 500 WPPM			Q_1 = 80 cc/min			Q_w = 8.00 gal min				S = .005 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.596	71247	.100	.030	.012	13706	22708	6789	2804	2.421	1.572
6.00	2.648	116279	.120	.027	.014	13706	27860	6336	3286	1.928	1.730
8.00	2.689	157439	.180	.028	.016	13706	42485	6724	3798	1.770	1.785
10.00	2.719	198994	.201	.030	.018	13706	48045	7142	4203	1.699	1.805
14.00	2.794	286276	.230	.033	.021	13706	56388	8042	5072	1.586	1.821
15.00	2.809	308371	.263	.035	.022	13706	64783	8669	5473	1.584	1.814
16.00	2.802	328110	.230	.033	.021	13706	56550	8077	5091	1.587	1.828
17.00	2.785	346501	.230	.033	.020	13706	56256	7988	4969	1.608	1.823
18.00	----	----	----	----	----	----	----	----	----	----	----

TABLE 38

BOUNDARY LAYER PARAMETERS FOR 800 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

C_1 = 800 WPPM Q_1 = 20 cc/min Q_w = 8.00 gal/min   S = .02 in.

X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.461	63707	.080	.025	.010	12928	16309	5063	2118	2.391	1.641
6.00	2.534	104954	.100	.030	.013	12928	20929	6307	2731	2.309	1.636
8.00	2.581	142535	.120	.032	.015	12928	25613	6836	3140	2.177	1.649
10.00	2.616	180584	.140	.032	.016	12928	30295	7040	3377	2.085	1.670
14.00	2.696	260550	.160	.036	.019	12928	35733	8501	4298	1.978	1.679
15.00	2.715	281128	.160	.040	.020	12928	35984	8928	4485	1.991	1.674
16.00	2.734	301968	.200	.040	.021	12928	45250	8978	4714	1.905	1.694
17.00	2.740	321545	.180	.038	.021	12928	40855	8560	4864	1.760	1.733
18.00	2.748	341454	.220	.037	.022	12928	50080	8444	4961	1.702	1.759

TABLE 39

BOUNDARY LAYER PARAMETERS FOR 800 WPPM POLYMER INJECTED AT V_1 = .0766 FT/SEC

C_1 = 800 WPPM			Q_1 = 80 cc/min			Q_w = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_{11}	Re_δ	$Re_{\delta 1}$	Re_{62}	H_{12}	H_{32}
3.75	2.467	61023	.100	.031	.012	12352	19566	5974	2343	2.550	1.597
6.00	2.539	100486	.120	.030	.014	12353	24117	6109	29090	2.100	1.670
8.00	2.582	136251	.140	.030	.016	12353	28613	6102	3202	1.906	1.728
10.00	2.611	172227	.160	.031	.018	12353	33068	6493	3658	1.775	1.760
14.00	2.687	248136	.230	.036	.022	12353	48833	7676	4609	1.665	1.778
15.00	2.706	267740	.220	.038	.023	12353	47122	8096	4922	1.645	1.778
16.00	2.721	287172	.220	.039	.023	12353	47383	8336	5059	1.648	1.779
17.00	2.729	306017	.240	.039	.024	12353	51843	8515	5264	1.618	1.784
18.00	2.747	326156	.262	.039	.024	12353	57055	8509	5187	1.640	1.780

TABLE 40

BOUNDARY LAYER PARAMETERS FOR 800 WPPM POLYMER INJECTED AT V_1 = .1532 FT/SEC

C1 = 800 WPPM		Q1 = 40 cc/min			Qw = 8.00 gal/min			S = .02 in.			
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.544	75861	.100	.025	.013	14892	24276	61754	3245	1.903	1.665
6.00	----	----	----	----	----	----	----	----	----	----	----
8.00	2.654	168836	.200	.032	.020	14892	50701	8105	5143	1.576	1.753
10.00	2.677	212873	.220	.035	.023	14892	56147	8908	5965	1.493	1.774
14.00	2.697	300249	.240	.037	.026	14892	61766	1.455	1.782	1.455	1.782
15.00	2.698	321815	.240	.035	.024	14892	61788	1.436	1.792	1.436	1.792
16.00	2.686	341742	.240	.034	.022	14892	51514	1.589	1.819	1.589	1.819
17.00	2.724	368238	.240	.035	.023	14892	62332	1.569	1.817	1.569	1.817
18.00	2.726	390186	.022	.034	.021	14892	57279	1.638	1.803	1.638	1.803

TABLE 41

BOUNDARY LAYER PARAMETERS FOR 800 WPPM POLYMER INJECTED AT V_1 = .6148 FT/SEC

C1 = 800 WPPM		Q1 = 40 cc/min			Qw = 8.00 gal/min				S = .02 in.	
X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.606	71521	.100	.033	.014	13706	22887	7664	3158	2.427	1.593
6.00	2.668	117157	.170	.032	.017	13706	39833	7554	3958	1.909	1.722
8.00	2.704	158317	.200	.034	.019	13706	47471	8159	4503	1.812	1.743
10.00	2.733	200019	.200	.034	.020	13706	47908	8338	4747	1.757	1.759
14.00	2.823	289248	.262	.037	.023	13706	64957	9198	5672	1.622	1.792
15.00	2.823	309908	.220	.037	.023	13706	54494	9171	5621	1.631	1.791
16.00	2.844	333028	.230	.039	.024	13706	57397	9709	6011	1.615	1.784
17.00	2.820	351354	.230	.036	.022	13706	57093	8929	5550	1.609	1.800
18.00	2.822	371758	.236	.036	.022	13706	58589	8986	5506	1.632	1.795

TABLE 42

BOUNDARY LAYER PARAMETERS FOR 2000 WPPM POLYMER INJECTED AT V_1 = .6148 FT/SEC

C_1 = 2000 WPPM Q_1 = 80 cc/min Q_w = 8.00 gal/min   S = .02 in.

X	UFS	Re_x	δ	δ_1	δ_2	Re_H	Re_δ	$Re_{\delta 1}$	$Re_{\delta 2}$	H_{12}	H_{32}
3.75	2.598	71302	.160	.032	.017	13706	36552	7257	3880	1.870	1.734
6.00	2.653	116498	.220	.039	.021	13706	51259	9124	4952	1.842	1.728
8.00	2.696	157849	.240	.043	.023	13706	56778	10085	5545	1.819	1.725
10.00	2.719	198994	.262	.044	.025	13706	62611	10438	6006	1.738	1.709
14.00	2.790	285866	.263	.047	.026	13706	64442	11575	6392	1.811	1.715
15.00	2.808	308261	2.40	.045	.026	13706	59186	11089	6322	1.754	1.730
16.00	2.820	330217	.239	.045	.026	13706	59291	11097	6317	1.757	1.727
17.00	2.813	349985	.240	.045	.026	13706	59390	11193	6354	1.763	1.726
18.00	2.819	371363	.240	.042	.025	13706	59319	10709	6182	1.732	1.739

APPENDIX E

VELOCITY PROFILE SAMPLES

TABLE 43

VELOCITY PROFILES FOR WATER INJECTED AT V_1 = .0383 FT/SEC

X inches Y inches	3.75	6.00	8.00	10.00	14.00	15.00	16.00	17.00	18.00
				VELOCITY (FT/SEC)					
.010	.880	.728	.763	.973	1.785	1.750	1.787	1.495	1.465
.020	1.410	1.217	1.172	----	1.996	1.998	1.970	1.830	1.807
.040	2.118	1.954	1.856	----	2.204	2.193	2.175	----	2.122
.060	2.510	2.426	2.348	2.402	2.351	2.311	2.304	2.341	2.291
.080	2.570	2.608	2.590	----	2.449	2.412	2.382	2.521	2.441
.100	2.580^{-10}	2.648	2.667	2.560	2.520	2.470	2.469	2.638	2.561
.120	2.581^{-10}	2.642	2.685	2.622	2.639	2.552	2.556	2.752	2.655
.140	2.577	2.652	2.686	2.683	2.701	2.627	2.649	2.758	2.737
.160	2.574^{+10}	2.650	2.685	2.708	2.744	2.682	2.682	2.693	2.764
.180	----	2.650	2.687	2.719	2.769	2.728	2.749	2.817	2.810
.200	2.569	2.649	2.689	2.729	2.791	2.757	2.799	2.825	2.833
.220	2.570^{+10}	2.650	----	2.731	2.793	2.782	2.822	2.825	----

NOTE: +10 - add .010 inches to y value

TABLE 44

VELOCITY PROFILES FOR 100 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

X_{inch} Y_{inches}	3.75	6.00	8.00	10.00	14.00	15.00	16.00	17.00	18.00
				VELOCITY (FT/SEC)					
.010	.741	.707	.793	.882	.965	1.128	1.414	1.308	1.127
.020	1.197	1.110	1.180	1.280	1.332	1.477	1.762	1.739	1.1795
.040	1.948	1.748	1.778	1.778	1.799	1.853	2.062	1.985	2.095
.060	2.412	2.229	2.237	2.172	2.119	2.134	2.179	2.135	2.229
.080	2.558	2.558	2.509	2.455	2.349	2.342	2.351	2.234	2.293
.100	2.571	2.635	2.631	2.623	2.559	2.511	2.497	2.359	2.385
.120	----	2.647	2.676	2.685	2.666	2.628	2.545	2.474	2.476
.140	2.571	2.653	2.691	2.715	2.726	2.714	2.644	2.590	2.553
.160	2.570	1.651+10	2.687+10	2.730	2.725	2.766	2.716	2.667	2.651
.180	----	----	----	2.737	2.777+10	2.793	2.751	2.732	2.704
.200	2.574	----	2.693	2.737	----	2.794	2.804	2.776	2.743
.220	2.572	----	----	----	2.802+10	2.813+10	2.805+10	2.817	2.775
.240	----	----	----	----	----	----	2.805	2.817	2.789

NOTE: +10 – add .010 inches to y value; +15 – add .015 inches to y value

TABLE 45

VELOCITY PROFILES FOR 200 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

X inches Y inches	3.75	6.00	8.00	10.00	14.00	15.00	16.00	17.00	18.00
			VELOCITY (FT/SEC)						
.010	.998	.875	$.832^{-2}$	.943	1.092	----	1.209	1.360	1.565
.020	1.469	1.345	1.284	1.318	1.437	1.435	1.543	1.762	1.913
.040	2.203	2.046	1.942	1.927	1.956	1.922	1.958	2.038	2.157
.060	2.514	2.460	2.385	2.379	2.304	2.290	2.244	2.232	2.277
.080	2.565	2.603	2.590	2.595	2.570	2.544	2.462	2.384	2.418
.100	2.567	2.630	2.651	2.683	2.693	2.682	2.606	2.515	2.540
.120	2.569	2.631	2.662	2.696	2.754	2.756	2.697	2.606	2.604
.140	2.570	2.624	2.664	2.702	2.772	2.787	2.751	2.681	2.679
.160	2.570	2.631^{+10}	2.662	2.705	2.775^{+10}	2.795	2.794	2.742	2.733
.180	2.570	----	2.660	2.704	----	2.804	2.795	2.779	2.788
.200	2.570	2.630	2.664	2.704	2.777	2.807	2.816	2.809	2.803
.220	2.570	2.630	2.663	2.700	2.87^{+10}	2.808	2.818	2.817	2.815

NOTE: +10 - Add .010 inches to y value; -2 - subtract .002 inches from y value

TABLE 46

VELOCITY PROFILES FOR 400 WPPM POLYMER INJECTED AT $V_1 = .0383$ FT/SEC

X_{Inches} Y_{Inches}	3.75	6.00	8.00	10.00	14.00	15.00	16.00	17.00	18.00
				VELOCITY (FT/SEC)					
.010	$.645^{+5}$	----	----	----	.949	1.004	1.138	1.458^{+5}	1.345
.020	.916	----	----	----	1.046^{-5}	1.285	1.369	1.559	1.676
.040	1.801	----	----	----	1.679	1.674	1.723	1.873	1.958
.060	2.285	----	----	----	2.063	2.034	2.019	2.063	2.114
.080	2.429	----	----	----	2.311	2.261	2.246	2.237	2.234
.100	2.447	----	----	----	2.491	2.451	2.400	2.365	2.337
.120	2.445	----	----	----	2.594	2.566	2.531	2.481	2.433
.140	2.449	----	----	----	2.648	2.643	2.617	2.572	2.532
.160	2.449	----	----	----	2.676^{+10}	2.693^{+10}	----	2.626	2.600
.180	2.449	----	----	----	----	----	2.704	2.678	2.663
.200	2.450	----	----	----	2.684	2.701	2.715	2.711	2.700

NOTE: +15 — add .005 inches to y value; +10 — add .010 inches to y value

TABLE 47

VELOCITY PROFILES FOR 500 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

X_{inches}	3.75	6.00	8.00	10.00	14.00	15.00	16.00	17.00	18.00
Y_{inches}				VELOCITY (FT/SEC)					
.010	.851	.994	.935	1.083	.883	.787	.804	1.109	.830
.020	1.326	1.380	1.313	1.522	1.140	1.297	1.136	1.402	1.619
.040	2.018	1.993	1.879	1.883	1.658	1.606	1.540	1.924	2.097
.060	2.355	2.337	2.270	2.229	2.063	----	2.068	2.098	2.235
.080	2.465	2.497	2.477	2.443	----	2.330	----	2.299	2.391
.100	2.480	2.541	2.566	2.582	----	2.528	2.527	2.490	2.526
.120	2.482	2.554	2.596	2.694	2.644	2.652	2.647	2.639	2.643
.140	2.481	2.557	2.603	2.692	2.714	2.711	2.720	2.716	2.708
.160	2.480	2.558	2.605	2.691	2.715	2.734	2.746	2.754	2.756
.180	2.481	2.556	2.604	2.708	2.723	2.738	2.761	2.769	2.782
.200	2.483	2.560	2.605	2.709	2.723	2.740	2.760	2.775	2.786

TABLE 48

VELOCITY PROFILES FOR 800 WPPM POLYMER INJECTED AT V_1 = .0383 FT/SEC

X inches Y inches	3.75	6.00	8.00	10.00	14.00	15.00	16.00	17.00	18.00
				VELOCITY (FT/SEC)					
.010	.713	.652	.742	.912	.875	.869	.952	1.194	1.175
.020	1.200	1.109	1.163	1.250	1.240	1.181	1.288	1.522	1.633
.040	1.940	1.772	1.725	2.746	1.679	1.686	1.717	1.826	1.931
.060	2.315	2.217	2.141	2.143	2.050	2.023	2.054	2.079	2.125
.080	2.441	2.433	2.409	2.395	2.311	2.277	2.288	2.277	2.255
.100	2.459	2.511	2.526	2.531	2.494	2.487	2.463	2.431	2.401
.120	2.460	2.529	2.568	2.581	2.597	2.588	2.579	2.540	2.513
.140	2.460	2.530	2.578	2.607	2.634	2.658	2.668[+10]	2.625	2.607
.160	2.460	2.531	2.581	----	2.677	2.689	2.693	2.677	2.669
.180	2.460	2.532	2.581	2.613[+10]	2.684	2.698	2.705	2.715	2.709
.200	2.460	2.534	2.581	2.614	2.689	2.709	2.727	2.724	2.720
.240	2.460	2.534	2.581	2.616	2.694	2.715	2.733	2.740	2.748

NOTE: +10 - add .010 inches to y value

APPENDIX F

COMPUTER PROGRAM SKINFRIC

```
$SCHED,CORE=48,SCR=4,CLASS=E
$FTNU(L,X,S)
      PROGRAM SKINFRIC
      REAL K,L,LAMBDA,INTGNO1,INTGNO2,INTGRL1,INTGRL2
      DIMENSION LAMBDA(256),X(256),CWX(256),YINTG1(256),YINTG2(256)
      DIMENSION Y1(50),Y2(50),OREX(50),DIFINT(50),CF(256),DELTA(256)
      DIMENSION REX(256),REXL(256),VVO(256),UTAU(256),DELTABL(256)
      DIMENSION CFT(256)
      COMMON K,B,VOS,ALPHA,GAMMA,U,RE,CX,XCT,DELT,CXL
    1 FORMAT(8F5.0,3F10.0,E10.0)
    2 FORMAT(I10,3F10.0,6F5.0)
    3 FORMAT(1H1,9X,'CI=',F7.2,/,10X,'XCT=',F6.3,/,10X,'VOS=',F5.3,/,
     A       10X,'ALPHA=',F5.3,/,10X,'GAMMA=',F5.3,/,10X,'B=',F5.3,/,
     B       10X,'K=',F5.3,/,10X,'NU=',E12.5,/,10X,'TEMP=',F5.1,/,
     C       10X,'REXT=',E12.5,/,10X,'XTRANS=',F6.3)
    4 FORMAT(' NO ROOT IN GIVEN RANGE')
    5 FORMAT(  9X,'X',9X,'REX',8X,'REXL',8X,'CW',5X,'DELTA',8X,'CF',
     X        4X,'LAMBDA',4X,'VS/VOS',6X,'UTAU',5X,'DELTABL',7X,'CFT')
    6 FORMAT(F10.2,2F12.2,F10.2,F10.3,F10.5,F10.3,2F10.4,F12.6,F10.5)
    7 FORMAT(1H1,18X,'X',11X,'INT1-INT2',16X,'OREX')
    8 FORMAT(5X,'(',F6.2,',',F6.2,')',2E20.8)
C
C         INPUTS
C
    9 CONTINUE
      READ 1,U,VOS,L,CI,ALPHA,GAMMA,B,K,T,XCT,CXL,REXT
      IF(U.EQ.0.0)GO TO 99
      READ 2,NITER,EPS,RES,EDREX,XB,XF,XINC,STEPX,XCTL,XCTH
C
C         INITIALIZATIONS
C
      V=3.60901355E-5+T*(-7.17973611E-7+T*(7.15890612E-9+
     1   T*(-3.50477166E-11+T*6.62377978E-14)))
      NSTEPS=(XF-XB)/STEPX+1.1
      UV=U/(V*12.)
      XTRANS=REXT/UV
      UVS=U/VOS
      NPT=XINC/STEPX+1.1
      GM=GAMMA
      AL=ALPHA
      IF(NPT.EQ.2*(NPT/2))NPT=NPT+1
      DELX=(NPT-1)*STEPX
      NINT=(XF-XB)/DELX+.00001
      REXO=UV*XB
      PREDIF=0.0
      DO 15 I=1,NSTEPS
      X(I)=XB+(I-1)*STEPX
      CWX(I)=CW(CI,X(I),L)
   15 CONTINUE
   10 CONTINUE
      GAMMA=0.0
      ALPHA=1.0
      JS=1
      SIGX=XB
      INTGRL1=0.0
      INTGRL2=0.0
C
C         GENERATE A TABLE OF X,CW,LAMBDA AND INTEGRANDS
C
```

```
      DO 30 I=1,NSTEPS
      CX=CWX(I)
      REX(I)=UV*X(I)
      IF(X(I).LE.XCT)GO TO 27
      REXL(I)=UV*(X(I)-XCT)
      RE=REXL(I)
      GAMMA=GM
      ALPHA=AL
      XL=5.
      F1=F(XL)
      DO 20 J=6,400
        XH=J
        F2=F(XH)
        IF(F1*F2.LT.0.0)GO TO 25
          F1=F2
          XL=XH
20      CONTINUE
      PRINT 6
      GO TO 99
25    CONTINUE
      CALL FALPOS(XL,XH,NITER,RES,EPS,LAMBDA(I))
      GO TO 29
27    CONTINUE
      REXL(I)=REX(I)
      LAMBDA(I)=1.735525336*REX(I)**.25
29    CONTINUE
      RE=REXL(I)
      YINTG2(I)=INTGNO2(LAMBDA(I),X(I))
      YINTG1(I)=INTGND1(LAMBDA(I),X(I))
      VVO(I)=UVS/LAMBDA(I)
      CF(I)=2./(LAMBDA(I)*LAMBDA(I))
      DELTA(I)=DELT
      UTAU(I)=VVO(I)*VOS
      DELTABL(I)=V*DELT*12./UTAU(I)
30    CONTINUE
C
C         PREFORM INTEGRATION
C
      DO 60 I=1,NINT
      BIGX=BIGX+DELX
      OREX(I)=BIGX*UV-REX0
      JE=JS+NPT-1
      M=0
      DO 55 J=JS,JE
        M=M+1
        Y1(M)=YINTG1(J)
        Y2(M)=YINTG2(J)
55      CONTINUE
      JS=JE
      CALL SIMPSON(NPT,STEPX,Y1,Y2,YIN1,YIN2)
      INTGRL1=INTGRL1+YIN1
      INTGRL2=INTGRL2+YIN2
      OIFINT(I)=INTGRL1-INTGRL2
60    CONTINUE
      RELER=(OIFINT(NINT)-PREDIF)/OIFINT(NINT)
      IF(ABS(RELER).LT.EDREX)GO TO 75
      DEL=OIFINT(NINT)-OREX(NINT)
      PREDIF=OIFINT(NINT)
      IF(DEL.GT.0.0)GO TO 65
        XCTH=XCT
        XCT=.5*(XCTL+XCTH)
        GO TO 10
```

```
65    CONTINUE
      XCTL=XCT
      XCT=.5*(XCTL+XCTH)
      GO TO 10
75 CONTINUE
      PRINT 3,CI,XCT,VOS,ALPHA,GAMMA,B,K,V,T,REXT,XTRANS
      PRINT 5
      DO 79 I=1,NSTEPS
      IF(X(I).GT.XCT) GO TO 76
          CFT(I)=CF(I)
          GO TO 78
76    CONTINUE
      IF(X(I).GT.XTRANS) GO TO 77
          GM=(X(I)-XCT)/(XTRANS-XCT)
          XLL=1.735525336*REX(I)**.25
          CFL=2./(XLL*XLL)
          CFT(I)=(1.-GM)*CFL+GM*CF(I)
          GO TO 78
77    CONTINUE
      CFT(I)=CF(I)
78    CONTINUE
      PRINT 6,X(I),REX(I),REXL(I),CMX(I),DELTA(I),CF(I),LAMBDA(I),
     X         VVO(I),UTAU(I),DELTABL(I),CFT(I)
79    CONTINUE
      PRINT 7
      BIGX=X8
      DO 80 I=1,NINT
      BIGX=BIGX+DELX
      PRINT 8,X8,BIGX,DIFINT(I),DREX(I)
80    CONTINUE
      GO TO 9
99 CONTINUE
      END
      SUBROUTINE FALPOS(XL,XH,N,R,E,X3)
C     SUBROUTINE FALPOS USES THE METHOD OF FALSE POSITION TO FIND THE ROOT OF
C         A NONLINEAR EQUATION . THE USER SUPPLIES THE NONLINEAR EQUATION IN THE
C         FORM OF A FUNCTION ROUTINE   F(X)=0.0
C     XL = LOW ESTIMATE OF ROOT
C     XH = HIGH ESTIMATE OF ROOT
C     N = NUMBER OF ITERATIONS
C     R = RESIDUAL,ABS(F(ANS)).LT. R)
C     E = TOLERANCE BETWEEN SUCCESSIVE APPROXIMATIONS , ABS(X1-X2) .LT.E
C     BOTH R AND E HAVE TO BE SATISFIED FOR CONVERGENCE
C     X3 = ROOT OF THE NONLINEAR EQUATION
1 FORMAT('  SOLUTION DOES NOT CONVERGE IN',I5,'  ITERATIONS')
      X1=XL
      X2=XH
      DO 10 I=1,N
      X21=X2-X1
      X3=X1+F(X1)/(F(X1)-F(X2))*X21
      Z=F(X3)
      IF(ABS(Z).GT.R)GO TO 7
      IF(ABS(X21).LT.E)RETURN
7 IF(2*(I/2).EQ.I)GO TO 9
      X1=X3
      GO TO 10
9 X2=X3
10 CONTINUE
      PRINT 1,N
      RETURN
      END
      SUBROUTINE SIMPSON(N,H,Y1,Y2,YINT1,YINT2)
```

```
      DIMENSION Y1(1), Y2(1)
      YINT1=Y1(1)+Y1(N)+4.*Y1(N-1)
      YINT2=Y2(1)+Y2(N)+4.*Y2(N-1)
      M=N-3
      DO 10 I=2,M,2
         YINT1=YINT1+4.*Y1(I)+2.*Y1(I+1)
         YINT2=YINT2+4.*Y2(I)+2.*Y2(I+1)
   10    CONTINUE
      YINT1=H*YINT1/3.
      YINT2=H*YINT2/3.
      RETURN
      END
      FUNCTION CW(CI,X,L)
      REAL L
      XL=X/L
      K=.01*CI+.0001
      GO TO (100,200,990,400,500,990,990,800),K
  100 CONTINUE
      CW=CI*(.97187+XL*(1.4495+XL*(-6.403+3.96539*XL)))
      IF(XL.GT..85)CW=.007*CI
      GO TO 900
  200 CONTINUE
      CW=CI*(.96372+XL*(1.5801+XL*(-5.84557+3.17897*XL)))
      IF(XL.GT..87)CW=.007*CI
      GO TO 900
  400 CONTINUE
      CW=CI*(.97238+XL*(1.2537+XL*(-4.6849+2.3182*XL)))
      IF(XL.GT..88)CW=.014*CI
      GO TO 900
  500 CONTINUE
      CW=CI*(1.00124+XL*(-.177746+XL*(.535748-1.6046*XL)))
      IF(XL.GT..93)CW=.008*CI
      GO TO 900
  800 CONTINUE
      CW=CI*(.99383+XL*(1.02215+XL*(-5.9056+3.9775*XL)))
      IF(XL.GT..89)CW=.025*CI
  900 CONTINUE
      IF(CW.GT.CI.OR.X.LT..4.)CW=CI
      RETURN
  990 CONTINUE
      STOP
      END
      FUNCTION F(LAMBDA)
      COMMON K,B,VOS,ALPHA,GAMMA,U,REX,CX,XCT,DELTA,CXL
      REAL LAMBDA,K
      ULV=U/LAMBDA/VOS
      IF (CX.GT. CXL) GO TO 20
      XK=K*ALPHA*CX**GAMMA
      GO TO 30
   20 CONTINUE
      XK=K*ALPHA*CXL**GAMMA
   30 CONTINUE
      F=LAMBDA*LAMBDA-4.395604396*ALOG(.06*REX*ULV**XK)**2
      RETURN
      END
      FUNCTION INTGN01(LAMBDA,X)
      REAL INTGN01,LAMBDA,K
      COMMON K,B,VOS,ALPHA,GAMMA,U,REX,CX,XCT,DELTA,CXL
      VV=U/LAMBDA/VOS
      ALV=ALOG(VV)
      IF (CX.GT. CXL) GO TO 20
      Z=ALPHA*CX**GAMMA
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      GO TO 30
   20 CONTINUE
      Z=ALPHA*CXL**GAMMA
   30 CONTINUE
      IF(X.GT.XCT)GO TO 10
      DELTA=5.*X/SQRT(REX)
      GO TO 15
   10 CONTINUE
      DELTA=EXP(K*(LAMBDA-B))*VV**(-K*Z)
   15 CONTINUE
      ADEL=ALOG(DELTA)
      ADL1=ADEL-1.
      F1=Z*ALV*(2./K*ADL1+2.*B+Z*ALV)
      F2=2./K*ADL1*(B-1./K)+1./(K*K)*ADEL*ADEL+B*B
      INTGNO1=DELTA*(F1+F2)
      RETURN
      END
      FUNCTION INTGNO2(LAMBDA,X)
      REAL INTGNO2,LAMBDA,K
      COMMON K,B,VOS,ALPHA,GAMMA,U,REX,CX,XCT,DELTA,CXL
      VV=U/LAMBDA/VOS
      ALV=ALOG(VV)
      IF (CX.GT. CXL) GO TO 20
      Z=ALPHA*CX**GAMMA
      GO TO 30
   20 CONTINUE
      Z=ALPHA*CXL**GAMMA
   30 CONTINUE
      IF(X.GT.XCT)GO TO 10
      DELTA=5.*X/SQRT(REX)
      GO TO 15
   10 CONTINUE
      DELTA=EXP(K*(LAMBDA-B))*VV**(-K*Z)
   15 CONTINUE
      ADEL=ALOG(DELTA)
      ADL1=ADEL-1.
      F1=DELTA*LAMBDA*GAMMA*Z/CX*ALV
      F2=1.+1./(K*K)*ADL1+B-1./K
      INTGNO2=F1*F2
      RETURN
      END
          FINIS
 $X.LGO
  2.8   .08  22.5 100. 2.3   .6  7.2   .4    64.5     5.       30.          3.5E5
        25   .001       .01       .01   2.   20. 1.   .1  1.   10.
  2.8   .08  22.5 200. 2.3   .6  7.2   .4    63.0     5.       30.          3.5E5
        25   .001       .01       .01   2.   20. 1.   .1  1.   10.
  2.8   .08  22.5 400. 2.3   .6  7.2   .4    65.0     5.       30.          3.5E5
        25   .001       .01       .01   2.   20. 1.   .1  1.   10.
  2.8   .08  22.5 500. 2.3   .6  7.2   .4    57.0     5.       30.          3.5E5
        25   .001       .01       .01   2.   20. 1.   .1  1.   10.
  2.8   .08  22.5 800. 2.3   .6  7.2   .4    68.0     5.       30.          3.5E5
        25   .001       .01       .01   2.   20. 1.   .1  1.   10.


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